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Local disalignment can promote coherent collective motion

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Abstract. When particles move at a constant speed and have the tendency to align their directions of motion, ordered large-scale movement can emerge despite significant levels of noise. Many variants of this model of self-propelled particles have been studied to explain the coherent motion of groups of birds, fish or microbes. Here, we generalize the aligning interaction rule of many classical self-propelled particle models to the case where particles after the interaction tend to move in slightly different directions away from each other, as characterized by a deflection angle α. We map out the resulting phase diagram and find that, in sufficiently dense systems, small local disalignment can lead to higher global alignment of particle movement directions. We show that in this dense regime, global alignment is accompanied by a grid-like spatial structure which allows information to rapidly percolate across the system by a 'domino' effect. Our results elucidate the relevance of disalignment for the emergence of collective motion in models with repulsive interaction terms.

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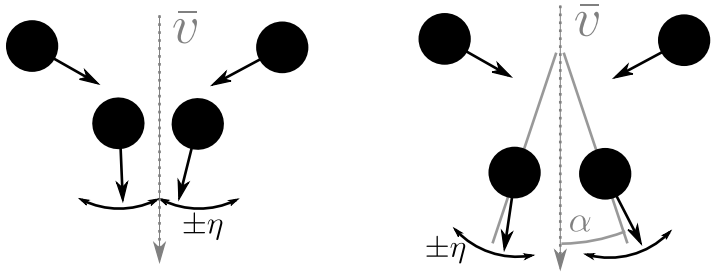
1. Introduction

The emergence of ordered motion in groups of interacting particles that move at a constant speed is reminiscent of the collective motion observed in many animate and inanimate systems [2, 9]. A wide variety of different models of such self-propelled particles (SPPs) have been explored with the goal of quantifying conditions for the global alignment of the movement directions of individual particles. These models have in common that they rely on local interaction rules, as they are thought to apply to many animal swarms [2–8], and that the movement directions of particles are continually perturbed by random noise.

Among the first models that have been developed to describe collective motion [9], the SPP model due to Vicsek *et al* [1] is particularly elegant and simple. It crucially relies on an explicit alignment interaction that adjusts each particle's movement direction to the average direction of its surrounding particles before a random perturbation is added, see figure 1(a). For sufficiently weak noise levels, coherent collective motion results from the local interaction rule by which particles self-organize into large groups, or 'flocks', moving in the same direction. For strong noise levels, the system inevitably fails to order globally. The amount of coherent collective motion can be measured by an 'order parameter' defined as the magnitude of the globally averaged velocity vector. As we change from strong to weak noise levels, the system undergoes a change from a disordered to an ordered phase. While an analogue of an equilibrium phase transition is obtained in the limit of zero velocities, the order-to-disorder transition is generally a unique non-equilibrium phenomenon as it is driven by the perpetual motion of the interacting entities [1]. Similar phase transitions are observed in variants of the classical Vicsek model that add cohesive and repulsive interaction terms [2, 6, 7, 10].

In contrast to the Vicsek model and its variants, a second group of SPP models [11] does not introduce an explicit alignment but only an isotropic repulsive force, repelling nearby particles. Surprisingly, an ordered phase can be observed even then: the perpetual motion of the SPPs leads to a weak alignment through each interaction and, when the effects of enough interactions are accumulated, order emerges given weak enough noise [2, 11].

The observation of local alignment causing global alignment, replicated many times, suggests that higher local alignment will always lead to stronger ordering. Furthermore, one might think that the Vicsek model, optimally aligning the SPPs locally, exhibits the highest



(a) Vicsek (b) Disalignment

Figure 1. SPP models describe the coherent motion of particles that move at a constant velocity and interact when they come close. In the classical model (a) due to Vicsek *et al* [1], interacting particles align their direction of motion. The gray dashed line in the figure indicates the direction of the mean movement velocity $\overline{v}$ of the particles prior to the interaction. The direction of motion of particles after the interaction is given by this mean velocity rotated by a random angle chosen from the interval $[-\eta, \eta]$. Hence, with equal chance, particles move toward and away from each other after the interaction. (b) Here, we study a more general model in which the velocity directions after the interaction deviate by an angle α from the averaged direction before being perturbed by noise. The case $\alpha = 0$ leads to the original Vicsek model. For $\alpha > 0$, particles tend to move away from each other.

levels of global order among all models with the same noise strength, particle density and interaction range.

