

Eruption versus intrusion? Arrest of propagation of constant volume, buoyant, liquid-filled cracks in an elastic, brittle host

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[1] When a volume of magma is released from a source at depth, one key question is whether or not this will culminate in an eruption or in the emplacement of a shallow intrusion. We address some of the physics behind this question by describing and interpreting laboratory experiments on the propagation of cracks filled with fixed volumes of buoyant liquid in a brittle, elastic host. Experiments were isothermal, and the liquid was incompressible. The cracks propagated vertically because of liquid buoyancy but were then found to come to a halt at a configuration of static mechanical equilibrium, a result that is inconsistent with the prediction of the theory of linear elastic fracture mechanics in two dimensions. We interpret this result as due to a three-dimensional effect. At the curved crack front, horizontal cracking is necessary in order for vertical propagation to take place. As the crack elongates and thins, the former becomes progressively harder and, in the end, impossible to fracture. We present a scaling law for the final length and breadth of cracks as a function of a governing dimensionless parameter, constructed from the liquid volume, the buoyancy, and host fracture toughness. An important implication of this result is that a minimum volume of magma is required for a volcanic eruption to occur for a given depth of magma reservoir.

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1. Introduction

[2] The initiation and propagation of magma-filled cracks is an important general topic, bearing on the mechanical and geochemical evolution of the lithosphere, rifting of the continental and oceanic crust, volcanic eruptions, or shallow intrusion of magmas. From a theoretical standpoint the subject is complex, involving fluid mechanics, solid deformation, fracture mechanics, and thermodynamic aspects such as thermal exchanges between magma and host rock and phase transitions within the magma [Lister and Kerr, 1991; Rubin, 1995]. This problem contains the same physics as other geological systems such as pockets of fluid released in sedimentary basins or injections of oil for the reactivation of a petroleum field. One cannot build a complete picture of behavior from geologic observations of frozen dikes exposed by erosion because no dynamic information is available [Delaney and Pollard, 1981; Jolly and Sanderson, 1995]. Also, one cannot make a complete picture from geophysical observations of actively propagating dikes because accurate information about dike shape

during propagation is not available [Brandsdóttir and Einarsson, 1979]. Laboratory experiments can bridge the gap between field observations and theory because both dynamic and geometric information can be acquired during crack propagation subject to well-known boundary conditions, enabling certain aspects of theoretical models to be tested. Nevertheless, in the limited size of the laboratory, nongeologic materials must be used which often have much simpler rheologies than magmas and host rocks. Here we describe new laboratory experiments on the propagation in a gelatine host of liquid-filled cracks of fixed volume. The purpose is to study some of the physics of dike propagation from a magmatic source driven by magma buoyancy, with a particular focus on the process of fracturing the host solid. One problem in nature is that the boundary conditions of magma supply from a source may vary considerably. When supply is long lived, a reasonable model approximation may be a constant flux. Alternatively, supply may cut off before a dike has reached the surface, in which case, one may consider a crack of fixed volume, and the details of establishing the crack are of secondary importance.

[3] We focus on the second case and particularly on the important unexpected result shown by our experiments that the cracks came to a halt after a regime of propagation under buoyancy became established. This result contrasts with the prediction of recent theoretical work in two dimensions and relates directly to the interesting volcanologic question of what determines whether a dike will propagate to the

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surface and feed an eruption or whether it will be emplaced as a shallow intrusion.

