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**Non-Gaussianity excess problem in classical bouncing cosmologies**

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The simplest possible classical model leading to a cosmological bounce is examined in the light of the non-Gaussianities it can generate. Concentrating solely on the transition between contraction and expansion, and assuming initially purely Gaussian perturbations at the end of the contracting phase, we find that the bounce acts as a source such that the resulting value for the post-bounce  $f_{\text{NL}}$  may largely exceed all current limits, to the point of potentially casting doubts on the validity of the perturbative expansion. We conjecture that if one can assume that the non-Gaussianity production depends only on the bouncing behavior of the scale factor and not on the specifics of the model examined, then many realistic models in which a nonsingular classical bounce takes place could exhibit a generic non-Gaussianity excess problem that would need to be addressed for each case.

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**I. INTRODUCTION**

The recently released Planck data [1,2] have set new standards as far as cosmological modeling is concerned, imposing very tight constraints on early universe physics [3,4] and discriminating [5,6] among numerous inflationary theories [7]. Bouncing cosmologies are among very few possibly viable alternatives to inflationary cosmology (see [8] for a review). This being said, the only relevant bouncing models worth investigating [8–10] are those that are able to reproduce the observed power spectra, both scalar and tensorial. In turn, these models have to face the most serious cosmological constraint to date, namely that imposed by the smallness of non-Gaussianities [11]. Whether or not generic bouncing models can successfully pass this test will decide on their viability. To a large extent, the non-Gaussianity parameter  $f_{\text{NL}}$  does not depend on the actual spectrum of first order perturbations, and is thus also independent of their initial conditions. This makes it an invaluable tool to assess the viability of any cosmological model.

The purpose of the present paper is to demonstrate, by means of an explicit calculation, itself drawing heavily on

the ones detailed in Ref. [12], that the non-Gaussianity produced during the transition from contraction to expansion, and thus by the bounce itself, may far exceed existing constraints on  $f_{\text{NL}}$ . Recalling that canonical single field slow-roll inflation naturally predicts small  $f_{\text{NL}}$ , our findings would tend to favor the inflationary paradigm by disqualifying one of its few alternatives.

The particular category of model studied in this paper is that for which the matter content is in the form of a strictly positive energy scalar field. The presence of a negative energy component being crucial for the obtention of a bounce, we take the spatial curvature to be positive, so that it acts as an *effective* negative energy component. While it is true that many bouncing models are constructed with a vanishing or negligible spatial curvature contribution, they necessarily involve other types of negative energy fields, which may cause serious instabilities, and hence also potentially produce large amounts of non-Gaussianities. Therefore, although the results which we present below apply, strictly speaking, to nonsingular bouncing models dominated at the bounce by the positive spatial curvature term in the Friedmann equation, and for which general relativity (GR) is valid all along, we conjecture that it could apply to a much wider set of similarly nonsingular models, hence raising a possibly generic problem with bouncing cosmologies. Note that we do not consider singular bounces for which GR does not apply throughout as no

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reasonable prediction can be made in such contexts without an explicit calculation within the framework of an (as of yet still unknown) theory of quantum gravity.

## II. THEORETICAL FRAMEWORK

We start from the GR action (we work in natural units in which  $8\pi G_N = c = \hbar = 1$ ),

$$S = \int d^4x \sqrt{-g} (-R + \mathcal{L}_{\text{mat}}), \quad (1)$$

where  $\mathcal{L}_{\text{mat}}$  describes the matter content and  $R$  is the Ricci scalar derived from the metric tensor  $g_{\mu\nu}$ . The metric itself is chosen to be that of a perturbed Friedmann-Lemaître line element, given in Poisson gauge<sup>1</sup> by

$$ds^2 = a^2(-e^{2\Phi}d\eta^2 + e^{-2\Psi}\gamma_{ij}dx^i dx^j), \quad (2)$$

where

$$\gamma_{ij} = \left(1 + \frac{1}{4}\mathcal{K}\delta_{mn}x^m x^n\right)^{-2} \delta_{ij}$$

is the background spatial metric which we take to be of constant positive curvature ( $\mathcal{K} = 1$ ). The fields

$$\Psi = \sum_i \frac{\Psi^{(i)}}{i!} \quad \text{and} \quad \Phi = \sum_i \frac{\Phi^{(i)}}{i!}$$

are the Bardeen potentials up to arbitrary order in perturbations and encode the scalar cosmological fluctuations in the metric. Note that here, one has, at first order,  $\Psi_{(1)} = \Phi_{(1)}$ .

