

SHARP – II. Mass structure in strong lenses is not necessarily dark matter substructure: a flux ratio anomaly from an edge-on disc in B1555+375

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ABSTRACT

Gravitational lens flux-ratio anomalies provide a powerful technique for measuring dark matter substructure in distant galaxies. However, before using these flux-ratio anomalies to test galaxy formation models, it is imperative to ascertain that the given anomalies are indeed due to the presence of dark matter substructure and not due to some other component of the lensing galaxy halo or to propagation effects. Here we present the case of CLASS B1555+375, which has a strong radio-wavelength flux-ratio anomaly. Our high-resolution near-infrared Keck II adaptive optics imaging and archival *Hubble Space Telescope* data reveal the lensing galaxy in this system to have a clear edge-on disc component that crosses directly over the pair of images that exhibit the flux-ratio anomaly. We find that simple models that include the disc can reproduce the cm-wavelength flux-ratio anomaly without requiring additional dark matter substructure. Although further studies are required, our results suggest the assumption that all flux-ratio anomalies are due to a population of dark matter sub-haloes may be incorrect, and analyses that do not account for the full complexity of the lens macro-model may overestimate the substructure mass fraction in massive lensing galaxies.

Key words: quasars: individual: CLASS B1555+375 – galaxies: structure.

1 INTRODUCTION

A common feature of dark-matter only simulations of structure formation is the presence of thousands of subhaloes that are associated with larger mass haloes (e.g. Springel et al. 2008). However, observations of the Local Group show many fewer satellite galaxies than are predicted by the simulations, even when taking into account the latest discoveries of new dwarf galaxies (Koposov et al. 2015; The DES Collaboration et al. 2015) and completeness corrections arising from the limited sky coverage imposed by the Galactic plane. This is the well-known *missing satellite problem* (Klypin et al. 1999; Moore et al. 1999; Strigari et al. 2007). In order to understand this discrepancy, it is imperative to build up a large sample of satellites/subhaloes in galaxies outside the Local Group. However, this process is challenging due to the faintness of the satellite galaxies. In fact, lower mass satellites may not be able to retain the gas needed for ongoing star formation (e.g. Papastergis et al. 2011), rendering them effectively dark.

It was first suggested by Mao & Schneider (1998) that the flux-ratio anomalies observed in the multiple radio-loud images of lensed quasars could be interpreted as a sign of the presence of substructure in lens galaxies. Indeed, small perturbations in the gravitational potential are sufficient to cause flux-ratio anomalies in gravitationally lensed systems (Metcalf & Madau 2001; Bradač et al. 2002; Dalal & Kochanek 2002), particularly for those systems with a close pair of merging images (Kochanek & Dalal 2004). At radio wavelengths, the lensed images are neither sensitive to micro-lensing from stars (Koopmans et al. 2003; although see Koopmans & de Bruyn 2000), nor to dust extinction. Also, any variation in the properties of the radio images that are due to propagation effects (e.g. interstellar scattering or free-free absorption) have a well understood frequency dependence that can be identified and accounted for (e.g. Biggs et al. 2003; Winn, Rusin & Kochanek 2004; Mittal, Porcas & Wucknitz 2007). This makes the flux-ratio anomalies of radio-loud lensed images a promising tool to detect and measure dark matter substructures in a large sample of distant galaxies (Dalal & Kochanek 2002). However, a recent comparison of the observational data (eight gravitational lens systems with four or more images) with the predictions from a set of cold dark matter

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simulations that are representative of the mass and redshift of the lens galaxies suggests that there is only a 1–4 per cent probability that the full observed flux-ratio distribution is produced only by substructure (Xu et al. 2015).

The *gravitational imaging technique*, which alternatively utilizes the surface brightness distribution of extended arcs and Einstein rings to detect low-mass substructures within lensing haloes, was introduced by Koopmans (2005) and Vegetti & Koopmans (2009). This method has given a measurement of the substructure mass fraction for a sample of 11 lenses that is currently in agreement with the theoretical expectations from cold matter matter simulations (Vegetti et al. 2012, 2014). The reason for the conflicting results between the flux-ratio anomaly and gravitational imaging techniques is currently not clear. While this could be the result of small sample statistics, it could also be an indication that the flux-ratio anomalies have a different origin than clumpy substructure (see Xu et al. 2015, for a discussion).