2. Experimental Approach

[4] We describe our experimental protocol in conjunction with Figure 1. The elastic host was prepared by pouring a gelatine solution into the tank and allowing it to set, typically for 48 h. Two copper plates fixed at the sides of the tank were heated up rapidly and removed from the tank once the gelatine had set, the remelted gelatine being replaced by cold water. Via this procedure, we obtained vertical, free surfaces at the edges of the tank parallel to the plane of the crack which reduced the resistance to opening because of the rigid tank walls. The densities of the gelatine and the water are very similar, so this technique does not perturb the initial hydrostatic condition. The initial pressure gradient is therefore hydrostatic everywhere. The temperature of the gelatine solid and the liquid injected into the crack were equal to that of the laboratory, which was controlled by an air-conditioning system. To avoid evaporation from the surface of the gelatine while it set to a solid, a thin layer of vegetable oil was placed onto the top of the gelatine solution just after it had been poured into the tank [Menand and Tait, 2002]. The authors demonstrate that on the time scale of the experiments gelatine behaves as a perfect elastic solid and gives an empirical relation between the fracture toughness and the Young's modulus,

$$K_c = 0.95\sqrt{E}. \quad (1)$$

The liquids used in the experiments were concentrated sugar solutions (dyed for purposes of visualization) and were denser than the gelatine. A crack was initiated by making a vertical cut in the upper surface of the gelatine and injecting the liquid into the crack. The initial length of the precut was a little greater than the buoyancy length (L_b),

$$L_b = \left(\frac{K_c}{\Delta\rho g} \right)^{2/3}. \quad (2)$$

This equation is derived by balancing the buoyancy pressure gradient ($\Delta\rho g$) against fracturing pressure on the basis of the concept of fracture toughness (K_c) in a two-dimensional geometry, i.e., for a crack of infinite length in the horizontal direction contained in the crack plane. A crack thus initiated in the experiments tended to spread first horizontally such that the breadth (b) increased faster than the length (l); subsequently, vertical propagation due to the negative buoyancy of the liquid took over, such that l increased with b remaining constant. Different initial aspect ratios were tested and were found not to influence the results. In one previous experimental study of constant volume cracks, Takada [1990] refers to a ‘‘critical crack height’’ that he interprets as the transition from a ‘‘subcritical’’ propagation regime to a brittle fracture; however, it appears likely to us that this critical height in fact corresponds to the buoyancy length, although not enough data is available in that paper for us to confirm this.

[5] We monitored the evolution of fissures by regularly taking photographs in the direction normal to the plane of

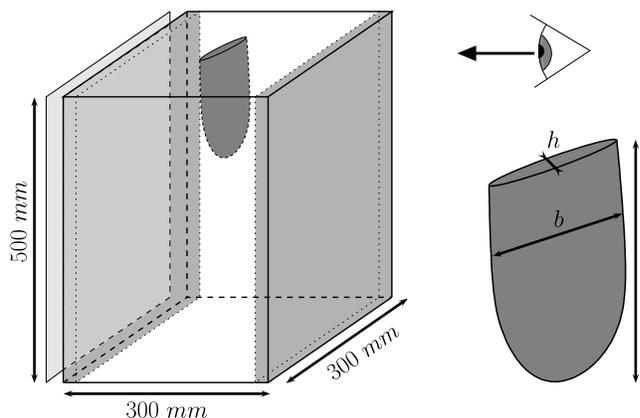


Figure 1. The experimental tank. Light grey represents the light table used for visualization. Dark grey at each side of the tank represents the free surface condition. These vertical sheets of water at the tank edges were each approximately 10 mm thick and were parallel to the plane of the crack (perpendicular to the direction of the opening).

the crack using a fixed camera. Time intervals between photographs could be flexibly adjusted during an experiment according to fissure velocity. The position and shape of each fissure as a function of time were later extracted by image analysis of the sequence of photographs. The photoelastic properties of the gelatine were exploited by placing sheets of polarizing film on either side of the illuminated tank. When gelatine is in a hydrostatic state of stress, one observes a homogeneous picture, but deviatoric stress in the gelatine makes it birefringent and gives rise to colored interference fringes that allow visualization of the stress field [Timoshenko and Goodier, 1970]. Table 1 lists all of the experiments and values of physical variables in each case.