The background metric, i.e. that obtained in the limit  $\Psi, \Phi \rightarrow 0$ , satisfies the Friedmann equations

$$\mathcal{H}^2 + \mathcal{K} = \frac{1}{3}a^2\rho, \quad (3)$$

where  $\rho$  is the fluid energy density and the conformal Hubble rate is  $\mathcal{H} \equiv a'/a$ , a prime meaning a derivative with respect to the conformal time  $\eta$ . The normalized energy density is defined through  $\Omega \equiv \rho a^2/(3\mathcal{H}^2)$ , and one may associate to the spatial curvature term  $\mathcal{K}$  a normalized energy density in a similar way through  $\Omega_{\mathcal{K}} \equiv \mathcal{K}/\mathcal{H}^2$ .

We set  $\mathcal{K} = 1$  for two reasons.

First, as stressed in the Introduction, the obtention of a bounce requires the presence of an effectively negative energy component. Positive spatial curvature is its simplest incarnation. It is free of the instabilities that may for

instance result from the introduction of ghost fields, and less speculative than for example the Galileon/ghost condensate implementation (see, e.g. Ref. [13] and references therein). Whether or not this latter implementation exhibits the large non-Gaussianity problem discussed in the present paper is still a matter of debate.

Second, spatial curvature is identically zero only in the special and entirely implausible situation where  $\Omega_{\mathcal{K}} = 0$  strictly. We would argue that  $\Omega_{\mathcal{K}} = 0$  can only be the result of extreme fine-tuning or occurs in specific theoretical frameworks (e.g. brane inflation in superstring theory where spatial flatness and isotropy are protected by symmetry). In general, in any realistic cosmology,  $\Omega_{\mathcal{K}} \neq 0$ , with current observational constraints to some extent favoring a slightly closed universe with  $\mathcal{K} = 1$  [2]. Furthermore, at the bounce, the Hubble parameter  $\mathcal{H}$  being equal to zero, it is the balance between the spatial curvature term and the energy contents of the cosmology which determines the dynamics. Under general conditions, spatial curvature can thus by no means be assumed negligible at the bounce point when otherwise only positive energy density components are present. In the case of a model that relies on a ghost condensate or some other effectively negative energy density component, the negligibility of the spatial curvature term can only be invoked *a posteriori*, i.e. if an explicit calculation of  $a_B$ , the scale factor at the bounce, demonstrates that it is indeed negligible.<sup>2</sup>

Although non-negligible at the bounce, the spatial curvature at late times can easily be made to agree with current limits on  $\Omega_{\mathcal{K}}$ . This can be achieved in two different ways. The first is the existence of a phase of inflation following the bounce [14,15]. The second is the existence of a phase of deflation prior to the bounce [16] with the added requirement that the bounce be close to symmetric (see [8]).

We now restrict attention to the specific case for which the matter consists in a single scalar field  $\phi$  with a canonical kinetic term and evolving in a potential  $V(\phi)$ . We therefore have

$$S = - \int d^4x \sqrt{-g} [R + (\partial\phi)^2 + V(\phi)]. \quad (4)$$

At the level of first order perturbations, introducing the variable  $u \propto a\Psi_{(1)}/\phi'$  and its Fourier modes, defined by  $\Delta u_k = -k^2 u_k$ , one finds [17]

$$u_k'' + [k^2 - V_u(\eta)]u_k = 0, \quad (5)$$

where the potential  $V_u(\eta)$  is sketched in Fig. 1, drawing on the specific functional shapes of  $V_u(\eta)$  obtained in previous works on the same model [14,15,18]. As shown in the

<sup>1</sup>This gauge is known to introduce potentially large and unphysical effects. The quantities calculated below however, being the ratios of spectra of first order perturbations, should not be plagued by this problem.

<sup>2</sup>Here and in what follows, the subscript ‘‘B’’ denotes a quantity evaluated at the time of the bounce.

figure, a typically asymmetric bouncing phase occurs at  $\eta_B$  and is generically preceded and followed by peaks in the potential with model-dependent amplitudes and widths. The peak that occurs prior to the bounce follows a regime in which  $V_u$  vanishes, in such a way that unambiguous vacuum initial conditions can be set. In contrast with what happens in inflation, for which modes cross the potential only once (e.g. the mode with wave number labeled  $k_3$  in Fig. 1), in a bouncing cosmology, modes may cross the potential three or more times (e.g. modes with wave numbers  $k_1$  or  $k_2$  in Fig. 1). The primordial spectrum is therefore modified for wave numbers  $k_1, k_2$ , with possibly superimposed oscillations [14,15] and, as will be shown below, the amplitude of the three-point function of cosmological perturbations generated by the bounce for such scales can consequently be very large [12].