We demonstrate in the following that, contrary to this intuition, local disalignment can even enhance global order. To show this, we generalize the Vicsek interaction rule such that the average velocity vectors after an interaction diverge by a small angle α. The deflection angle α is chosen to point away from the center of mass of the interacting particles, which results in a repulsive interaction, see figure 1(b). For any finite deflection angle, our interaction rule tends to reduce local order compared to the classical Vicsek model. Yet, our numerically determined phase diagram shows that *global* orientational order can be increased for a non-zero disalignment, for certain densities and noise levels. We argue that this effect is ultimately the manifestation of reduced density fluctuations in the presence of disalignment, which leads to the formation of a more efficient mechanism of information transmission across the system.

2. The disalignment model

Our model is a generalization of the classical Vicsek model [1]. In two dimensions, the orientation of each particle can be characterized by an angle Φ, measured in the counter-clockwise direction. In each timestep, the angle Φ_i corresponding to a focal particle i is updated according to the rule

$$\Phi_i(t + \Delta t) = \overline{\Phi}_i^{(r)}(t) \ \pm\alpha \ + \Delta\Phi \tag{1}$$

as illustrated in figure 1(b). Here, the angle $\overline{\Phi}_i^{(r)}(t)$ characterizes the average orientation of all particles within a circle of radius r centered at the focal particle. The random noise term $\Delta\Phi$ is

chosen uniformly from the interval $[-\eta, \eta]$. Both the level of noise η and the deflection angle $0 \leqslant \alpha \leqslant 180^\circ$ are measured in arcdegrees. All angles are measured in the counter-clockwise direction. Finally, the parameter $\alpha \geqslant 0$ is the deflection angle—the new parameter of our model. The sign in front of α is chosen such that the particles tend to move away from the line defined by the center of mass and the average movement direction of all particles inside the interaction radius[4]. As a consequence, the α term drives particles away from one another, as illustrated in figure 1(a). Note that the original Vicsek model is obtained when the deflection angle α is set to zero. The parameter α can also be viewed as a tuning wheel by which the symmetry of the unbiased alignment interaction can be broken systematically.

The position $\vec{x}_i$ of particle i is consequently updated as

$$\vec{x}_i(t + \Delta t) = \vec{x}_i(t) + v \begin{pmatrix} \cos(\Phi_i(t + \Delta t)) \\ \sin(\Phi_i(t + \Delta t)) \end{pmatrix} \Delta t. \tag{2}$$

We choose the magnitude of the particle velocity constant as $v = 0.1$ (except figure 4), the timestep $\Delta t = 1$ and the interaction radius $r = 1.0$. The N particles move in a square cell with periodic boundaries of length L. The particle density is given by $\rho = N/L^2$.

We measure the degree of global alignment by the order parameter φ,

$$\varphi = \frac{1}{Nv} \left| \sum_i^N \vec{v}_i \right| , \tag{3}$$

which represents the normalized average particle velocity.

3. Snapshots and phase diagrams

After randomizing the initial particle positions and orientations, the particles start to move and interact. For small deflection angle α, each interaction tends to align particles that come close. In the original Vicsek model, dense groups of aligned particles form and move jointly through the system [1], see figure 2(a). The abundance of particles within an interaction range inside a dense group reduces the effect of noise by averaging the movement directions of many particles. Depending on the noise strength and density, these groups can further align among each other, thus leading to a non-zero order parameter φ. The smaller the number and size of the groups, the less frequent are the interactions between them.

As one increases α to 1°, the formation of dense groups is suppressed due to the repulsive effect of the disalignment interaction. More loosely connected groups form instead, which occupy a much larger area of our simulation box. On the one hand, this leads to more frequent interactions between clusters, as can be seen in figure 2(b). On the other hand, the number of particles within an interaction range decreases inside clusters, thus amplifying the effect of noise.

As we further increase α to 10°, dense groups become very rare and a grid-like structure forms that spans most of the simulation area (figure 2(c)). Such a system spanning grid can, however, only form for large enough mean densities, $\rho \gtrsim 1$. For very low densities, the repulsive interaction disintegrates any cluster of particles such that solitary particles move in random directions. We also note that at very high densities, several particles can occupy a single grid site (see appendix B).

