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Protoplanets with core masses below the critical mass fill in their Roche lobe

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ABSTRACT

We study the evolution of a protoplanet of a few earth masses embedded in a protoplanetary disc. If we assume that the atmosphere of the protoplanet, i.e. the volume of gas in hydrostatic equilibrium bound to the core, has a surface radius smaller than the Roche lobe radius, we show that it expands as it accretes both planetesimals and gas at a fixed rate from the nebula until it fills in the Roche lobe. The evolution occurs on a time-scale shorter than the formation or migration time-scales. Therefore, we conclude that protoplanets of a few earth masses have an atmosphere that extends to the Roche lobe surface, where it joins on to the nebula. This is true even when the Bondi radius is smaller than the Roche lobe radius. This is in contrast to the commonly used models in which the static atmosphere extends up to the Bondi radius and is surrounded by a cold accretion flow. As a result, any calculation of the tidal torque exerted by the disc on to the protoplanet should exclude the material present in the Roche lobe, since it is bound to the protoplanet.

Key words: planets and satellites: atmospheres – planets and satellites: formation – planets and satellites: general – protoplanetary discs.

1 INTRODUCTION

The recent release of *Kepler* data reporting more than 1200 planet candidates transiting their host star confirms that the core accretion scenario for forming planets is ubiquitous. Indeed, most of these objects have a radius that is less than half the radius of Jupiter, which indicates that they are at most Neptune-size (Borucki et al. 2011). Such low-mass objects are probably not the result of gravitational instabilities, but are more likely formed through the accumulation of a metal-rich core followed by the capture of a more or less massive envelope of gas (Lissauer 1993, and references therein).

In this model, the core is believed to be formed through the solid-body accretion of kilometre-sized planetesimals. For typical disc parameters, when the core reaches about $0.1 M_{\oplus}$, it starts binding the gas of the nebula in which it is embedded and a gaseous atmosphere forms around it. As long as the core is less massive than the so-called *critical mass*, the energy radiated from the atmosphere is compensated for by the gravitational energy that the planetesimals entering the atmosphere release when they collide with the surface of the core. During this stage of evolution, the atmosphere is therefore in quasi-static and thermal equilibrium and grows slowly in mass along with the core (Perri & Cameron 1974; Mizuno 1980).

By the time the core reaches the critical mass, the atmosphere has become too massive to be supported at equilibrium by the energy released by the planetesimals. At that point, it starts contracting and the subsequent runaway accretion of gas leads to the formation of giant planets (Bodenheimer & Pollack 1986; Pollack et al. 1996).

At the same time as they form, protoplanets migrate through the nebula as a result of tidal interaction with the surrounding gas (Goldreich & Tremaine 1979, 1980; Lin & Papaloizou 1979, 1993, and references therein; Papaloizou & Lin 1984; Ward 1986, 1997). Calculations performed in isothermal discs show that cores of several earth mass migrate inwards on a relatively short time-scale, shorter than the planet formation time-scale (Ward 1997; Tanaka, Takeuchi & Ward 2002; Bate et al. 2003). However, more recent calculations including detailed energy balance in non-isothermal discs show that migration can be slower or even outwards (Paardekooper & Mellema 2006, 2008; Baruteau & Masset 2008; Kley & Crida 2008; Paardekooper & Papaloizou 2008). Usually, the torque exerted by the disc on the protoplanet is calculated by excluding the gas comprised in its Roche lobe. However, it has been suggested by D'Angelo, Kley & Henning (2003; see also Crida et al. 2009) that the gas present in the Roche lobe but not bound to the protoplanet may contribute significantly to the total torque and slow down migration. This happens when the atmosphere of the protoplanet, i.e. the volume of gas at hydrostatic equilibrium that is bound to the core, does not fill in its Roche lobe, and is surrounded by a cold (Bondi type) accretion flow.

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Models of atmospheres of protoplanets that do not fill in the Roche lobe have been considered by different authors. Papaloizou & Nelson (2005) have constructed models assuming that the only source of energy for the atmosphere is that produced by the gravitational contraction of the gas. This corresponds to the runaway gas accretion phase, i.e. after the core has reached the critical mass, as the luminosity due to the accretion of planetesimals is then negligible. The radius of the protoplanet atmosphere may become smaller than the Roche lobe radius in this phase if the protostellar disc cannot supply gas to the atmosphere rapidly enough. Such models have also been considered by Mordasini et al. (2011). Lissauer et al. (2009; see also Movshovitz et al. 2010) have calculated models of atmospheres prior to the runaway gas accretion phase by assuming that the surface radius is either the Bondi radius or a fraction of the Roche lobe radius depending on whether the Bondi radius is smaller or larger than the Roche lobe radius. This is based on the results of three-dimensional numerical simulations indicating that only gas within about one quarter of the Roche lobe remains bound to the protoplanet. The gas accretion rate on to the atmosphere is then taken to be the value required to match the outer radius to the desired value. To calculate the volume of gas which is bound to the protoplanet from the numerical simulations, they identify the trajectories of tracer particles that are trapped inside the gravitational potential of the protoplanet. We comment that this approach may severely underestimate the amount of gas that can be accreted as in reality particles can collide with the protoplanet and become bound if energy is dissipated into shocks, as in the process of star formation. If the protoplanet fills in a significant part of its Roche lobe, then collisions may become frequent.