3. Results

[6] In this paper, we emphasize the result that fissures stopped after propagating vertically over a certain distance, despite the fact that $l > L_b$. Figure 2 shows representative curves of crack length against time, which reveal three different sections: an initial phase during which the crack is establishing its shape, mostly by horizontal spreading; a second phase during which the vertical propagation of the crack closely follows a power law; and a final horizontal plateau which testifies that the fissures eventually stopped moving altogether. The theory of buoyancy-driven, two-dimensional crack propagation in the framework of linear elastic fracture mechanics predicts that once $l > L_b$, a fissure should propagate indefinitely in accordance with a power law $l \sim t^{1/3}$ [Roper and Lister, 2007]. In some experiments (with a large volume of fluid injected) this law gave a good fit, but in others exponents different from 1/3 were observed (Figure 2).

[7] Previous experimental studies reported that fixed volume cracks released in gelatine (as host solid) propagated at constant velocities [Takada, 1990; Heimpel and Olson, 1994; Dahm, 2000]; however, hydrophobic fluids (e.g., oils,

Table 1. Conditions for the Constant Volume Experiments^a

Experiment	V (10^{-6} m ³)	E (Pa)	$\Delta\rho$ (kg m ⁻³)	L_f (10^{-3} m)	B_f (10^{-3} m)
1	5	986	200	288	—
2	10	986	200	407	—
3	10	1224	180	250	—
4	10	2592	245	281	—
5	20	2592	245	434	—
6	30	4061	245	400	—
7	30	5352	240	380	116
8	10	5352	240	241	82
9	20	1544	103	215	74
10	15	1544	240	371	96
11	10	6950	240	176	67
12	44	2338	102	156 ^b	114
14	30	1787	102	148 ^b	81
15	26,1	2129	102	141 ^b	80
16	5	1903	373	359	62
17	3	1903	373	192	48
20	5	1843	255	169	58
22	6	2770	373	267	70,1
23	6	2770	373	280	72,5
24	1	2770	373	80	33
25	4	1102	373	302	60
26	10	3402	373	378	85
28	10	3402	186	187	76

^aWe chose different volumes of liquid and varied its buoyancy by changing the concentration of the sugar solution. We varied the fracture toughness of the gelatine by adjusting the concentration of the gelatine. L_f and B_f correspond to the final static length and breadth, respectively.

^bExperiments 12, 14, and 15 correspond to the injection of oil; in those cases, L_f corresponds to the head length.

alkanes, mercury, air) were used. For comparison, we carried out a few experiments with oils and also observed that each crack propagated at a constant velocity (Table 1). Our interpretation is that hydrophobic liquids are unable to wet a thin “tail” region of the cracks; hence, this region completely closes and expels the fluid. We confirmed this by our observations of color. Figure 3 presents an experiment performed with a hydrophobic fluid. We note that *Takada* [1990, Figure 3b] and *Heimpel and Olson* [1994, Figure 8]

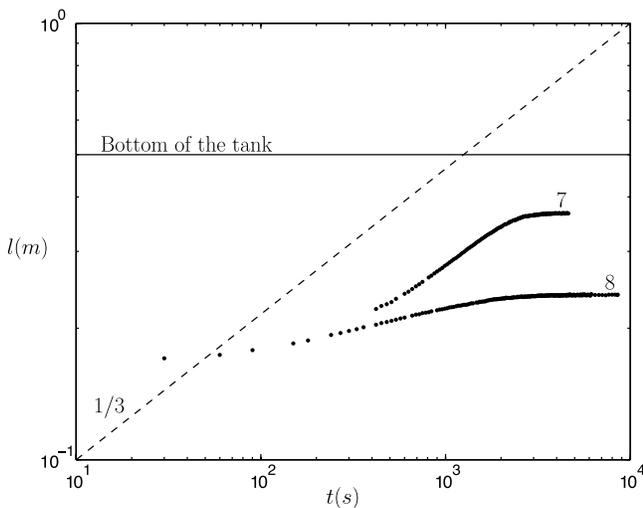


Figure 2. Temporal evolution of fissure length. The solid line represents the bottom of the tank and the dashed line represents the prediction of the two-dimensional theory, $l \propto t^{1/3}$. The number of the experiment is shown above each curve. See Table 1 for more details.