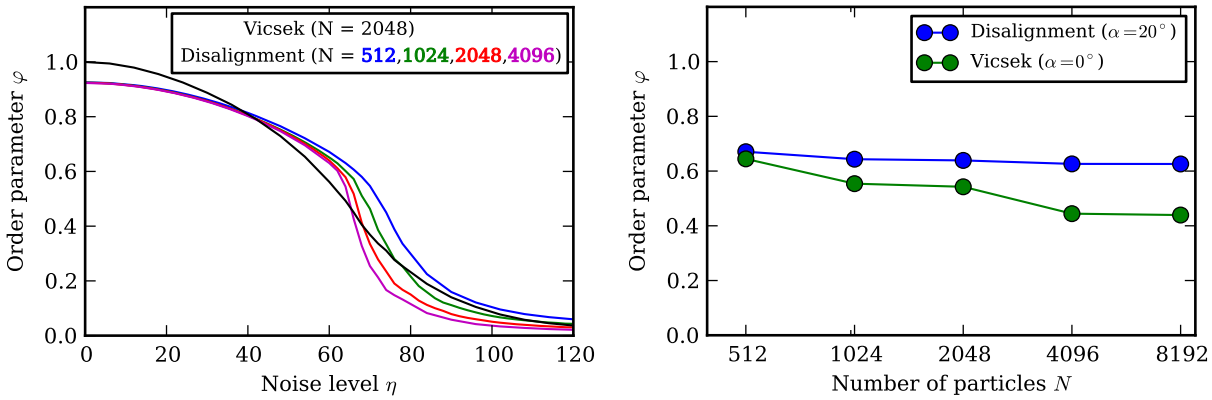
The phase diagrams in figures 3–5 summarize the behavior of our model. The first plot (figure 3(a)) shows the behavior of the order parameter φ as a function of noise level η for the

[4] We set $\alpha = 0$ when a particle is alone within its interaction range.



(a) $\alpha = 0°$ (b) $\alpha = 1°$ (c) $\alpha = 10°$

Figure 2. Characteristic snapshots of the distribution of particle positions and movement directions (arrows) in our disalignment model. (a) For vanishing deflection angle α, the original Vicsek model is obtained with its characteristic flocking structure. As the deflection angle is increased (b) and (c), the distribution of particles changes markedly—density fluctuations are suppressed. As we argue in this paper, the resulting homogeneous distribution of particles leads to a change in the mechanism that drives global order (other simulation parameters are $N = 2048$, $\rho = 1.0$, $\eta = 25°$ and $v = 0.1$).



(a) (b)

Figure 3. Phase diagrams summarize the global orientational order in our disalignment model, and show that collective motion can be promoted by a small degree of disalignment. (a) Order parameter φ as a function of noise η in the classical Vicsek model ($\rho = 2.0$, $\alpha = 0°$, $v = 0.1$ black) and the disalignment model with different particle numbers N ($\rho = 2.0$, $\alpha = 20°$, $v = 0.1$ colored). For medium noise levels, the disalignment model has a higher order parameter and exhibits a sharper transition to the disordered phase. The effect seems to be stable for large system sizes. In (b), we display the order parameter φ as a function of the number of particles N for fixed noise strength, $\eta = 60°$, which is within the zone where the disalignment model exhibits higher order than the Vicsek model (parameters: $\rho = 2.0$, $v = 0.1$). Again, system size does not seem to change this effect.