Three-dimensional numerical simulations of protoplanets of a few earth masses in protoplanetary discs usually start with a core surrounded by a static atmosphere that extends at most up to the Bondi radius. The Keplerian flow located within and around the Roche lobe, and in which the protoplanet is embedded, is then perturbed and material from the disc rains down on the protoplanet from above and below, because the gravitational potential from the protoplanet is not balanced (D'Angelo et al. 2003; Paardekooper & Mellema 2008; Machida et al. 2010). This results in the protoplanet being surrounded by a cold isothermal accretion flow. In this paper, we show that if a core below the critical mass is surrounded by an atmosphere that does not fill in entirely the Roche lobe and is embedded in an accretion flow, it is going to evolve into a static model expanded to the Roche lobe radius on a very short time-scale, even when the Bondi radius is smaller than the Roche lobe radius.

In the calculations presented here, we include the accretion of planetesimals on to the core, as we focus on the stage of the evolution before runaway gas accretion starts. In Section 2, we calculate the mass of the atmosphere that can be supported at hydrostatic and thermal equilibrium around a core of a given mass, assuming the surface radius of the atmosphere to be fixed. We consider a range of surface radii from a few core radii up to the Roche lobe radius. In Section 3, we consider the evolution of a core that starts accreting gas at a given rate from a radius much smaller than its Roche lobe radius. In contrast to what is usually assumed, we suppose here that the gas accretion rate on to the protoplanet is fixed, as it is likely to be constrained by the disc's physical parameters (e.g. density, temperature). We calculate a sequence of quasi-static equilibria along which the mass of the protoplanet increases. We show that the atmosphere has to expand very rapidly to accommodate the gas that is being accreted, even if the accretion rate is relatively small. We argue that, even when the Bondi radius is smaller than the Roche

lobe radius, the protoplanet will eventually fill in its Roche lobe. Finally, in Section 4, we summarize and discuss our results.

2 DEPENDENCE OF THE ATMOSPHERE MASS ON THE ATMOSPHERE SURFACE RADIUS

We consider an atmosphere at hydrostatic and thermal equilibrium and calculate its mass for a fixed core mass, accretion rate of planetesimals and surface radius of the atmosphere.

2.1 Structure of the protoplanet atmosphere

The equations governing the structure of the protoplanet atmosphere at hydrostatic and thermal equilibrium are the same as the equations of stellar structure, with the nuclear energy rate being replaced by the rate at which planetesimals that enter the planet atmosphere and collide with the core surface release their energy. These equations have been presented in Papaloizou & Terquem (1999), and we recall them below.

We assume that the protoplanet is spherically symmetric and non-rotating. We denote r the radius in spherical coordinates in a frame with origin at the centre of the protoplanet. The equation of hydrostatic equilibrium is

$$\frac{dP}{dr} = -g\rho. \quad (1)$$

Here, P is the pressure, $g = GM(r)/r^2$ is the acceleration due to gravity, with $M(r)$ being the mass interior to radius r (this includes the core mass if r is larger than the core radius) and G is the gravitational constant. The relation between $M(r)$ and the mass density per unit volume ρ is

$$\frac{dM}{dr} = 4\pi r^2 \rho. \quad (2)$$

We adopt the equation of state for a hydrogen and helium mixture given by Chabrier et al. (1992) for mass fractions of hydrogen and helium of 0.7 and 0.28, respectively. The standard equation of radiative transport gives the luminosity L_{rad} that is transported by radiation through the atmosphere as a function of the temperature gradient dT/dr :

$$\frac{dT}{dr} = \frac{-3\kappa\rho}{16\sigma T^3} \frac{L_{\text{rad}}}{4\pi r^2}, \quad (3)$$

where κ is the opacity, which in general depends on both ρ and T , and σ is the Stefan–Boltzmann constant.

As a large part of the atmosphere interior is unstable to convection, only a fraction of the total luminosity is transported by radiation. In the models presented here, we assume that the only energy source comes from the planetesimals that are accreted by the protoplanet and release their gravitational energy as they collide with the surface of the core. They generate a total core luminosity L_{core} given by

$$L_{\text{core}} = \frac{GM_c \dot{M}_c}{r_c}, \quad (4)$$

where M_c and r_c are, respectively, the mass and the radius of the core and \dot{M}_c is the planetesimal accretion rate.

The radiative and adiabatic temperature gradients, ∇_{rad} and ∇_{ad} , are given by

$$\nabla_{\text{rad}} = \left(\frac{\partial \ln T}{\partial \ln P} \right)_{\text{rad}} = \frac{3\kappa L_{\text{core}} P}{64\pi\sigma G M T^4} \quad (5)$$

and

$$\nabla_{\text{ad}} = \left(\frac{\partial \ln T}{\partial \ln P} \right)_s, \quad (6)$$

with the subscript s denoting evaluation at constant entropy.