At this stage in the discussion, it is possible to make one more argument, at the level of first order perturbations, towards the genericity of the analysis presented here, and its nonspecificity to spatial curvature dominated bounces.

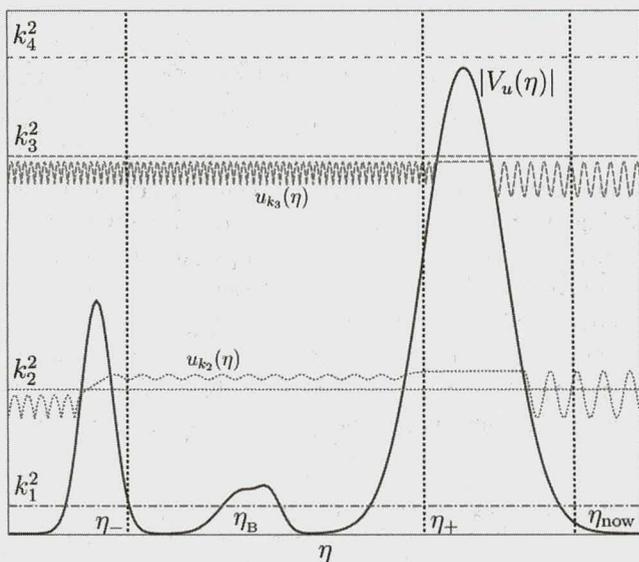


FIG. 1 (color online). Prototypical potential  $V_u(\eta)$ , and wave number squared (see [14,15,18] for explicit examples). The bounce itself occurs between  $\eta_-$  and  $\eta_+$ . At the level of the two-point statistics, small scale perturbations (e.g. those of wave number  $k_4$ ) remain unaffected, while long wavelength perturbations ( $k_1, k_2$  or  $k_3$ ) can be spectrally modified in different ways. For illustrative purposes, the time evolution of two modes,  $u_{k_2}$  and  $u_{k_3}$ , is also shown. As shown in this paper, the bounce produces large non-Gaussianities for any  $\{k_1, k_2, k_3\}$  configuration. The first peak before  $\eta_-$  might represent an initial source for primordial perturbation enhancement, as e.g. a matter or ekpyrotic contraction, while the second peak, after  $\eta_+$ , could be understood as an inflationary stage subsequent to the bounce. Although during both these phases, further non-Gaussianity could be produced, we restrict attention here to the seemingly more harmless period between  $\eta_-$  and  $\eta_+$ , i.e. the bounce itself.

The shape of the potential  $V_u(\eta)$  was discussed in detail in Ref. [18]. In a Taylor expansion in the vicinity of the bounce, the potential for the rescaled Bardeen variable  $u$  at the bounce is characterized by its width and height, each given by Eqs. (52) and (53) of that paper. From these equations, it is easily seen that the potential depends mainly on the kinetic term  $(1/2)(\phi')^2$  and on the logarithmic derivatives of  $V(\phi)$ . It does not depend crucially on spatial curvature. In fact as shown in Refs. [14,15], spatial curvature enters in the potential of first order perturbations through a constant term equal to 4. It can also be noted that taking the limit  $\mathcal{K} \rightarrow 0$  in the final results obtained below yields exactly the same conclusions.

### III. MODELING THE BOUNCE

In this paper, we focus on the calculation of the amount of non-Gaussianity produced by the bouncing phase only. It is thus sufficient for our purpose to expand the scale factor around the bounce in powers of conformal time  $\eta$ ,

$$\frac{a}{a_0} = 1 + \frac{1}{2} \left( \frac{\eta}{\eta_c} \right)^2 + \lambda_3 \left( \frac{\eta}{\eta_c} \right)^3 + \frac{5(1 + \lambda_4)}{24} \left( \frac{\eta}{\eta_c} \right)^4 + \dots, \quad (6)$$

where  $\eta_c$  is the characteristic time scale of the bounce, and to compute the production of non-Gaussianity between an initial spatial hypersurface at time  $\eta_-$  satisfying  $-\eta_c \lesssim \eta_- < 0$  and a final spatial hypersurface at time  $\eta_+$  satisfying  $0 \lesssim \eta_+ < \eta_c$ . In Eq. (6), we have set the bounce conformal time  $\eta_B = 0$  for convenience. The two additional constants  $\lambda_3$  and  $\lambda_4$  parametrize deviations from a de Sitter bounce at cubic and quartic order in  $\eta$  respectively while  $\eta_c$  is an overall deviation in the bouncing time scale from the de Sitter bouncing time scale.