In this Letter, we explore the idea that not all flux-ratio anomalies are due to the presence of clumpy substructure, but that some originate from more complex lens galaxy mass distributions than have initially been considered. To this end, we use new infrared and radio interferometric imaging data from the Strong lensing at High Angular Resolution Programme (SHARP; McKean et al. 2007; Lagattuta, Auger & Fassnacht 2010; Lagattuta et al. 2012; Vegetti et al. 2012, Chen et al. 2016). The main goal of SHARP is to investigate galaxy structure and other astrophysical topics through the re-observation of known gravitational lens systems at high angular resolution, making use of observations with (1) adaptive optics (AO) on the W. M. Keck telescope, (2) the *Hubble Space Telescope* (*HST*), and (3) large radio and mm interferometric arrays.

Here we focus on CLASS B1555+375 (Marlow et al. 1999), which was discovered as part of the Cosmic Lens All-Sky Survey (Browne et al. 2003; Myers et al. 2003). B1555+375 has four lensed images with a maximum separation of 426 mas in a fold configuration. The two merging lensed images show a strong and persistent flux-ratio anomaly at radio wavelengths, and previous radio monitoring observations ruled out microlensing effects as the cause of the anomaly (Koopmans et al. 2003). As B1555+375 was included in the observational and theoretical analyses of both Dalal & Kochanek (2002) and Xu et al. (2015), understanding its mass distribution is of key importance to assess any systematics with the inferred level of substructure in the lensing halo with the flux-ratio anomaly method. The SHARP imaging data are introduced in Section 2 and we present a revised lens model using these data in Section 3. The results of this analysis and their implications are discussed in Section 4. Note that the redshifts of both the lens galaxy

Table 1. Summary of the B1555+375 observations.

Telescope	Camera	Band	Date	t_{exp} (s)
VLBA		1.66 GHz	2000 Mar 21	10800
<i>HST</i>	WFPC2	<i>F555W</i>	2000 Oct 09	5200
<i>HST</i>	WFPC2	<i>F814W</i>	2000 Oct 09	5200
<i>HST</i>	NICMOS/NIC2	<i>F160W</i>	2003 Nov 02	5376
Keck-II	NIRC2 AO	<i>K'</i>	2012 May 16	1800

and the lensed source are unknown for B1555+375, so we assume redshifts of $z_l = 1$ and $z_s = 1.5$, respectively, due to their red colours (Marlow et al. 1999). This assumption has no effect on the derived magnifications or flux ratios of our analysis.

2 OBSERVATIONS AND DATA REDUCTION

Our data set consists of optical and infrared imaging taken with the W. M. Keck-II Telescope and *HST*, and high-resolution radio imaging taken with the Very Long Baseline Array (VLBA), which are discussed below and are summarized in Table 1.

2.1 Keck adaptive optics and *HST* imaging

B1555+375 was observed as part of SHARP using the NIRC2 camera on the W. M. Keck-II Telescope on 2012 May 16 (PI: Fassnacht). The AO system was used, with the corrections derived from the laser guide star and a $R = 14.4$ -mag tip-tilt star located ~ 45 arcsec from the lens. The narrow camera mode was used, giving a field of view of ~ 10 arcsec on a side and a pixel scale of 10 mas. Six dithered 300 s exposures were obtained in the K' filter. The data were reduced with the standard SHARP pipeline, which is a PYTHON-based package that is a refinement of the process described by Auger et al. (2011). A cutout of the final reduced image is shown in Fig. 1 and also in Fig. 2a, where the contours from 5-GHz Multi-Element Radio Link Interferometer Network (MERLIN) imaging by Marlow et al. (1999) and 1.66-GHz VLBA imaging (see below) are overlaid.