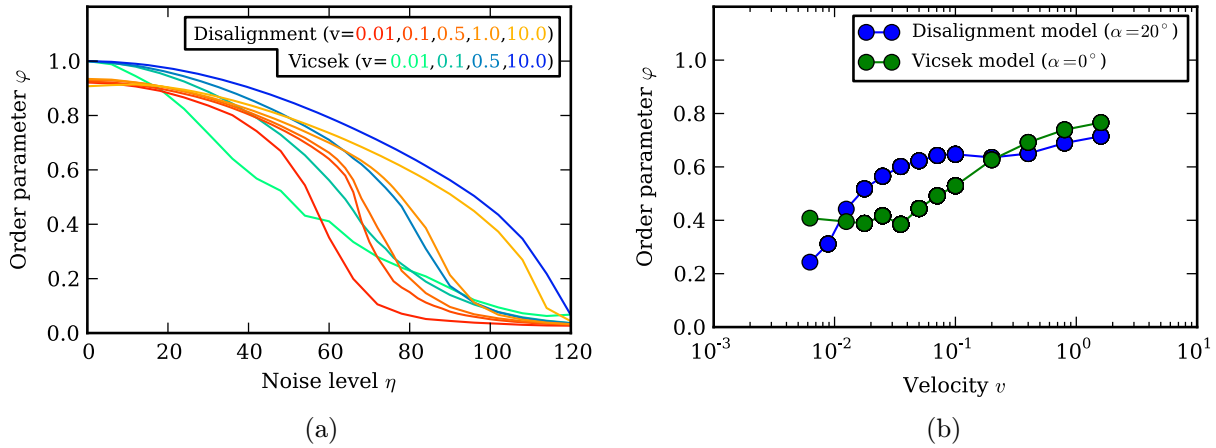


Figure 4. As in figure 3(a), we display the order parameter φ as a function of the noise level η ($N = 2048$, $\rho = 2.0$), but this time for a set of different particle velocities. (a) The results of the original Vicsek model ($\alpha = 0^\circ$) are shown for different velocities in colors from turquoise to blue (turquoise: small; blue: large). The red to orange curves correspond to the disalignment model ($\alpha = 20^\circ$) for different velocities (red: small; orange: large). Note that the order enhancing effect of the disalignment interaction is confined to the low-velocity regime with $v = 0.1$ and 0.01. In (b), we show the development of the order parameter φ as we change v for $\eta = 60^\circ$ which is close to the point of maximum gain of order through the disalignment interaction for the parameters $N = 2048$ and $\rho = 2.0$. The Vicsek model leads to greater order for velocities of $v \gtrsim 0.2$.

original Vicsek model with $\alpha = 0$ and for finite disalignment with angle $\alpha = 20^\circ$ and different particle numbers N. Both systems are highly ordered for small noise and become disordered as noise levels are increased. At zero noise, the Vicsek model approaches perfect global order with $\varphi = 1$ while the disalignment model retains less order, as one might expect. At medium noise levels, however, the disalignment rule for $\alpha = 20^\circ$ leads to higher global order than for $\alpha = 0^\circ$ for all particle numbers N. This counterintuitive behavior at intermediate noise levels is the focus of our discussion below. The effect seems to be stable for large system sizes as can be seen from figure 3(b). Also note that the transition from the ordered to the disordered phase appears to be sharper for disalignment than for alignment, all the more so for larger system sizes (see figure 3(a)).

As can be seen from figures 4(a) and (b), the impact of disalignment on order also depends on the velocity of the particles. At large speeds, the disalignment model closely follows the behavior of the Vicsek model albeit at slightly lower values of the order parameter. The effect of disalignment seems to be weak in this 'mean-field' regime, probably because the effect of disalignment is averaged out since particles interact with many other particles. The structural differences of the two models, which is quite striking at low speeds (cf figure 2), therefore vanish and disalignment tends to decrease order. At speeds $v \lesssim 0.1$, the disalignment interaction results in a sharper transition from order to disorder as a function of noise (cf figure 4(a)) and the appearance of an optimal disalignment angle (which itself is a function of the noise strength). Lower speeds shift the critical noise strength to lower values in both models.