At the level of the background cosmology, introducing the parameter  $\Upsilon = \phi_B^2/2$ , one may use the Einstein equations to express the bouncing time scale as  $\eta_c = (1 - \Upsilon)^{-1/2} \geq 1$ . Two additional parameters  $\varepsilon_V = (V_{,\phi}/V)|_B$  and  $\eta_V = (V_{,\phi\phi}/V)|_B$  can be related to  $\Upsilon$ ,  $\lambda_3$  and  $\lambda_4$  in Eq. (6) through the Einstein equations, with the de Sitter bounce being recovered in the limit  $\Upsilon \rightarrow 0$  [12,14,18] (recall that one expects the de Sitter solution to be an attractor for this dynamical system). In terms of  $\Upsilon$ ,  $\varepsilon_V$  and  $\eta_V$ , the bounce is seen to be controlled by the kinetic energy of  $\phi$  and the flatness of the potential  $V(\phi)$ .

The equation of motion for the Fourier modes of perturbation at the  $i$ th order reads

$$\mathcal{D}\Psi_{(i)} = \mathcal{S}[\Psi_{(i-1)}], \quad (7)$$

where

$$\mathcal{D} = \partial_\eta^2 + F(\eta)\partial_\eta + k^2 + W(\eta)$$

(the subscript “ $k$ ” on the modes is not written explicitly but is instead implicitly assumed for notational simplicity), with

$$F(\eta) = 2(\mathcal{H} - \frac{\phi''}{\phi'})$$

and

$$W(\eta) = 2(\mathcal{H}' - \mathcal{H}\frac{\phi''}{\phi'} - 2\mathcal{K}).$$

The source term  $S[\Psi_{(i-1)}]$  is vanishing for  $i = 1$  and its explicit form for  $i = 2$ , not essential for the present discussion, was computed in [12] and depends on quantities computed at all previous orders.

#### IV. NON-GAUSSIANITIES

The series solution of Eq. (7) for  $\Psi_{(1)}$  up to order  $\eta^2$  can be written in terms of two mode functions  $v_1(k, \eta)$  and  $v_2(k, \eta)$  normalized at the prebounce time  $\eta_-$  (see Fig. 1) in such a way that  $v_1(k, \eta_-) = 1$ ,  $v_1'(k, \eta_-) = 0$ ,  $v_2(k, \eta_-) = 0$  and  $v_2'(k, \eta_-) = 1$  [12]. In this basis, the initial conditions are given in terms of a set of random variables  $\hat{x}_a \equiv \{\Psi_{(1)}(\eta_-), \Psi'_{(1)}(\eta_-)\}$  providing the initial conditions of the first order perturbation and its time derivative on the initial spatial hypersurface. As we are interested in the amount of non-Gaussianity produced during the bouncing phase, we shall assume that the variables  $\hat{x}_a$  follow Gaussian statistics. The  $\hat{x}_a$  in turn define a spectral matrix  $\mathbf{P}$  at  $\eta_-$  by  $\langle \hat{x}_a(\mathbf{k}_1)\hat{x}_b(\mathbf{k}_2) \rangle = \delta_{\mathbf{k}_1\mathbf{k}_2} P_{ab}(k)$ , where the indices  $a, b$  represent either  $\Psi$  or  $\Psi'$ . It is important to note that, in general, and in contrast to the more usual inflationary case, all four entries in  $\mathbf{P}$  are necessary to calculate the amount of non-Gaussianity produced by the bouncing phase since we cannot assume the mode to have reached the constant

super-Hubble value which is characteristic of the more usual inflationary evolution. Note also that the background spacetime being of constant positive curvature, all calculations are performed on the three-sphere  $\mathbb{S}^3$  and the wave vectors consist in three integer numbers:  $n > 1$ , giving the amplitude  $k^2 = n(n+2)$ ;  $\ell > 0$ ; and  $m \in [-\ell, \ell]$ , while  $\delta_{\mathbf{k}_1\mathbf{k}_2}$  is the product of three Kronecker delta functions  $\delta_{n_1 n_2}$ ,  $\delta_{\ell_1 \ell_2}$ , and  $\delta_{m_1 m_2}$ .