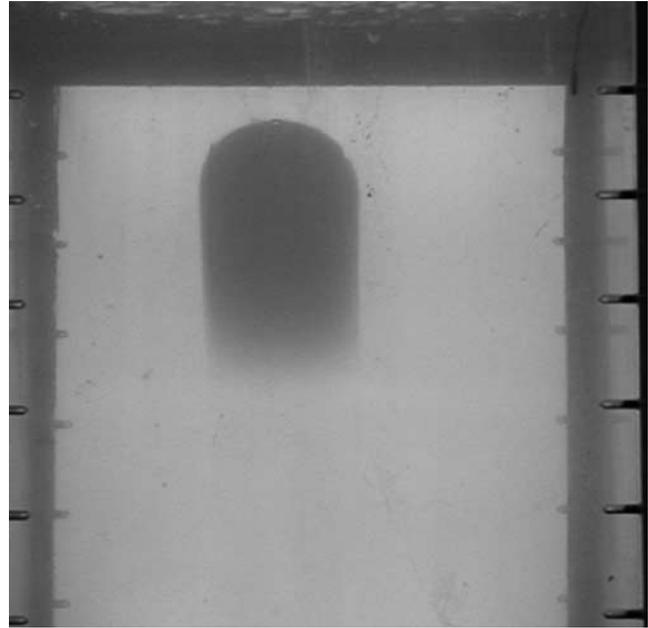


Figure 3. Photo of a fissure filled with a constant volume of oil (experiment 15). Note that unlike the fissures shown in Figure 4, the tail of the fissure is colorless and, thus, can be considered as completely closed. We attribute this to a high surface energy between oil and gelatine. For scale, the distance between two successive screws is 5 cm.

show the same thing. The crack can thus maintain a constant shape and move at a constant velocity. The shape of such cracks in which the tail pinches shut was analyzed in the pioneering work of *Weertman* [1971a, 1971b]. In the current experiments with aqueous solutions, the front of the crack steadily loses fluid to the lengthening tail region which does not pinch off (see Figure 4). Therefore, the balance between buoyancy and viscous forces steadily evolves to give a decreasing velocity. As shown by *Roper and Lister* [2007], reasoning with these two forces on the scale of the whole crack leads to the $t^{1/3}$ scaling law. The questions raised by these observations about crack dynamics will be addressed in more detail in another paper. We concentrate now on the striking observation that fissures came to a halt after propagation in the buoyancy-driven regime had become established. Photographs of an experiment taken in the direction parallel to the plane of the crack reveal the geometry of the stress lobes associated with the crack tip (Figure 5). The final photograph was taken when the crack had ceased propagating, but stress lobes are still present.

[8] The physical variables describing the system (K_c , $\Delta\rho$, and V being the fracture toughness, the density difference between the fluid and the solid, and the volume of injected fluid, respectively) can be combined into one dimensionless parameter, expressed here as dimensionless fracture toughness and defined as

$$\hat{K} = \frac{K_c}{\Delta\rho g V^{1/2}}. \quad (3)$$

[9] Table 2 shows the typical range of values for \hat{K} calculated for our experiments and for natural conditions. Note that in deriving \hat{K} , we made use of the fact that