B1555+375 was also observed with the *HST* in three broadbands. The optical data were obtained with the Wide-Field Planetary Camera 2 (WFPC2) in the *F555W* and *F814W* bands (GO-8804; PI: Falco), while the Near Infrared Multi-Object Spectrograph (NICMOS) was used to observe the system in the *F160W* band (GO-9744; PI: Kochanek). The NICMOS observations were obtained with the NIC2 camera. We reduced all of the archival *HST* data with the standard MULTIDRIZZLE pipeline, producing final drizzled images with pixel-sizes of 50 mas. The reduced images are also shown in Fig. 1. The lens and the lensed images are not detected

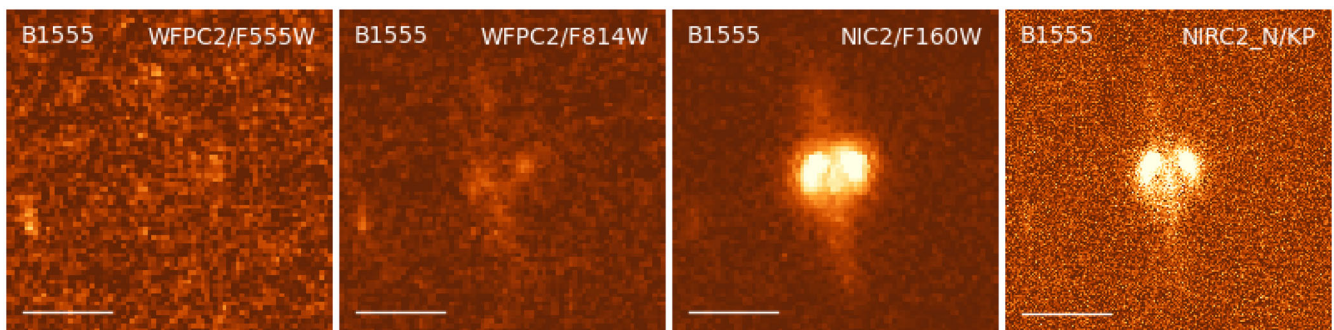


Figure 1. High-resolution multi-band imaging of B1555+375 with *HST*/WFPC2 at *F555W* (left) and *F814W* (middle-left), with *HST*/NICMOS/NIC2 at *F160W* (middle-right) and with Keck adaptive optics at K' -band (right). All of the panels are 3.7 arcsec on a side and the scale-bar represents 1 arcsec.

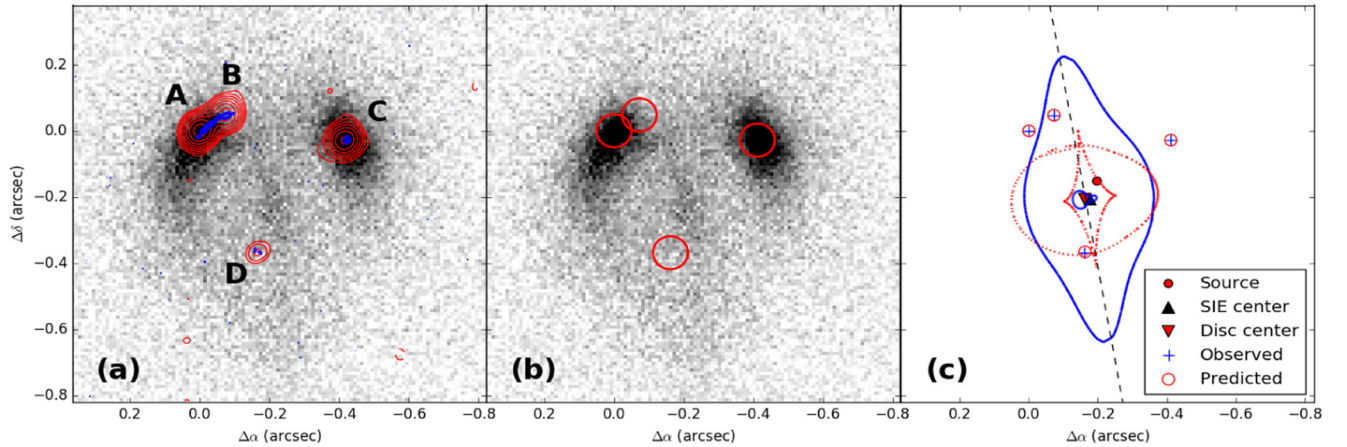


Figure 2. (a) K' -band AO imaging of B1555+375 with contours from the MERLIN observations (red) of Marlow et al. (1999) and the VLBA observations presented here (blue) overlaid. (b) K' -band AO image with the locations of the four radio components marked with circles. (c) Observed radio positions: blue pluses; model-predicted positions: red open circles; lens-plane critical curves: blue solid curves; source-plane caustics: red dotted curves. The source position is at $(-0.1953, -0.1494)$, which is marked by the red filled circle. The centroid positions of the SIE and ExpDisc components of the lens are marked by the black and red filled triangles, respectively. The disc middle plane measured with NICMOS image is shown with the black dashed line.

at high significance in the optical bands, but are clearly seen in the NICMOS $F160W$ image.