In the regions where $\nabla_{\text{rad}} < \nabla_{\text{ad}}$, there is stability to convection and therefore all the energy is transported by radiation, i.e. $L_{\text{rad}} = L_{\text{core}}$. In the regions where $\nabla_{\text{rad}} > \nabla_{\text{ad}}$, there is instability to convection and therefore part of the energy is transported by convection, i.e. $L_{\text{core}} = L_{\text{rad}} + L_{\text{conv}}$, where L_{conv} is the luminosity associated with convection. Using the mixing length theory (Cox & Giuli 1968), we obtain

$$L_{\text{conv}} = \pi r^2 C_p \Lambda_{\text{ml}}^2 \left[\left(\frac{\partial T}{\partial r} \right)_s - \left(\frac{\partial T}{\partial r} \right) \right]^{3/2} \sqrt{\frac{1}{2} \rho g \left| \left(\frac{\partial \rho}{\partial T} \right)_p \right|}, \quad (7)$$

where $\Lambda_{\text{ml}} = |\alpha_{\text{ml}} P / (dP/dr)|$ is the mixing length, α_{ml} being a constant of order unity, $(\partial T / \partial r)_s = \nabla_{\text{ad}} T (d \ln P / dr)$, C_p is the specific heat and the subscript P denotes evaluation at constant pressure. The different thermodynamic parameters needed in the above equation are given by Chabrier et al. (1992), and we fix $\alpha_{\text{ml}} = 1$.

2.2 Boundary conditions

We have to solve the above equations for the three variables P , M and T as a function of r . Accordingly, we need three boundary conditions.

Following Bodenheimer & Pollack (1986) and Pollack et al. (1996), we take for the mass density of the core $\rho_c = 3.2 \text{ g cm}^{-3}$. The inner boundary of the atmosphere is equal to the core radius, r_c , given by

$$r_c = \left(\frac{3M_c}{4\pi\rho_c} \right)^{1/3}. \quad (8)$$

The first boundary condition is that $M(r_c) = M_c$.

We denote r_{atm} the outer boundary of the atmosphere, and we assume this radius to be fixed. In the calculations presented below, we will take r_{atm} to be either a few core radii or the Roche lobe radius r_L , which is assumed to be the same as the Hill radius, and is given by

$$r_L = \frac{2}{3} \left(\frac{M(r_{\text{atm}})}{3M_*} \right)^{1/3} a, \quad (9)$$

where a is the orbital radius of the protoplanet and M_* is the mass of the central star. Throughout the paper, we take $M_* = 1 M_\odot$.

When $r_{\text{atm}} < r_L$, we assume that the space above the atmosphere and in the Roche lobe of the protoplanet is filled with a cold accretion flow. Then the temperature T_s and the pressure P_s at the surface $r = r_{\text{atm}}$ can be approximated by the standard photospheric conditions (e.g. Kippenhahn & Weigert 1990; Bodenheimer, Hubickyj & Lissauer 2000):

$$T_s = \left(\frac{L_{\text{core}}}{4\pi\sigma r_{\text{atm}}^2} \right)^{1/4}, \quad (10)$$

and

$$P_s = \frac{GM(r_{\text{atm}})}{r_{\text{atm}}^2} \frac{2}{3\kappa_s}, \quad (11)$$

where $\kappa_s = \kappa(\rho_s, T_s)$ and ρ_s is the mass density at $r = r_{\text{atm}}$. Similar boundary conditions have been used by Palla & Stahler (1992) for computing the evolution of a protostar accreting from a disc. The validity of these boundary conditions will be discussed in Section 3.1. Note that in any case the structure of the atmosphere does not depend significantly on T_s (Papaloizou & Terquem 1999).

We also present below some calculations with $r_{\text{atm}} = r_L$, which are done using the boundary conditions presented in Papaloizou & Terquem (1999).

2.3 Calculations

For a given core mass M_c , planetesimal accretion rate \dot{M}_c and atmosphere radius r_{atm} , we solve the equations (1), (2) and (3) with the boundary conditions described above to get the structure of the atmosphere and its mass M_{atm} . We start the integration at $r = r_{\text{atm}}$ with $T = T_s$, $P = P_s$ and an initial guess for $M(r_{\text{atm}}) = M_c + M_{\text{atm}}$. In principle, to calculate P_s , we need to know ρ_s , which we derive from the pressure. However, for the values of T_s we obtain, which are smaller than 1000 K, the opacity κ_s does not depend on ρ_s . We can therefore self-consistently ignore ρ_s when calculating κ_s . The integration is carried through down to $r = r_c$. In general, the condition that $M(r_c) = M_c$ will not be satisfied. An iteration procedure is then used to adjust the value of $M(r_{\text{atm}})$ until $M(r_c) = M_c$ to a specified accuracy. When the core mass is larger than the critical mass M_{crit} , no solution can be found, i.e. no atmosphere at equilibrium can be supported at the surface of a core whose mass exceeds M_{crit} .