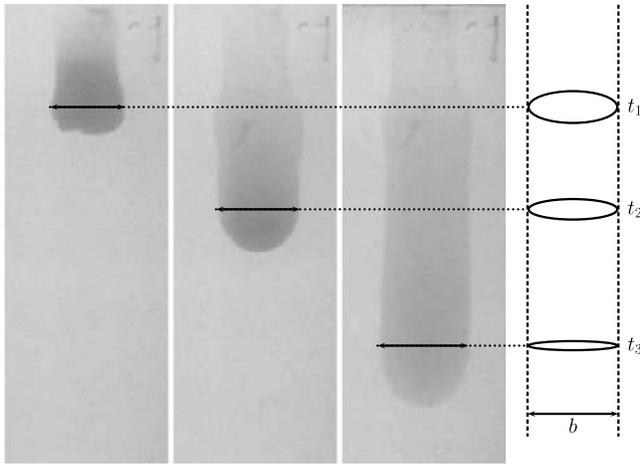


Figure 4. Photographs showing the evolution of fissure shapes for experiment 8. After an initial phase of horizontal spreading, propagation was essentially vertical. Color intensity (seen here in terms of tones of grey) generally decreased from the tip to the tail and gave an idea of the thickness of the crack. A schematic representation of the horizontal cross section is given on the right. Note the fact that the crack thins as it elongates.

gelatine behaves, on the time scale of the experiments, as a brittle elastic solid, such that there is a direct relationship (equation (1)) between the elastic modulus and the fracture toughness [Menand and Tait, 2002]. Hence, these two parameters are not independent, and one less physical variable is necessary for dimensional analysis. We eliminated Young’s modulus and retained fracture toughness. For a given natural system \hat{K} can be seen as a ratio between a constant (whose value is fixed by the fracture toughness, the density of the surrounding medium, and the density of the magma) and the volume of injected magma. The volume is

the key parameter that can change between two successive magmatic pulses. Final lengths (Figure 6a) and final breadths (Figure 6b), made dimensionless with the buoyancy length, are shown as a function of \hat{K} . These results can be expressed in terms of simple laws for the dependence on \hat{K} of final lengths and breadths of cracks:

$$\frac{L_f}{L_b} \sim 54 \hat{K}^{-3/2} \tag{4}$$

$$\frac{B_f}{L_b} \sim 5 \hat{K}^{-5/6}. \tag{5}$$

Assuming that the volume of the crack can be expressed as $V = L_f B_f H_f$, the combination of equations (4) and (5) leads to the following expression for the final width:

$$\frac{H_f}{L_b} \sim \frac{1}{54 \times 5} \hat{K}^{1/3}. \tag{6}$$

4. Discussion and Volcanologic Application

[10] The final static state revealed by our results poses an intriguing question. Viewed in a two-dimensional geometry, once a crack is vertically long enough that the buoyancy is sufficient to fracture the elastic host, it should not stop moving [Roper and Lister, 2007]. Viscous resistance to motion causes cracks to decelerate but should not cause them to stop completely. An interaction with the rigid base of the tank can also be excluded as the reason for the arrest of cracks, first, on the grounds that fissures stopped moving at very different distances from that tank bottom (obtained in Table 1 by subtracting the final length from the total height of the tank, 0.5 m) and, second, because of the