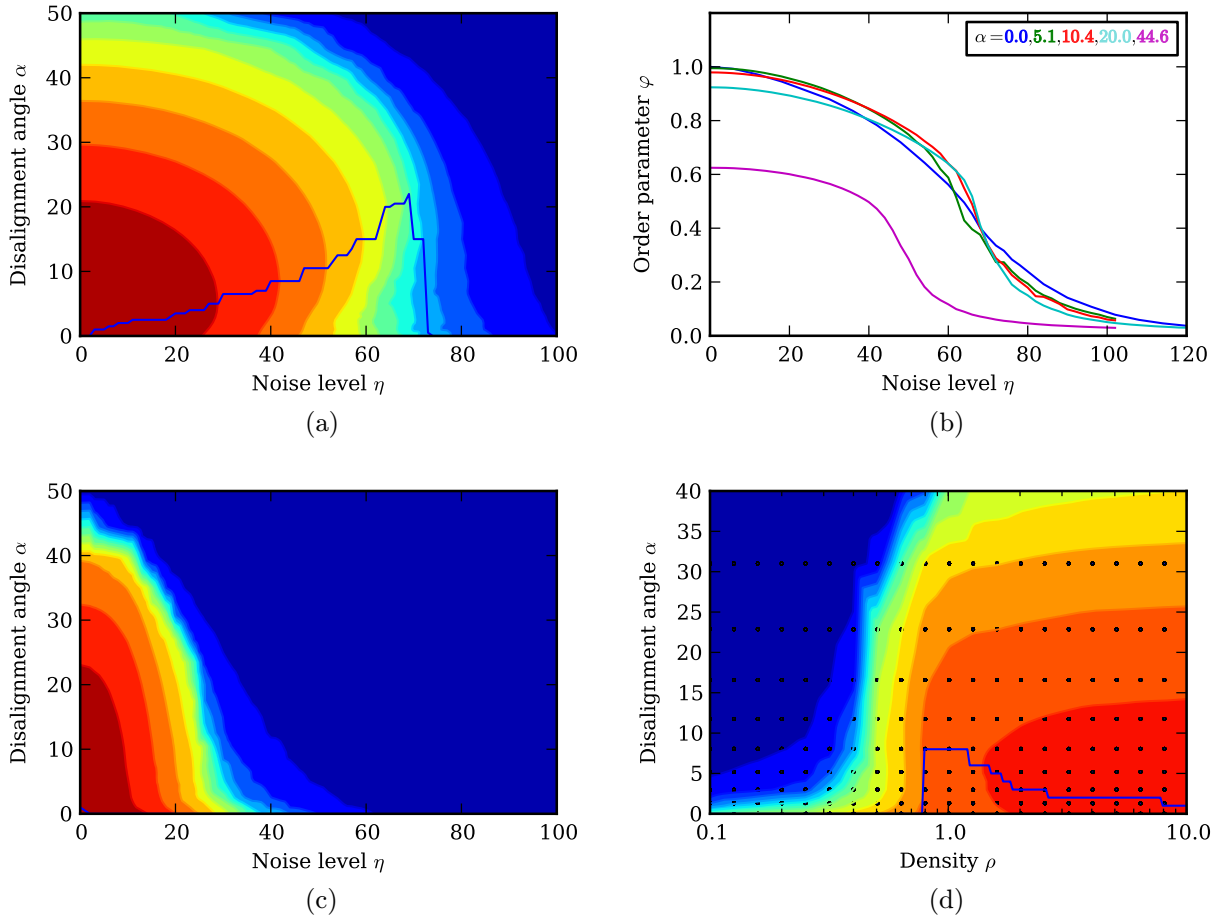


Figure 5. Two-dimensional (2D) phase diagrams. In the heat plots (a)–(c), the order parameter φ is indicated by color (red = 1, blue = 0) as a function of deflection angle α and noise η (a, b) and as a function of α and ρ (d), respectively. Panels (a) and (b) differ in their densities (a) $\rho = 2.0$; (b) $\rho = 0.5$). In (c), the noise level was fixed at $\eta = 20^\circ$. The optimal deflection angle (maximum in the vertical direction) is indicated by the blue line. Measurement points are shown in (c) as black dots. Other parameters are $N = 2048$ and $v = 0.1$. Panel (d) shows various one-dimensional horizontal cuts through (a) for fixed angles to illustrate the change of the phase transition curve.

The heat plot in figure 5(a) shows a 2D phase diagram, in which the order parameter is indicated as a function of both the noise level and the deflection angle, for a similar density as in figures 3 and 4. Slices of figure 5(a) at various fixed deflection angles are shown in figure 5(b). Again, the asymptotic behavior follows intuition: high noise levels and large deflection angles together prevent order. Highest order is achieved for zero noise levels and zero deflection angles. The intermediate behavior, however, again shows the surprising phenomenon of an 'optimal' deflection angle that leads to the highest order for a given noise level η. This angle is indicated as a blue line and increases with increasing noise levels.

The simulations indicate that disalignment can only promote global alignment when densities are of order $\approx$1 or larger. For lower densities, the order parameter is always largest for vanishing disalignment, as can be appreciated from figure 5(b) ($\rho = 0.5$). At these densities,

smaller noise levels suffice to break down order, even more so for finite deflection angles. The heat plot in figure 5(d) depicts this dependence of the order on both the deflection angle and the particle density, for a fixed level of noise. As one increases the density of the system, an optimal deflection angle appears at $\rho \approx 1$, and decreases as one further increases the density of the system. Our results on the phase behavior together with the structurally different collective motion visible from the snapshots in figure 2 suggest that the spatial distribution of the particles, in dense groups or a grid-like structure, could play a crucial role in explaining our results.

4. Information transmission

4.1. Two mechanisms of information transmission

In models of collective motion, global coherent order emerges from an interplay between aligning and random disaligning forces. The aligning forces allow orientational information to be transmitted from one particle to its close neighbors. The global effect of these driving forces on order does not only depend on the degree of alignment in an interaction. It also depends to a large extent on the ability of the system to exchange information about movement directions between all particles: a particle that never interacts with a group of aligned particles will never align with them, no matter how strong the aligning force might be. From this view, low density therefore should generally decrease the order of a system because the number of interactions between particles is reduced such that information about the orientation of a particle travels more slowly through the system while noise deteriorates the information during the transmission. This behavior has often been observed in previous studies [1, 12, 13] and also characterizes our model, see figure 5(d).