The opacity is taken from Bell & Lin (1994). This has contributions from dust grains, molecules, atoms and ions. It is written in the form $\kappa = \kappa_i \rho^a T^b$, where κ_i , a and b vary with temperature.

We take \dot{M}_c to be in the range 10^{-8} to $10^{-5} M_\oplus \text{ yr}^{-1}$, as this gives time-scales for building up a $10 M_\oplus$ core of 10^6 – 10^9 years. These values are consistent with the calculation of \dot{M}_c by Pollack et al. (1996) and, more recently, by Movshovitz et al. (2010).

Fig. 1 shows the mass of the atmosphere *versus* that of the core for $\dot{M}_c = 10^{-5}$, 10^{-6} , 10^{-7} and $10^{-8} M_\oplus \text{ yr}^{-1}$, and for an atmosphere radius $r_{\text{atm}} = 5r_c$, $10r_c$ or r_L . When $r_{\text{atm}} = 5r_c$, the photospheric temperature and pressure are $T_s = 555.3 \text{ K}$ and $P_s = 3.73 \text{ erg cm}^{-3}$ for $\dot{M}_c = 10^{-5} M_\oplus \text{ yr}^{-1}$, and $T_s = 98.7 \text{ K}$ and $P_s = 20.42 \text{ erg cm}^{-3}$ for $\dot{M}_c = 10^{-8} M_\oplus \text{ yr}^{-1}$. The curves corresponding to $r_{\text{atm}} = r_L$ are calculated assuming the temperature and pressure in the mid-plane of the nebula to be 33 K and $0.025 \text{ erg cm}^{-3}$, respectively, which correspond to a standard steady-state disc model with $\alpha = 10^{-2}$ and gas accretion rate $\dot{M} = 10^{-8} M_\odot \text{ yr}^{-1}$, assuming the protoplanet is at 5 au from the central star (see Papaloizou & Terquem 1999).

For $r_{\text{atm}} = 10r_c$, the critical core mass is 36, 24, 16 and $10 M_\oplus$ for $\dot{M}_c = 10^{-5}$, 10^{-6} , 10^{-7} and $10^{-8} M_\oplus \text{ yr}^{-1}$, respectively. For $r_{\text{atm}} = 5r_c$, the critical core mass is 55, 37, 25 and $16 M_\oplus$ for these values of \dot{M}_c . We note that, if the core is at 5 au from the central star, then $r_{\text{atm}} = 10$ and $5r_c$ represent about 1 and 0.7 per cent, respectively, of the Roche lobe radius of the protoplanet when its core reaches the critical mass.

For comparison, in a standard steady-state disc model with $\alpha = 10^{-2}$ and gas accretion rate $\dot{M} = 10^{-8} M_\odot \text{ yr}^{-1}$, at 5 au from the central star, the critical core mass of a protoplanet with $r_{\text{atm}} = r_L$ is 24, 15, 9 and $5 M_\oplus$ for $\dot{M}_c = 10^{-5}$, 10^{-6} , 10^{-7} and $10^{-8} M_\oplus \text{ yr}^{-1}$, respectively. These values are almost independent of the conditions in the nebula, and therefore do not change when \dot{M} and α are varied.

We see that the critical core mass increases when r_{atm} decreases. This is because the gas in the atmosphere has less (negative) gravitational energy when r_{atm} is smaller, i.e. it is more bound, and therefore, for a fixed core accretion luminosity, it can be supported at equilibrium up to larger core masses. Also, the temperatures at the bottom of the atmosphere tend to be lower for smaller r_{atm} , so that more mass can settle near the surface of the core, which also helps increase the critical core mass. Finally, because the surface of the atmosphere is smaller, radiative losses are smaller, which helps in