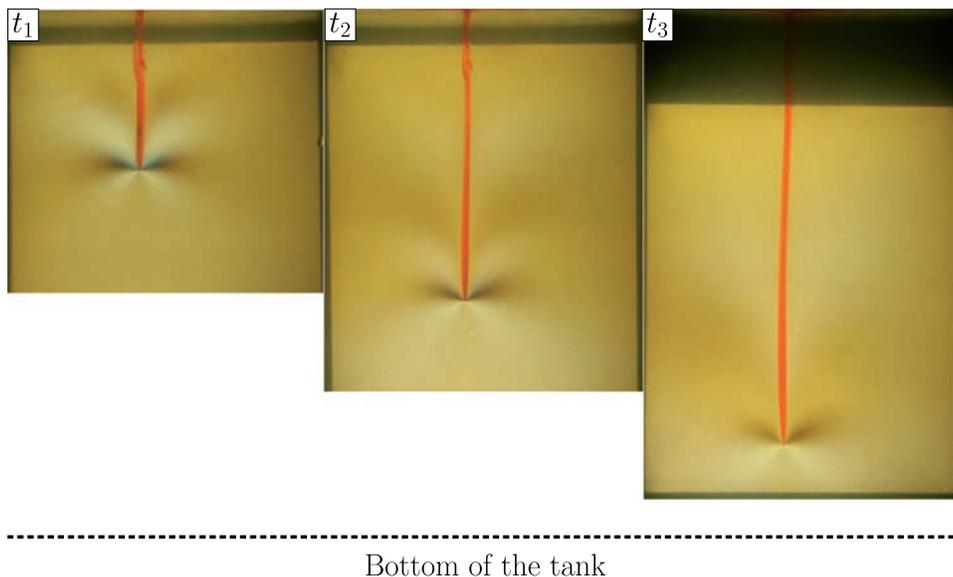


Figure 5. Photographs showing the evolution of the photoelastic fringes at the tip of a constant volume of fluid released in an elastic medium. Note that on the last photograph, despite the fact that the fissure has permanently stopped moving, we nevertheless observe photoelastic fringes.

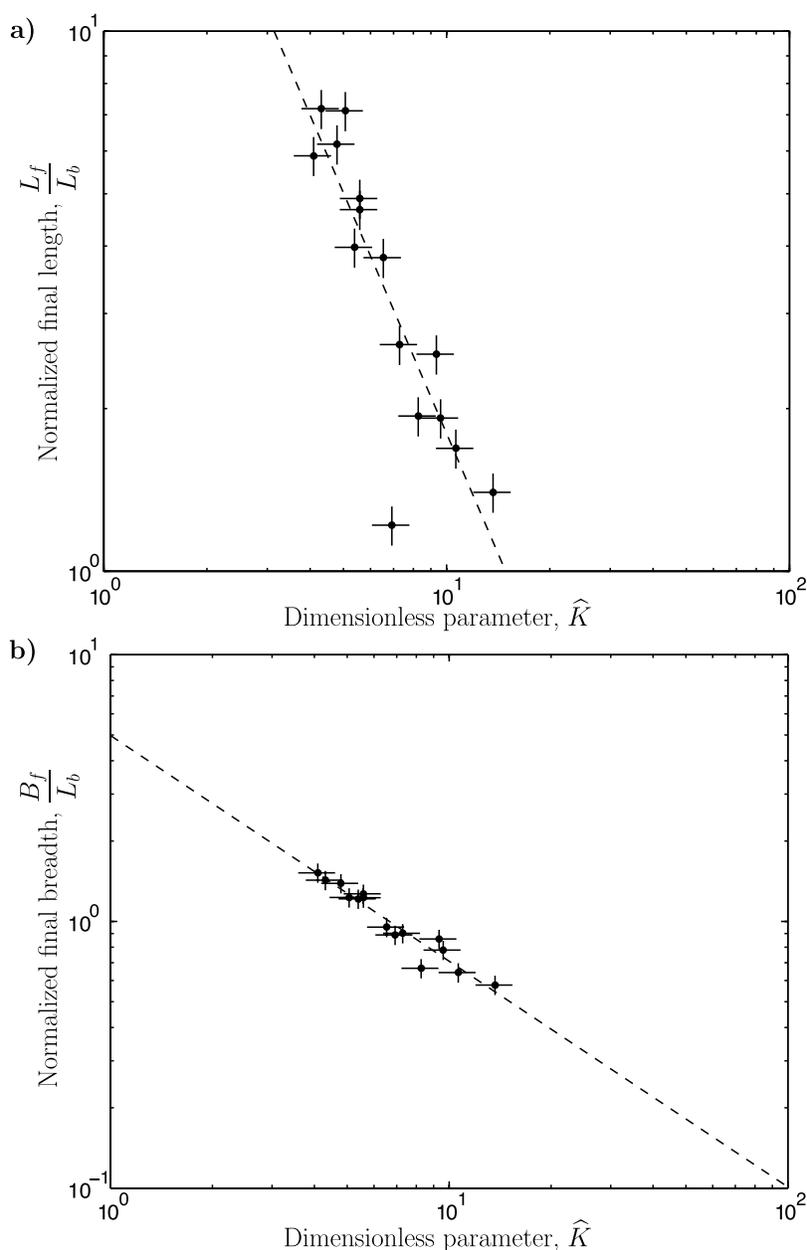


Figure 6. (a) Final length normalized by the buoyancy length ($\frac{L_f}{L_b}$) as a function of dimensionless fracture toughness (\hat{K}). The dashed line represents a slope of $-3/2$. (b) Final breadth normalized by the buoyancy length ($\frac{B_f}{L_b}$) as a function of dimensionless fracture toughness (\hat{K}). The dashed line represents a slope of $-0.83 \sim -5/6$.