In models of collective motion, information about movement directions can spread through two distinct mechanisms [14].

1. Neighboring particles closer than the interaction distance directly adjust their movement directions with respect to one another. The orientation of particles within a connected cluster can be aligned after a few timesteps by this direct interaction through a 'domino' effect. First neighbors within an interaction range align and encode the information about the directions of their neighbors in their own new direction. In the next step, they propagate this information to their nearest neighbors such that information spreads until there are no more new particles within an interaction range of the considered cluster of particles. While the positions of all particles change during this process, the information transmission does not rely on particle movement (in contrast to the second mechanism of information transmission, below)—it is also present in the limit of zero velocities. In a grid-like structure, such a 'domino' network is further characterized by the large number of different chains that connect two particles.

2. The second type of information flow in models of collective motion depends on the movement of particles. This property fundamentally distinguishes SPP models from equilibrium models. A particle can move to a distant location and exchange orientational information with a particle there. Or in the context of the whole system, a particle can be used as a 'messenger' that exchanges orientational information between two or more other particles. In contrast to the above described direct interaction, this mechanism works over large distances even without a continuous chain of neighbors that are separated by at most one interaction distance. However, since information deteriorates by noise while a particle is on its way, single particles can transmit information only over short distances. Therefore, dense groups of interacting particles, or 'flocks', play an important role for preserving orientational information against randomization

through noise: a group continually averages the orientations of all of its members and can thus retain its average velocity over longer times and travel distances. A clustered structure as in the original Vicsek model is, therefore, crucial for this type of long distance information transmission [3, 15–17]).

Information transmission through the directed motion of SPPs is suppressed in the presence of a disalignment rule because the formation of groups is hampered by the repulsive character of the interaction. On the other hand, the disalignment model generates a grid-like structure at large enough densities ($\rho > 1$), which allows the whole system to receive orientational information through the first 'domino' mechanism. Information can then percolate through the system in a similar manner as in the equilibrium 'XY' spin model.

When the densities are decreased, a point is reached where the grid structure breaks apart. We hypothesize that order then down because there are not enough neighbors to ensure a continuous chain of information transmission.

4.2. The maximum speed of information transmission

We expect that the domino mechanism of information transmission is typically much faster than information transmission via moving flocks, simply because the domino effect is independent of the slow motion of the particles. Thus, the speed of information transmission may be a useful quantity to discriminate the structure of the interaction network generated by our model.

To put an upper bound on the speed of information transmission, we measured how quickly a particle interacts with all other particles through a chain of successive interactions. This can be done by choosing a focal particle and following through time how other particles become influenced via such an interaction chain to this particle. Operationally, we define the set of 'influenced' particles as follows: (i) At time t_0 no particle is influenced by the focal particle, yet. (ii) At $t > t_0$, particles become influenced by the focal particle if they interact either directly with the focal particle, or indirectly via an already influenced particle. The number of influenced particles increases over time as interactions occur. Note that we do not measure the magnitude of the influence the focal particle exerts on other particles but only how quickly influence can spread. Thus, we obtain an upper limit for the speed at which the focal particle is able to transmit information to all other particles in the population.

In the snapshots of figure 6, the color of a particle codes for the time until that particle was influenced by a randomly chosen focal particle for a single realization of (a) the Vicsek model and (b) the disalignment model with $\alpha = 20^\circ$, where the focal particle is marked by a red dot. Particle positions are shown at time t_0. Note that the influence of the focal particle spreads much more quickly in the presence of disalignment. Furthermore, the influence spreads from group to group in the Vicsek model. In the disalignment model, on the other hand, the influence of the focal particle spreads in a wave-like manner and reaches the last particles much faster. Note that our measure of the speed of network formation does not entail any statement about the amount of transmitted information, which decreases through the continual perturbation by noise. Nevertheless, it demonstrates the qualitative difference between the two mechanisms of information transmission that we described above.