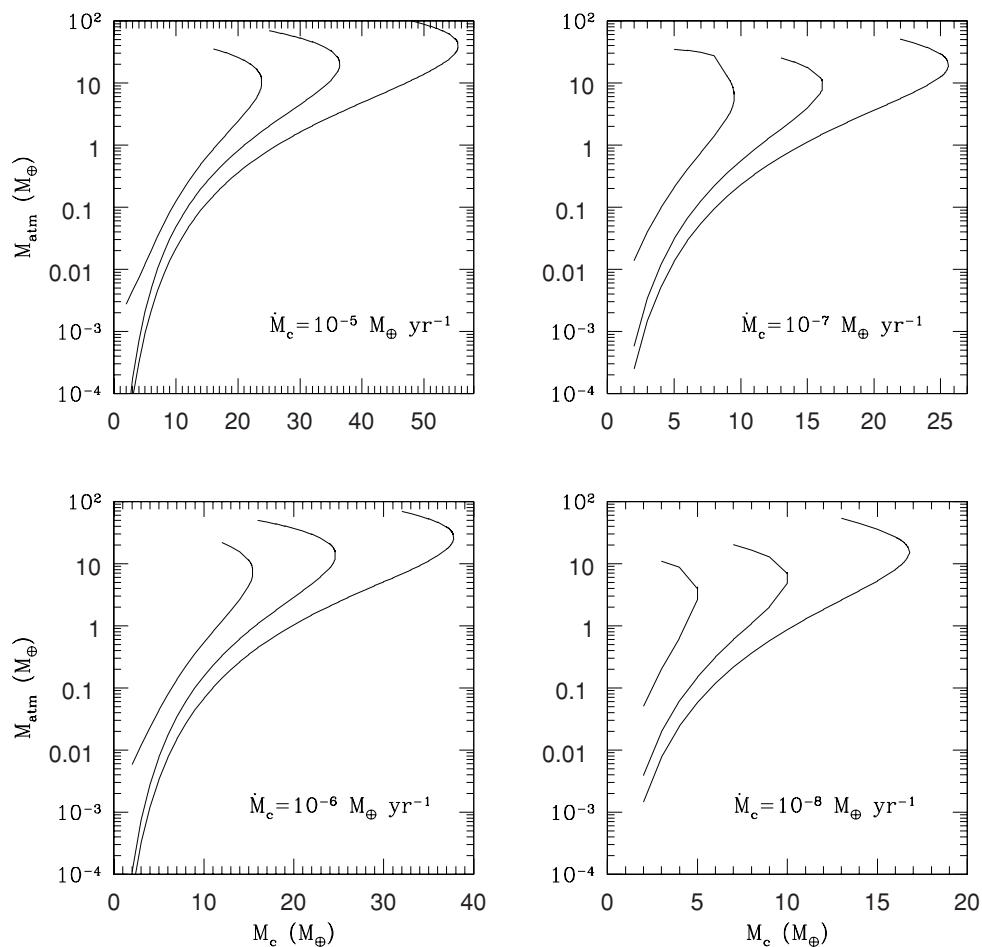


Figure 1. Atmosphere mass, M_{atm} , in units of M_{\oplus} , versus core mass, M_c , in units of M_{\oplus} , for different planetesimal accretion rates \dot{M}_c . The different frames correspond to $\dot{M}_c = 10^{-5} M_{\oplus} \text{ yr}^{-1}$ (top left), $10^{-6} M_{\oplus} \text{ yr}^{-1}$ (bottom left), $10^{-7} M_{\oplus} \text{ yr}^{-1}$ (top right) and $10^{-8} M_{\oplus} \text{ yr}^{-1}$ (bottom right), respectively. Each frame contains three curves corresponding, from left to right, to an atmosphere radius $r_{\text{atm}} = r_L$, $10r_c$ and $5r_c$, respectively. When $r_{\text{atm}} = r_L$, the calculation is done for a protoplanet embedded in a standard disc model at a distance of 5 au from the central star. The critical core mass is the mass beyond which no atmosphere at equilibrium can be supported at the surface of the core.

supporting the atmosphere at equilibrium up to larger core masses. However, for a given core mass and core accretion luminosity, the mass of the atmosphere decreases as r_{atm} decreases.

In all the calculations displayed on Fig. 1, r_L is smaller than the Bondi radius r_B and represents the largest radius the static atmosphere of the protoplanet can expand to. If the mid-plane temperature of the protoplanetary disc were larger, then r_B would become smaller than r_L . This is the case, for instance, for protoplanets less massive than $16 M_{\oplus}$ when the mid-plane temperature is 140 K, which corresponds to a disc model with $\alpha = 10^{-2}$ and gas accretion rate $\dot{M} = 10^{-7} M_{\odot} \text{ yr}^{-1}$, at 5 au from the central star.

3 EVOLUTION OF A PROTOPLANET ACCRETING FROM A PROTOPLANETARY DISC

We are now going to consider the evolution of a protoplanet which has an atmosphere that does not fill in its Roche lobe.

3.1 Accretion on to a core

Numerical simulations of low-mass protoplanets (\sim a few M_{\oplus}) embedded in protoplanetary discs and which do not fill entirely their

Roche lobe show that they significantly perturb the Keplerian motion of the fluid in the vicinity of and within their Roche lobe. High-resolution two-dimensional computations by D’Angelo, Henning & Kley (2002) indicate that circumplanetary discs may form within the Roche lobe of cores more massive than about $5 M_{\oplus}$. Subsequent three-dimensional simulations by the same authors (D’Angelo et al. 2003; see also Bate et al. 2003), using an isothermal equation of state, show that the circumplanetary disc features are much less marked around low-mass cores due to the vertical motion of the fluid around them. Gas entering the Roche lobe from above and below the protoplanet falls down directly on to it instead of joining a circumplanetary disc in the mid-plane. However, in these calculations, the resolution is limited to be several per cent of the Hill radius of the protoplanet, which is between one and two orders of magnitude larger than the radius of the core itself. Therefore, from these simulations, the formation of a circumplanetary disc around low-mass cores cannot be ruled out. Simulations by Machida et al. (2008) and Machida (2009) of circumplanetary disc formation around protoplanets show indeed that these discs tend to be rather compact, with an outer radius smaller than about 50 core radii.