The bispectrum  $\mathcal{B}_\Psi$  produced during the bouncing phase (i.e., in the interval  $\eta_-$  to  $\eta_+$ , as shown on Fig. 1) is defined through the three-point function of the perturbation  $\Psi$ , evaluated at  $\eta_+$  [12],

$$\langle \Psi_{\mathbf{k}_1} \Psi_{\mathbf{k}_2} \Psi_{\mathbf{k}_3} \rangle = \frac{1}{2} \mathcal{G}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3} \mathcal{B}_\Psi(k_1, k_2, k_3), \quad (8)$$

where  $\mathcal{G}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}$  is a geometrical form factor generalizing the flat case  $\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$  to  $\mathbb{S}^3$ ; it is given by an integral over the product of three hyperspherical harmonics. The bispectrum is used to define the nonlinearity parameter  $f_{\text{NL}}$ , obtained by expressing the non-Gaussian signal in terms of the sum of squares of the two-point functions for wave numbers  $k_1, k_2$  and  $k_3$  through

$$\mathcal{B}_\Psi(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}} [P_{\Psi\Psi}(k_1)P_{\Psi\Psi}(k_2) + P_{\Psi\Psi}(k_2)P_{\Psi\Psi}(k_3) + P_{\Psi\Psi}(k_3)P_{\Psi\Psi}(k_1)]. \quad (9)$$

Using the results obtained in [12], we now calculate  $f_{\text{NL}}$  at leading order in  $\Upsilon$ ,  $\varepsilon_V$  and  $\eta_V$  and in the limit of large wave numbers  $k$ . This latter assumption is justified because the range of observationally accessible physical wave numbers today is  $10^{-2}h \text{ Mpc}^{-1} \lesssim k_{\text{phys}} \lesssim 10^3 h \text{ Mpc}^{-1}$  and corresponds to a range of comoving wave numbers  $10^2 \lesssim k \lesssim 10^8$  for a conservative value  $\Omega_\kappa \sim 10^{-2}$  [2] (Planck latest results indicating  $\Omega_\kappa \lesssim 5 \times 10^{-3}$ ). We find

$$f_{\text{NL}} = -\frac{5(k_1 + k_2 + k_3)}{3\Upsilon K_3(k_1, k_2, k_3)} \left[ \prod_{\sigma(i,j,\ell)} (k_i + k_j - k_\ell) \right] \left\{ \sum_{\sigma(i,j,\ell)} \frac{K_1(k_i)K_1(k_j)}{k_\ell^2} - 4 \left[ \frac{K_1(k_i)K_2(k_j)}{k_j^2 k_\ell^2} + \frac{K_1(k_j)K_2(k_i)}{k_i^2 k_\ell^2} \right] \right\} \\ + \frac{5}{3\Upsilon K_3(k_1, k_2, k_3)} \sum_{\sigma(i,j,\ell)} \left[ \frac{7}{3} + \frac{2}{3} \left( \frac{k_i^2 + k_j^2}{k_\ell^2} \right) - 3 \left( \frac{k_i^2 - k_j^2}{k_\ell^2} \right)^2 \right] K_1(k_i)K_1(k_j) + \dots, \quad (10)$$

where the dots denote subleading terms in inverse powers of  $k$  and higher order in  $\Upsilon$ ,  $\varepsilon_V$  and  $\eta_V$ . In Eq. (10), the relevant functions of the initial spectra are

$$K_1(k) = 6P_{\Psi\Psi}(k) + 7P_{\Psi\Psi'}(k) + 2P_{\Psi'\Psi'}(k), \quad K_2(k) = 7P_{\Psi\Psi}(k) + 11P_{\Psi\Psi'}(k) + 4P_{\Psi'\Psi'}(k), \quad (11)$$

and

$$K_3(k_1, k_2, k_3) = 81 \sum_{\sigma(i,j)} P_{\Psi\Psi}(k_i)P_{\Psi\Psi}(k_j) + 108 \sum_{\sigma(i,j)} P_{\Psi\Psi}(k_i)P_{\Psi\Psi'}(k_j) + 36 \sum_{\sigma(i,j)} P_{\Psi\Psi}(k_i)P_{\Psi'\Psi'}(k_j) \\ + 144 \sum_{\sigma(i,j)} P_{\Psi\Psi'}(k_i)P_{\Psi\Psi'}(k_j) + 48 \sum_{\sigma(i,j)} P_{\Psi\Psi'}(k_i)P_{\Psi'\Psi'}(k_j) + 16 \sum_{\sigma(i,j)} P_{\Psi'\Psi'}(k_i)P_{\Psi'\Psi'}(k_j), \quad (12)$$

so in the general case, the non-Gaussianity parameter  $f_{\text{NL}}$  depends not only on the spectrum of curvature perturbations  $P_{\Psi\Psi}$  but also on that of its time derivative  $P_{\Psi'\Psi'}$  as well as on the cross spectrum  $P_{\Psi\Psi'}$ , both usually assumed irrelevant in the usual inflationary framework.