Figure 6(c) shows the measured distributions of influence times obtained from many runs by averaging over the choice of the focal particle and the influenced particle. Note that the difference between the case with disalignment and with strict alignment is particularly prominent at $\rho \approx 1$, where the Vicsek model exhibits a broad class of particles that need many timesteps to become connected to the focal particle. The distribution is peaked at the

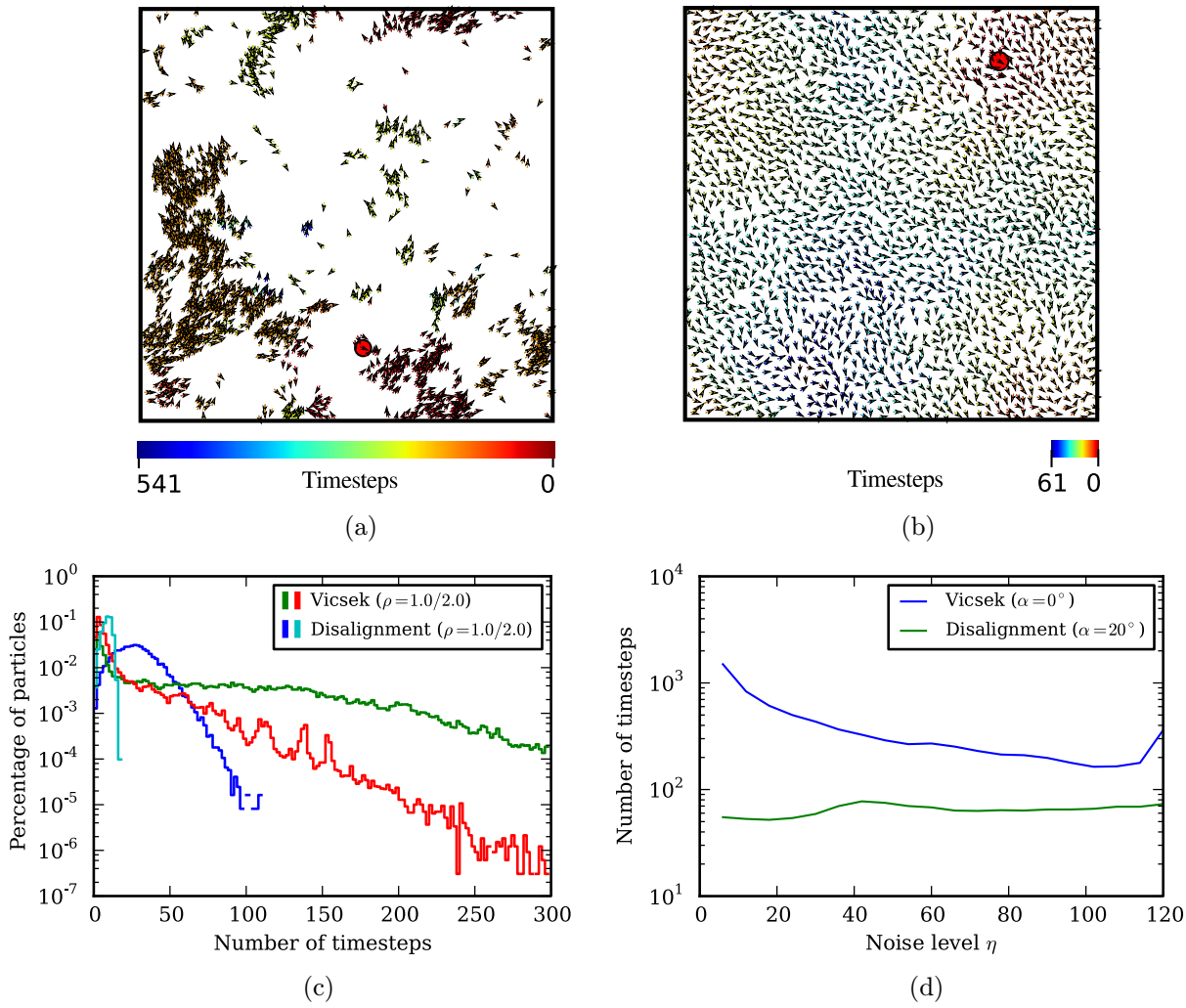


Figure 6. Information spreads differently with and without disalignment. Panels (a) and (b) show snapshots of particle positions and movement directions (arrows) at a given point in time. The color of a particle codes for the additional delay time until it is 'influenced' (as defined in the main text) by the current state of a focal particle (red dot). In (a), original Vicsek model ($\rho = 1.0$, $N = 2048$, $\eta = 30°$, $v = 0.1$); influence spreads in chunks from group to group. In (b), disalignment model ($\rho = 1.0$, $N = 2048$, $\eta = 30°$, $\alpha = 20°$, $v = 0.1$); influence spreads evenly and much faster through the system, which is spatially organized in a grid structure. Panel (c) shows distributions of influence times for the Vicsek and the disalignment model ($\alpha = 20°$) at two different densities and equal noise ($N = 2048$, $\eta = 40°$, $v = 0.1$). For both densities, the distribution corresponding to the Vicsek model exhibits a long time tail and a peak, which is due to interactions within and between 'flocks', as we argue in the main text. These features are absent in the grid-structured regime of the disalignment model, where particles connect through a different mechanism. Panel (d) shows the maximal influence times for different noise levels and many realizations. Note that disalignment (with $\alpha = 20°$) can reduce these maximal influence times by orders of magnitudes (other parameters: $N = 2048$, $\rho = 1.0$, $v = 0.1$).

characteristic size of particle groups, which exchange information on a fast time scale. In figure 6(d), we report the time until the last particle of the system became connected to a focal particle for many realizations. This maximum interaction time is for low noise, orders of magnitude larger than in the disalignment model. Interestingly, the disalignment model displays a visible change of the maximum influence time around the critical noise level where global order breaks down.

5. Conclusions