Further three-dimensional numerical simulations by Paardekooper & Mellema (2008), including radiative transfer, show significant differences compared to the isothermal case.

In particular, the accretion rate on to the protoplanet is much lower due to the compression of gas within the planetary atmosphere (which, in these calculations, is the unresolved region around the core). The authors comment that the formation of a hot bubble around the protoplanet limits the gas flow towards its surface. In these simulations, the resolution is again on the order of 1 per cent of the Hill radius, so that the potential formation of a circumplanetary disc close to the core could not be seen or dismissed.

We suppose here that the protoplanet accretes through either a cold Bondi-type accretion flow or a circumplanetary disc. The surface of the protoplanet atmosphere is at a radius r_{atm} which is the transition from the (spherical or disc-like) accretion flow to a hydrostatic model. The value of the gas accretion rate \dot{M}_{gas} on to the protoplanet is very uncertain. The simulations by D'Angelo et al. (2003) indicate an infall rate on to a $5 M_{\oplus}$ core of the order of $10^{-4} M_{\oplus} \text{ yr}^{-1}$, but computations incorporating radiative transport give a value which is an order of magnitude smaller (Paardekooper & Mellema 2008). Computations with higher resolution would give even smaller values as compression of the gas near the core would be larger on smaller scales.

When the gas accreted encounters the surface of the protoplanet atmosphere, it releases the luminosity $L_{\text{acc}} = [G(M_c + M_{\text{atm}})\dot{M}_{\text{gas}}]/r_{\text{atm}}$. We assume that essentially all this luminosity is converted into heat in and radiated away from the accretion shocks that form at the surface of the protoplanet (we neglect the energy that is converted into rotational energy of the protoplanet). The gas that is accreted though has to bring some entropy deep inside the atmosphere, since the temperature and pressure rise there as the atmosphere mass increases, and it does so by inducing some gravitational contraction of the atmosphere. This results in an increase of the internal energy of the atmosphere which in turn drives the expansion of its surface radius. However, since the mass of gas which is accreted is brought to the surface of the atmosphere, the resulting increase in the protoplanet luminosity is not very important (see discussion in section 7.2 of Papaloizou & Nelson 2005). We will neglect it here and will therefore consider that the accretion of gas does not modify the luminosity of the protoplanet, which is always given by L_{core} .

In Section 2.2, we have assumed that the temperature and pressure at the surface of the static atmosphere were given by the photospheric values. This is a valid approximation as long as the temperature and pressure of the accretion flow are low compared to the photospheric values for the planet models. Note that if some of the accretion luminosity were not radiated away, which may well be the case, the surface temperature T_s would be higher than the photospheric value. However, we remark that the structure of the atmosphere is not very sensitive to T_s for the masses considered here, and, in any case, an increase of T_s would make the atmosphere more extended and therefore would strengthen the point we make in this paper. As far as the surface pressure is concerned, the only way the infalling gas contributes to it is through its ram pressure P_{ram} , because its motion is supersonic (e.g. Paardekooper & Mellema 2008). When the accretion is spherically symmetric, the ram pressure is given by

$$P_{\text{ram}} = \frac{\dot{M}_{\text{gas}}}{4\pi r_{\text{atm}}^2} \left[\frac{2G(M_c + M_{\text{atm}})}{r_{\text{atm}}} \right]^{1/2}, \quad (12)$$

where we have assumed that the gas is accelerated to free-fall velocity by the gravitational potential of the protoplanet. In the calculations discussed in Section 3.2, we have checked that P_{ram} is always

smaller (and in most cases much smaller) than the photospheric pressure P_s .

How exactly the gas is accreted by the protoplanet is not known, but we assume that once the gas is accreted it is redistributed in a spherically symmetric way at the surface of the atmosphere. As we mentioned above, the gas that is accreted induces some gravitational contraction of the atmosphere. However, as the accretion rate is low, we can consider that the atmosphere stays at quasi-hydrostatic and thermal equilibrium as it accretes. The time-scale for establishing hydrostatic equilibrium is indeed very short. If the temperature everywhere in the atmosphere were equal to that in the mid-plane of the protoplanetary disc (this gives a lower limit on the real value), i.e. between 30 and 140 K for a typical disc at a distance of 5 au from the central star, then the sound crossing time through a distance of 10^{12} cm, which is a typical value of the Hill radius of a few earth mass protoplanet, would be of the order of a year. The amount of gas accreted on such a time-scale is a tiny fraction of the mass of the atmosphere, and so it can be assumed that it is brought to equilibrium almost instantaneously.

We now consider the evolution of the protoplanet as it accretes gas at constant luminosity, assuming quasi-hydrostatic and thermal equilibrium.