consistent behavior of the whole set of results expressed by Figures 6a and 6b. The fact that the elastic fringes visible in Figure 5 are not interacting with the bottom of the tank is another argument to exclude the potential influence of the proximity of this rigid boundary. The experiments of *Takada* [1990] and *Heimpel and Olson* [1994], performed with fluid denser than gelatine and referred to in section 3, do not show an arrest close to the bottom of the tank, which would be expected behavior if the distance to the rigid bottom boundary were to play an important role. The experiments involving hydrophobic fluids and gelatine produce results that are unlikely to be relevant to the case of magmatic dikes. This is because a thin zone of frozen magma should always form in contact with the host rock;

hence, we should not expect a surface energy effect to be present.

[11] Our physical interpretation is that in the final state, crack buoyancy is indeed balanced by the fracture tough-

Table 2. Value of the Dimensionless Parameter \hat{K} for Natural and Experimental Systems^a

	Natural Systems	Experimental Systems
K_c in Pa m ^{1/2}	10^6	10
$\Delta\rho$ in kg m ⁻³	10^2	10^2
V in m ³	10^5 – 10^7	10^{-6} – 10^{-4}
\hat{K}	0.3–3	1–10

^a \hat{K} can be thought of as a dimensionless fracture toughness. The value of K_c for natural rocks comes from *Atkinson* [1984, Table 2].

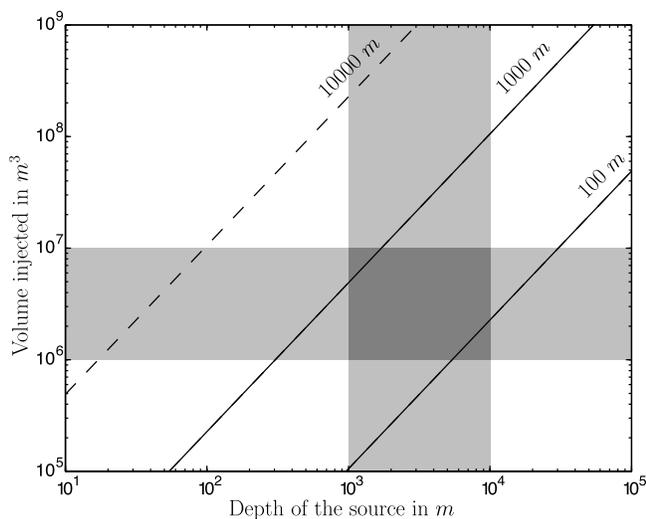


Figure 7. Relationship between minimum eruptible magma volume and final fissure length. The lines correspond to three different values of the buoyancy length and are calculated using equation (7) assuming that the final length (L_f) equals the depth of the source. Shaded areas correspond to typical observations of emitted volume and a range of estimated depth of the source at Piton de la Fournaise volcano, Reunion Island [Peltier *et al.*, 2009]. If we assume that the smallest emitted volumes are close to the minimum eruptible volume, then L_f should be close to the depth of the magmatic source. Values of minimum erupted volumes and the depth to the source for this example correspond to a value of the order of 100 m for the buoyancy length, a reasonable value for the values of physical quantities at this kind of volcano.