In Eqs. (10) and (12), the sums and products are taken over all possible permutations of  $i, j$  and  $\ell$  with  $\sigma(i, j, \ell)$  denoting  $(i, j, \ell) \in \{(1, 2, 3), (1, 3, 2), (2, 3, 1)\}$ , and  $\sigma(i, j)$  denoting  $(i, j) \in \{(1, 2), (1, 3), (2, 3)\}$ . In the equilateral ( $k_1 = k_2 = k_3 = k$ ) and squeezed ( $k_i = k_j = k$  and  $k_\ell = p \ll k$ ) configurations and at leading order, Eq. (10) simplifies to

$$f_{\text{NL}}^{\text{equi}} = -\frac{15k^2}{\Upsilon} \frac{K_1^2(k)}{K_3(k, k, k)}, \quad (13)$$

$$f_{\text{NL}}^{\text{sq}} = -\frac{20k^2}{3\Upsilon} \frac{K_1^2(k) + K_1(k)K_1(p)}{K_3(k, k, p)}, \quad (14)$$

so that the non-Gaussianity parameter is of order  $k^2/\Upsilon$ . In the folded configuration ( $k_2 = k_3 = \frac{1}{2}k_1$ ), the first non-vanishing term is given in the second line of Eq. (10) and simplifies to

$$f_{\text{NL}}^{\text{fold}} = \frac{40}{9\Upsilon} \frac{K_1(k)[K_1(k) - 16K_1(2k)]}{K_3(k, k, 2k)}. \quad (15)$$

The square of the wave number does not appear in the numerator of Eq. (15) so that the folded configuration is in general subdominant relative to the equilateral and squeezed configurations.

## V. DISCUSSION

Given that the matrix  $\mathbf{P}$  is unknown, the  $K$ 's are also unknown, and thus no definite conclusion can be drawn from the above calculations as far as the actual values of  $f_{\text{NL}}$  are concerned. Some information on the dominant shapes of non-Gaussianity produced at the bounce can however be extracted from Eq. (10) by making plausible assumptions on the matrix elements of  $\mathbf{P}$ . In this paper, we provide two such examples which also highlight the dependence of the shapes of non-Gaussianities on the initial conditions at  $\eta_-$ .

Let us first assume that the functions of the original spectra are all roughly equal, i.e.  $K_1(k_i)K_{1,2}(k_j) \simeq K_3(k_1, k_2, k_3)$ , an approximation that should be roughly valid in many cosmologically relevant situations. With this simplifying assumption, one obtains from Eq. (10) that

$$f_{\text{NL}} \simeq \frac{5}{3\Upsilon} [\mathcal{B}(x_2, x_3) - k_1^2 \mathcal{C}(x_2, x_3)], \quad (16)$$

where the dimensionless characteristic shape functions  $\mathcal{B}$  and  $\mathcal{C}$ , which depend only on the ratios  $x_2 = k_2/k_1$  and  $x_3 = k_3/k_1$ , are given by

$$\begin{aligned} \mathcal{B}(x_2, x_3) \equiv & 7 + \frac{2}{3} \left( \frac{1+x_2^2}{x_3^2} + \frac{1+x_3^2}{x_2^2} + x_2^2 + x_3^2 \right) \\ & - 3 \left[ \left( \frac{1-x_2^2}{x_3^2} \right)^2 + \left( \frac{1-x_3^2}{x_2^2} \right)^2 + (x_2^2 - x_3^2)^2 \right], \end{aligned} \quad (17)$$

and

$$\begin{aligned} \mathcal{C}(x_2, x_3) = & (1+x_2+x_3)(1+x_2-x_3)(1+x_3-x_2) \\ & \times (x_2+x_3-1) \left( 1 + \frac{1}{x_2^2} + \frac{1}{x_3^2} \right). \end{aligned} \quad (18)$$

These shape functions are displayed in the upper plots of Fig. 2 where, without loss of generality, we have ordered the variables by assuming  $x_3 \leq x_2 \leq 1$ , with the triangle inequality given by  $x_2 - x_3 \leq 1 \leq x_2 + x_3$ . The left-hand plot shows the function  $\log(|\mathcal{B}|)$  and suggests that non-Gaussianities proportional to  $1/\Upsilon$  peak in the folded configuration. The right-hand plot shows the function  $\mathcal{C}$  and suggests that non-Gaussianities proportional to the overall factor  $k_1^2/\Upsilon$  produced in the bouncing phase peak in the equilateral, take intermediate values in the squeezed, and are small in the folded configuration.