There are several notable features in the high-resolution AO and *HST* data. These include the nearly complete lack of emission associated with lensed images B and D, (Fig. 2b), and the faint, but clearly visible emission from the lensing galaxy that appears to be an edge-on disc, with a position angle of ~ 10 deg and an axis ratio $q \sim 0.2$. We note that the position angle of the lensing galaxy is such that the disc emission lies close to the lensed images B and D. Therefore, the lack of emission from these rest-frame optical images is likely due to dust extinction and additional demagnification due to lensing.

2.2 Very long baseline array imaging

B1555+375 was observed with the 10 telescopes of the VLBA at a central observing frequency of 1.66 GHz on 2000 March 21 (BN0009; PI: Norbury). The data were recorded at 128 Mbits s^{-1} and then correlated to produce two spectral windows with 8 MHz bandwidth, 16 channels and both circular polarizations (RR and LL). The observations were phase referenced using J1544+398 every 3 min over the total observing time of 3 h. The data were reduced in the standard manner using the VLBAUTLIS analysis pipeline that is part of the Astronomical Image Processing Software (AIPS). Imaging was done with the CLEAN algorithm in AIPS using natural weighting of the visibilities, and restored using an elliptical beam of size $9.7 \times 6.9 \text{ mas}$ at a position angle of -7.6° east of north. The final map (also shown in Fig. 2a) has an rms of $78 \mu\text{Jy beam}^{-1}$.

We find that lensed images A and B have been resolved into a gravitational arc that is $\sim 100 \text{ mas}$ in length. Image C is still compact on mas-scales, whereas image D is only marginally detected ($\sim 6\sigma$). The VLBA imaging shows that image B has a smaller angular size than image A, which is consistent with a perturbation in the mass model being responsible for the flux-ratio anomaly (recall that gravitational lensing conserves surface brightness), as opposed to free-free absorption or interstellar scattering.

3 LENS MODELLING

The flux-ratio anomaly and relative sizes of the images at radio-wavelengths imply that there must be some form of perturbation to

the gravitational lens mass model, which has thus far been attributed to dark matter substructure within the lensing galaxy or along the line of sight to the lensed images (Dalal & Kochanek 2002; Xu et al. 2012, 2015). However, our high-resolution near-infrared imaging suggests the intriguing possibility that the mass perturbation could be due to an edge-on disc component within the lens. To test this possibility, we model the system with a disc component in order to see if a plausible mass model can explain the flux-ratio anomaly without the need for additional substructure.

We use the lens modelling code GRAVLENS (Keeton 2001) to model the compact radio components. The inputs to the model are the observed image positions measured from the MERLIN radio observations of Marlow et al. (1999) as all of the images are significantly detected in these data. The flux-ratio measurements are taken from Koopmans et al. (2003), who used half a year of MERLIN monitoring to obtain flux-ratio curves. In total, there are 11 constraints to the mass model provided by the observational data. As a first trial of modelling, we re-create the singular isothermal ellipsoid without external shear model (SIE; 7 free parameters) of Marlow et al. (1999) to check the performance of a simple lensing potential. Consistent with previous studies (Marlow et al. 1999; Xu et al. 2015), a single elliptical potential model cannot reproduce the observed flux ratios in B1555+375 (see Table 2).

The next step is to test if a physically plausible edge-on disc can cause a perturbation in the strong lensing mass model that produces the observed flux ratio anomaly of merging images A and B. An exponential disc component is added to the smooth lens model. Previous study has shown that exponential disc profile best describes the light distribution of the disc component in spiral galaxies (Keeton & Kochanek 1998). The free parameters of this model are the SIE Einstein radius (b), centroid position, ellipticity ($e = 1 - q$), position angle (θ), the exponential disc intrinsic central density (κ_0) and scale length (R_s). A detailed definition of these mass profiles is described by Keeton (2001). The source position adds two additional model parameters.