ness of the gelatine but that we must take into account the three-dimensional geometry of the crack. We propose that the effective fracture toughness of the gelatine changes as fracture shape evolves. As the tip of a crack propagates, the trailing tail or body of the crack never completely closes. This is expected from theory, but we also confirmed this experimentally from the distribution of color in the crack (Figure 4). The front of the crack has some curvature, which means that in order for the crack to advance vertically, cracking of the gelatine in the horizontal direction must also take place. The elastic stress available to achieve this is proportional to the aspect ratio (h/b) of the fissure in a horizontal cross section (Figure 4). Each crack contains a finite volume of fluid; hence, as the cross-sectional area of the crack (viewed in the crack plane) increases, the crack thickness h must decrease, and the ratio h/b should also decrease. Eventually, the stress available should become insufficient to crack the host horizontally. At this point, the fissure can no longer maintain its width by horizontal cracking; hence, it cannot advance vertically either, and it is forced to a halt. Quantitative modeling of the propagation of a crack in three dimensions is difficult. Our physical arguments remain qualitative for the moment. Nevertheless, the fact that all of the data are well correlated with one dimensionless number involving just fracture toughness

(K_c), buoyancy ($\Delta\rho g$), and volume (V) places a strong constraint on other interpretations that might be proposed.

[12] Our results suggest that natural magmatic dikes may be stopped in the crust before arriving at the Earth's surface, depending on the relative magnitudes of buoyancy, fracture toughness, and magma volume. Although for magmatic dikes this result will be quantitatively modified by thermal effects and changes of buoyancy that can occur, these differences will not alter the fundamental prediction that dikes may stall despite being longer than the buoyancy length. All else being equal, the smaller the volume of magma, the more likely it is that the dike will stall. We now use our experimentally derived scaling law to estimate the order of magnitude of this effect and to compare it to natural observations. We express the result as a minimum volume of magma necessary in order for an eruption to occur from a magma chamber at a given depth, i.e., such that L_f is greater than the depth to the magma chamber. For fixed buoyancy and (two-dimensional) host fracture toughness, the volume of magma must be greater than some threshold to be eruptible. In Figure 7, we show the minimum volume of magma that we would expect to be eruptible as a function of magma chamber depth for the given rock fracture toughness and magma buoyancy. Figure 7 is obtained by combining equations (2), (3), and (4) as follows:

$$L_f \sim 54 \frac{V^{3/4}}{L_b^{5/4}}. \quad (7)$$

The observations available in the literature are, unfortunately, sparse, probably because determining the volume of the smallest eruption from a given volcano is not a priority. Nevertheless, the values obtained on the basis of our model are plausible in light of the data available for the well-studied system of Piton de la Fournaise [Peltier *et al.*, 2009].

[13] The propagation of a dike close to the surface raises the question of cohesive strength of the ambient host rock due to the decreasing value of the lithostatic pressure. Such effects have been treated numerically [Agnon and Lyakhovskiy, 1995]. The fact that the excess pressure at the tip of the propagating dike can become greater than the cohesive forces implies a change in the dynamics of propagation which can lead to a storage at shallow depth and the formation of a graben-like structure. This storage prevents eruption at the surface by a deviation of the dike trajectory rather than an arrest of vertical propagation as shown in this paper. Nevertheless, the present study does not take into account this shallow effect which will increase the value of the minimum volume injected for an eruption to occur. Another effect that will tend to increase the minimum volume is the freezing of magma at the dike tip.

5. Conclusion

[14] When a magma reservoir cracks and magma is released toward the surface, a key issue is whether a volcanic eruption will ensue or whether a shallow intrusion of magma will take place. Aborted eruptions are probably not uncommon, but physical models that give a basis to predict whether eruption or intrusion will occur have not

been well developed. In this paper, we describe a novel effect that can stop the progression of a propagating crack that arises in the case of buoyant cracks of finite volume, and we suggest that this is due to an effective three-dimensional fracture toughness of the host medium. This predicts that a minimum volume of magma must be released into a crack in order for an eruption to occur, although the precise values are expected to be modified in the natural application because of the additional complexities of thermal effects and volatile exsolution.

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