Another way to determine the shapes of non-Gaussianities produced in a bouncing phase in a largely model-independent way consists in assuming the Bardeen potential to have reached, at  $\eta = \eta_-$ , the frozen state characteristic of super-Hubble inflationary evolution, so that one has  $\Psi' \ll \Psi$ , leading to  $P_{\Psi'\Psi'} \ll P_{\Psi\Psi'} \ll P_{\Psi\Psi}$ . Denoting for simplicity  $P(k_i) \equiv P_{\Psi\Psi}(k_i)$ , this then leads to

$$f_{\text{NL}}^{\text{frozen}} = \frac{180}{243\Upsilon} \frac{\mathcal{F}[P(k_1), P(k_2), P(k_3), x_2, x_3] - k_1^2 \mathcal{G}[P(k_1), P(k_2), P(k_3), x_2, x_3]}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)}, \quad (19)$$

where

$$\begin{aligned} \mathcal{F}[P(k_1), P(k_2), P(k_3), x_2, x_3] = & \left[ \frac{7}{3} + \frac{2}{3} \left( \frac{1+x_2^2}{x_3^2} \right) - 3 \left( \frac{1-x_2^2}{x_3^2} \right)^2 \right] P(k_1)P(k_2) + \left[ \frac{7}{3} + \frac{2}{3} \left( \frac{1+x_3^2}{x_2^2} \right) - 3 \left( \frac{1-x_3^2}{x_2^2} \right)^2 \right] P(k_1)P(k_3) \\ & + \left[ \frac{7}{3} + \frac{2}{3} (x_2^2 + x_3^2) - 3(x_2^2 - x_3^2)^2 \right] P(k_2)P(k_3), \end{aligned} \quad (20)$$