More sophisticated mass models (i.e. external shear or additional halo term) require additional constraints, such as those from the lensed extended emission in either the NIR or radio imaging. While we have successfully used the former approach in the past (e.g. Vegetti et al. 2012), for B1555+375 the extinction in the AO imaging is

Table 2. A summary of the observational constraints and the predicted lens-model parameters. The lensed image positions in the radio (Marlow et al. 1999) (in arcsec) and average flux ratios (Koopmans et al. 2003) are with respect to image A. Uncertainties in the position measurements for images A, B and C are 0.001 arcsec and for image D are 0.006 arcsec.

Image	Observed			Model-predicted			
	East	MERLIN 5 GHz North	Flux ratio	East	SIE+ExpDisc North	Flux ratio	SIE-only Flux ratio
A	0	0	1	+0.000	+0.000	1	1
B	-0.0726	+0.0480	0.620 ± 0.039	-0.0715	+0.0487	0.606	0.971
C	-0.4117	-0.0280	0.507 ± 0.030	-0.4115	-0.0278	0.445	0.312
D	-0.1619	-0.3680	0.086 ± 0.024	-0.1693	-0.3704	0.123	0.106

Table 3. Best-fitting model parameters and uncertainties for our two-component SIE + ExpDisc lens model (our single SIE model is effectively equivalent to the model in Marlow et al. 1999 and so is not shown here.) The positions are measured in arcsec, relative to the MERLIN radio position of image A. The position angles are measured in degrees east of north. The Einstein radius b and disc scale radius (R_s) are measured in arcsec.

Parameter	SIE component	ExpDisc component
ΔRA	-0.177 ± 0.003	-0.162 (fixed)
ΔDec	-0.205 ± 0.004	-0.206 (fixed)
b	0.178 ± 0.004	...
κ_0	...	0.273 ± 0.003
e	0.25 ± 0.05	0.83 (fixed)
θ	4.9 ± 1.3	8.0 (fixed)
R_s	...	0.236 ± 0.003

so severe that components B and D are not visible, and the arc that connects A and B appears to be truncated. The low signal-to-noise ratio in the ring away from the lensed AGN and the strong extinction in the lens galaxy prevent us from using this gravitational imaging approach. Significantly deeper observations are needed at a higher bandwidth with very long baseline interferometric arrays, which can test more complex models in the future by imaging the extended gravitational arc and more significantly detecting image D.

We use GRAVLENS to obtain the best-fitting SIE+ExpDisc lens model. The best-fitting parameters from Marlow et al. (1999) are chosen as the starting point of the SIE profile. We have used the NICMOS *F160W* imaging, in which the disc component has the highest signal-to-noise ratio among our data, to fix the disc centroid position relative to image A to $(-0.162, -0.206)$, its ellipticity to $e = 0.83$, and its position angle to $\theta = 8^\circ$. The total number of free parameters is 9. After obtaining the best-fitting lens model, we run a Markov-Chain Monte Carlo (MCMC) analysis to obtain the uncertainties.

Our model predicted results are shown in Table 2 with comparison to the observed data and the single SIE lens model. The best-fitting model parameters and 68 per cent confidence level (CL) errors are listed in Table 3, and the lens model is illustrated in Fig. 2c. We find that adding an elongated exponential disc to a smooth SIE model successfully reproduces the flux ratio anomaly in this system. The positions are all in agreement, and the flux ratios are consistent at the 2σ level, with excellent agreement now obtained for the two merging images A and B. The remaining differences in the fluxes may be due to the real lensed source being slightly extended rather than the point source that we used in our model. The model requires that the disc component makes up about 15 per cent of the total projected mass within the Einstein radius. Investigations of edge-on spiral lenses, for example from the SWELLS survey, show that the disc mass fraction within the Ein-

stein radius ranges from 5 to 30 per cent (Treu et al. 2011; Brewer et al. 2012, 2014).