3.2 Evolution of a $5 M_{\oplus}$ protoplanet

For illustrative purposes, we consider a $5 M_{\oplus}$ core accreting from a protoplanetary disc. We calculate a sequence of quasi-static equilibria along which the mass of the protoplanet increases. As the gas accretion rate is fixed, constrained by the physics in the disc, the surface radius r_{atm} of the atmosphere has to adjust to accommodate the mass increase. If the mass of the protoplanet atmosphere, with initially $r_{\text{atm}} < r_L$, increases rapidly relative to that of its core, then, for a constant luminosity, we see from Fig. 1 that r_{atm} has to increase, i.e. the atmosphere of the protoplanet expands. In that case, the evolution is along almost vertical lines going from the lower curve ($r_{\text{atm}} = 5r_c$ on these plots) to the upper curve, which corresponds to $r_{\text{atm}} = r_L$.

For all reasonable values of \dot{M}_{gas} and \dot{M}_c , we have found that a $5 M_{\oplus}$ core always undergoes such an evolution and that its atmosphere has to fill in its Roche lobe after a rather short time-scale. To illustrate this point, we now develop in detail some particular cases, starting with a core that has $r_{\text{atm}} = 5r_c$.

If $\dot{M}_c = 10^{-6} M_{\oplus} \text{ yr}^{-1}$, such a value of r_{atm} corresponds to $M_{\text{atm}} = 3.3 \times 10^{-3} M_{\oplus}$. We assume that the atmosphere accretes at a rate $\dot{M}_{\text{gas}} = 10^{-5} M_{\oplus} \text{ yr}^{-1}$. After 10^3 years, the core mass has hardly changed, but M_{atm} has grown significantly, being about $1.3 \times 10^{-2} M_{\oplus}$, which corresponds to $r_{\text{atm}} = 30r_c$. After 4×10^3 years, the atmosphere mass has grown to $4.3 \times 10^{-2} M_{\oplus}$, which corresponds to $r_{\text{atm}} = r_L$.

Let us now consider the case $\dot{M}_c = 10^{-5} M_{\oplus} \text{ yr}^{-1}$, which gives $M_{\text{atm}} = 9.2 \times 10^{-4} M_{\oplus}$ for $r_{\text{atm}} = 5r_c$, and $\dot{M}_{\text{gas}} = 5 \times 10^{-7} M_{\oplus} \text{ yr}^{-1}$. After 5×10^3 years, $M_c = 5.05 M_{\oplus}$ and $M_{\text{atm}} = 3.4 \times 10^{-3} M_{\oplus}$, which corresponds to $r_{\text{atm}} = 25r_c$. After 2.5×10^4 years, $M_c = 5.25 M_{\oplus}$ and $M_{\text{atm}} = 1.3 \times 10^{-2} M_{\oplus}$, which corresponds to $r_{\text{atm}} = r_L$.

Only when \dot{M}_c or \dot{M}_{gas} gets very small may we have models with $r_{\text{atm}} < r_L$. This is the case, for instance, when $\dot{M}_c = 10^{-5} M_{\oplus} \text{ yr}^{-1}$ and $\dot{M}_{\text{gas}} < 10^{-7} M_{\oplus} \text{ yr}^{-1}$, or when $\dot{M}_c = 10^{-8} M_{\oplus} \text{ yr}^{-1}$ and $\dot{M}_{\text{gas}} < 5 \times 10^{-5} M_{\oplus} \text{ yr}^{-1}$.

Therefore, *the static atmosphere of a protoplanet of a few earth masses has to fill in the Roche lobe as gas is being accreted*. There is no Bondi-type accretion flow around the protoplanet. The rate at

which gas penetrates from the nebula through the Roche lobe surface in the atmosphere depends on the rate at which the core grows. As the core grows in mass from the accretion of planetesimals, gas from the nebula is allowed to penetrate in its atmosphere to keep it at equilibrium. Unless the core grows very fast or the protoplanet is in a very low density part of the nebula, there will always be enough gas coming through the Roche lobe surface to supply the atmosphere so that it always fills in the Roche lobe. Only when the core reaches the critical core mass may the atmosphere contract so that $r_{\text{atm}} < r_L$. That happens if the nebula cannot deliver gas fast enough to the collapsing atmosphere. We therefore conclude that *a protoplanet with a core mass below the critical mass always fills in the Roche lobe.*

Note that the point we are making in this paper would still be valid if the luminosity of the protoplanet were not generated by the accretion of planetesimals but by the contraction of the atmosphere, provided the Kelvin–Helmholtz time-scale were long enough so that the atmosphere were at quasi-hydrostatic and thermal equilibrium over the accretion time-scale.

We have assumed here that the atmosphere can freely expand up to the Roche lobe radius. We now give an argument to show that this is always the case, even when the Bondi radius is smaller than the Roche lobe radius.