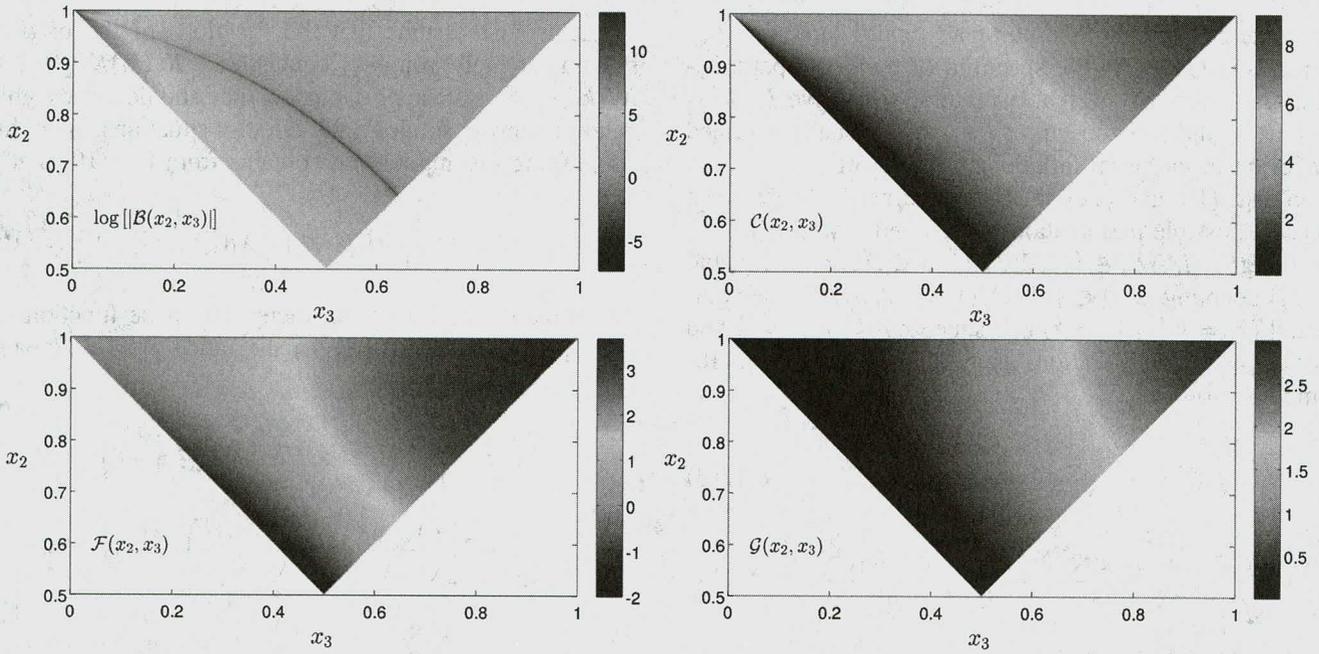


FIG. 2 (color online). Shape functions derived from Eq. (10) showing the relative contributions of the various possible non-Gaussian configurations. Top figures:  $\log(|\mathcal{B}|)$  (left) and  $\mathcal{C}$  (right) obtained assuming  $K_1(k_i)K_{1,2}(k_j) \simeq K_3(k_1, k_2, k_3)$ . Lower figures: almost scale-invariant “frozen” state approximation ( $P_{\Psi\Psi'} \ll P_{\Psi\Psi} \ll P_{\Psi\Psi} \propto k^{n_s-4}$  with  $n_s = 0.9603$ ); the figures are the shape functions obtained by combining Eqs. (20) to (24). In the figures on the left,  $f_{\text{NL}} \propto \Upsilon^{-1}$ . In the figures on the right,  $f_{\text{NL}} \propto (k_1^2/\Upsilon)$ . In all four figures,  $x_2 = k_2/k_1$  and  $x_3 = k_3/k_1$ . The differences in the amplitude as a function of the configuration  $\{k_1, k_2, k_3\}$  highlight the dependence of the shape function on the details of  $\mathcal{P}$ .

and

$$\mathcal{G}[P(k_1), P(k_2), P(k_3), x_2, x_3] = (1 + x_2 + x_3)(1 + x_2 - x_3)(1 + x_3 - x_2)(x_2 + x_3 - 1) \times \left[ \frac{P(k_1)P(k_2)}{x_3^2} + \frac{P(k_1)P(k_3)}{x_2^2} + P(k_2)P(k_3) \right]. \quad (21)$$

In order to go one step further and actually evaluate the non-Gaussianities produced during the contraction-to-expansion transition, we assume, as is often done, that the spectrum produced during the contraction phase not only passed through the bounce unchanged but also that it

is in agreement with the data. Assuming observational constraints to be those of Planck, we obtain that, in our notations, this requires the power spectrum to behave as a power law  $P(k) \propto k^{n_s-4}$ , with [1]  $n_s = 0.9603 \pm 0.0073$ . The ratios of power spectra in Eq. (19) then read

$$\frac{P(k_1)P(k_2)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)} = \left[ 1 + \left( \frac{x_3}{x_2} \right)^{n_s-4} + x_3^{n_s-4} \right]^{-1}, \quad (22)$$

$$\frac{P(k_1)P(k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)} = \left[ 1 + \left( \frac{x_2}{x_3} \right)^{n_s-4} + x_2^{n_s-4} \right]^{-1}, \quad (23)$$

$$\frac{P(k_2)P(k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)} = [1 + (x_2)^{4-n_s} + (x_3)^{4-n_s}]^{-1}. \quad (24)$$

The shape functions that can be formed by combining Eqs. (20) to (24) are shown in the lower panel of Fig. 2. In this case, the equilateral configuration is favored while both the squeezed and folded configurations are subdominant.

To conclude, let us discuss two interesting limiting behaviors of the model. The first is the quasi-de Sitter approximation which, as mentioned before, is equivalent to having  $Y \ll 1$ . In this limit, and contrary to the single field slow-roll inflationary situation, Eqs. (13)–(15) show that large amounts of non-Gaussianities are produced in all possible shapes, with  $f_{\text{NL}} \propto Y^{-1} \gg 1$ . Thus, although large non-Gaussianities in inflation often stem from a violation of slow roll, in the bouncing case, the closer one is to a de Sitter bounce, the more non-Gaussianities are produced. The second limiting behavior is perhaps more relevant for comparison with observational data, as it is not based on any prerequisite regarding the structure of the bounce. As seen from Eqs. (13) to (15), the parameter  $f_{\text{NL}}$  is scale dependent, and in particular, is proportional to  $k^2$  in the equilateral and squeezed configurations. In a cosmological background with closed spatial sections and with a present

value of  $\Omega_{\mathcal{K}}$  of the order of  $10^{-2}$ , the mode numbers are, as discussed above, in the range  $[10^2, 10^8]$ , so the expected non-Gaussianities are predicted to be extremely large right after the bouncing phase. In both limits, the amount of non-Gaussianity produced greatly exceeds the current observational limits and the validity of the perturbative expansion may be brought into question. We conjecture that this is likely to be a generic and potentially serious problem for nonsingular bouncing cosmologies.

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