4 DISCUSSION AND CONCLUSIONS

Although flux-ratio anomalies provide a powerful tool to investigate low-mass substructure in lensing haloes out to cosmological redshifts, it is clear that some care is needed in interpreting the results. It is now well established that observations of radio-loud gravitational lenses may provide the most robust observational constraints because they are less affected by dust extinction and microlensing, and monitoring can determine the best estimate of the intrinsic flux ratios. This has led to an oft-studied sample of eight radio-loud gravitational lenses with well-defined flux-ratio measurements (Dalal & Kochanek 2002; Kochanek & Dalal 2004; Xu et al. 2015). In general, lens models that are based only on the locations of lensed radio-loud images have large degeneracies due to the limited number of constraints that are provided by 50–200 mas imaging of the unresolved sources (e.g. Kochanek 1991). Furthermore, observed flux-ratio anomalies may not be due exclusively to a population of low-mass ‘dark’ substructures, as already seen in the MG J0414+0534 and MG J2016+112 systems, where there is evidence for luminous companion dwarf galaxies that can account for the observed flux ratios, and even observed astrometric anomalies (Ros et al. 2000; Chen et al. 2007; More et al. 2009; see also McKean et al. 2007; Jackson et al. 2010).

Here, we find evidence for an edge-on disc in the flux-ratio anomaly gravitational lens B1555+375 that can account for the required perturbation in the mass model. Although a constraint on the disc mass from photometry is currently not available due to the low signal-to-noise ratio in the AO imaging and the lack of redshift information, our model indicates that the flux-ratio anomaly in B1555+375 has a strong link to the presence of an edge-on disc. Such high-order deviations from a smooth mass distribution have already been suggested previously (Evans & Witt 2003; Congdon & Keeton 2005), but this is the first time that an observed disc has been explicitly modelled to account for a flux-ratio anomaly. However, it is important to realize that by including observationally motivated modifications to a ‘smooth’ mass distribution, such as luminous dwarf companions or edge-on discs, in the mass models, we are still limited in our interpretation if we can not determine how much of the anomaly are due to these components. For our current analysis of B1555+375, we have shown that the entire anomaly could be caused by the edge-on disc. That is not to imply this is a unique model, but rather that the disc component is a plausible explanation of the anomaly.

In future it may be possible to independently constrain the mass of the disc through kinematic measurements (most likely from mm-wavelength resolved emission line observations), which would test

whether the disc actually makes a sufficient contribution to the mass model (e.g. as in the case of B1933+503; Suyu et al. 2012). Also, in the blue contours of Fig. 2a, B1555+375 shows evidence for an extended ~ 100 mas gravitational arc at radio wavelengths that crosses the edge-on disc. Further high resolution imaging with very long baseline interferometry will provide the observational constraints required to fully test whether the disc is sufficient to cause the anomaly.

Our analysis of B1555+375 demonstrates that the choice of the macromodel used to describe the ‘smooth’ lens galaxy mass distribution, combined with the limited information provided by the position of compact lensed images, can lead to a misinterpretation of the origin of flux-ratio anomalies in some systems. Special care may be needed for small-separation lens systems, in which the lensing galaxy is likely to be a spiral (e.g. Turner, Ostriker & Gott 1984; Fukugita et al. 1992). This is because a disc with a high inclination angle can significantly affect the lensing cross-section for such systems (Maller, Flores & Primack 1997; Keeton & Kochanek 1998; Möller, Hewett & Blain 2003). The current sample of lensed quasars with radio-wavelength flux ratio measurements is small. Therefore, having even a few systems with edge-on discs may be sufficient to introduce a bias into the substructure analysis. In fact, our near-infrared AO imaging of other lenses in the SHARP sample shows that B1555+375 is not the only system that has a significant disc component that crosses the merging lensed images (Fassnacht et al., in preparation). These gravitational lenses will be investigated in future work, but the images alone suggest that other systems may have a similar link between flux-ratio anomalies and disc components. This potential bias introduced by edge-on discs may be avoided by post-selection as sample sizes grow in the future. Therefore, it is critical to obtain high-resolution imaging lenses to detect additional mass features such as edge-on discs or even massive luminous satellites. This result is timely, as upcoming large-scale surveys (e.g. the Dark Energy Survey, Kilo-Degree Survey, *Euclid*, and the Large Synoptic Survey Telescope) will deliver orders of magnitude more gravitationally lensed quasars for which flux ratios can be measured using narrow lines (e.g. Nierenberg et al. 2014).

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