3.3 Roche lobe radius versus Bondi radius

It is usually assumed that a protoplanet embedded in a protoplanetary disc cannot expand beyond the Bondi radius defined as

$$r_B = \frac{G(M_c + M_{\text{atm}})}{c^2}, \quad (13)$$

where c is the sound speed in the protoplanetary disc (e.g. Bodenheimer et al. 2000). The argument is that the gas located at a distance from the protoplanet larger than r_B has a thermal energy larger than the gravitational energy that would bind it to the protoplanet, and therefore cannot be accreted by it. However, if a molecule of gas located in the protoplanetary disc beyond r_B is accelerated towards the protoplanet and collides with it, it may become bound (and therefore be accreted) like any other molecule coming from within the Roche lobe as described in Section 3.1. Accretion of this molecule only requires that it loses at least part of its (kinetic plus thermal) energy into shocks during the collision. If the molecule hits and settles into a circumplanetary disc before falling on to the planet, accretion will happen as energy is radiated away from the disc. When the molecule becomes bound to the protoplanet, the pressure gradient in the atmosphere adjusts itself to balance the gravitational attraction of the protoplanet, i.e. hydrostatic equilibrium is maintained. At fixed luminosity, the protoplanet has to expand to accommodate this extra mass, and its final surface radius does not have to be limited by r_B . For a given mass M_{atm} and surface temperature T_s of the atmosphere, an equilibrium atmosphere can be constructed with $r_{\text{atm}} > r_B$.

This is exactly similar to the process of star formation. In a molecular cloud that collapses on to a star, when particles hit the surface of the star they have a positive energy. This is because they have some thermal energy in addition to the kinetic energy due to the acceleration by the gravitational potential of the forming star. Accretion is possible because at least part of this energy is dissipated into shocks. The particle that is accreted may or may not bring some entropy into the stellar envelope, depending on whether all or only some of the energy is radiated away, and the star readjusts

to equilibrium (Palla & Stahler 1991, 1992; Hosokawa, Yorke & Omukai 2010).

We conclude that *the protoplanet can always expand to fill in its Roche lobe, even when the Bondi radius is smaller than the Roche lobe radius.*

4 DISCUSSION AND CONCLUSION

We have shown above that if a $5 M_{\oplus}$ protoplanet has a static atmosphere that does not fill in the Roche lobe but is embedded in a cold accretion flow, it has to expand as it accretes both planetesimals and gas from the nebula until it does fill in the Roche lobe. The evolution occurs on a time-scale shorter than the formation or migration time-scales. The value of $5 M_{\oplus}$ was chosen to illustrate the point, but the conclusion of course holds for any smaller protoplanets. Therefore, we conclude that protoplanets of a few earth masses have an atmosphere with a static structure all the way up to the Roche lobe surface, where it joins on to the nebula, and that remains the case until the core reaches the critical mass or the gas in the nebula gets depleted.

This result holds whether or not the Bondi radius is smaller than the Roche lobe radius. Therefore, the Bondi radius does not determine the extent of the static structure around the core, although it gives the scale on which the density increases in the atmosphere. For radii smaller than the Bondi radius, the structure of the atmosphere is almost independent of the conditions in the nebula, whereas beyond the Bondi radius the density does not vary much and the structure is determined by matching the pressure and temperature in the nebula.

Numerical simulations which show that low-mass protoplanets affect the flow in the surrounding protoplanetary disc inside their Roche lobe start with artificial initial conditions. The flow in the Roche lobe of the protoplanet is not at equilibrium to start with, as the gravitational attraction of the core is not balanced. Therefore, as seen in the simulations, material from the disc rains down on the protoplanet (D’Angelo et al. 2003; Paardekooper & Mellema 2008). When radiative effects are taken into account, this infall is slowed down as the gas is being compressed around the protoplanet (Paardekooper & Mellema 2008; Ayliffe & Bate 2009). When the luminosity generated by this compression is high enough, an almost spherical atmosphere forms around low-mass protoplanets, ultimately filling in their Roche lobe (Ayliffe & Bate 2009). This is in agreement with the results presented in this paper, where the luminosity is not being produced by the gas compression but by the accretion of planetesimals. The results reported in this paper indicate that numerical simulations of low-mass cores in discs, when they cannot result in the build-up of a static atmosphere around the core (like in the isothermal case), should have as initial conditions a core surrounded by a static atmosphere extending all the way up to the Roche lobe radius. In such a case, there will be no cold accretion flow around the protoplanet.

It has been argued that the orbital migration of low-mass cores depends very significantly on the torque exerted by the gas within the Roche lobe of the protoplanet (D’Angelo et al. 2003; Crida et al. 2009). The results presented above indicate that this is probably not the case, as the protoplanet itself fills in its Roche lobe.

In this paper, we have considered the evolution of a protoplanet which accretes planetesimals from the protoplanetary disc. Such an accretion may be reduced significantly before the gas in the nebula has disappeared, with the consequence that the core reaches the critical mass and the atmosphere has to contract to provide the energy radiated at its surface. In that case, its evolution is that calculated by Papaloizou & Nelson (2005).

Finally, we comment that we have not taken into account here the rotation of the protoplanet on to itself. Although this would not affect the main conclusion of our paper, we point out that there may be some boundary layer at the surface of the protoplanet's atmosphere due to a finite relative velocity (shear) between the atmosphere and the surrounding disc material. The size of the boundary layer would be determined by the disc's viscosity.

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