We have demonstrated that changing the unbiased aligning interaction of the classical Vicsek model to a slightly disaligning interaction can lead to an increase in global orientational order. The ordering effect of the disalignment term on global order is most prominent for densities close to ~ 1, when the spatial structure of the population was found to be fundamentally changed by the repulsive disalignment interaction. Isolated groups or flocks, as found in the classical Vicsek model, were unstable and disintegrated in the presence of the disaligning interaction. As a consequence, a homogeneous grid-like spatial structure formed that spanned the whole system. Within this structure, different parts of the population can interact without movement through a domino effect, by passing information sequentially through the intervening neighbors via exclusively short-ranged interaction. This is in contrast to the ordered state in the Vicsek model where information can spread over large distances through the motion of dense groups of particles ('flocks'). A small disalignment was sufficient to change the qualitative behavior of the system from flock-like to grid-like. There is an abundance of systems that are modeled with repulsive terms in nature [4, 6, 8, 18] (e.g. many particles avoid collisions), in which this effect could be relevant.

Since the mechanisms by which different parts of the population interact are very different in the grid-like and the flocking regime, it is not evident that the phase transition from order to disorder is of the same type in both regimes. Even for the well-studied Vicsek model, the question whether the phase transition is continuous or discontinuous (in the thermodynamic limit) has been intensely debated in the recent literature [2, 15, 16, 19]. The disalignment model shows a much sharper transition from our result which would conform to the discontinuous phase transition observed for an isotropically repulsive interaction [2].

Grid-like structures have been observed before in models with alignment and cohesion force [4, 20]. In this case, small groups with positional and directional order ('crystallites') formed in the regime of low densities when cohesion and disalignment interaction were strong. In our model, a system spanning grid appears in the regime of large densities and small velocities. Although our simulations suggest that the directional order remains finite as the system size is increased, it is not entirely clear how the system achieves true long-range order: assuming orientational information spreads in a similar manner to the XY-model, noise should destroy long-range order as predicted for 2D equilibrium systems through the Mermin–Wagner theorem [14, 21] if this was the only mechanism of alignment. The observed order could still be quasi-long-range (i.e. correlations decay according to a power law) as in the low-temperature phase of the XY-model in two dimensions. Answering these questions would be the virtue of more extensive simulations than were possible for this first study. Moreover, an analytical treatment [14, 22–24] as well as a detailed scaling analysis [12] could help to clarify the key control parameters in our system.

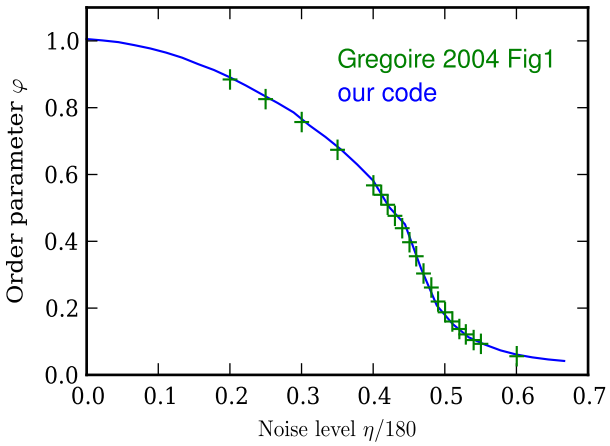
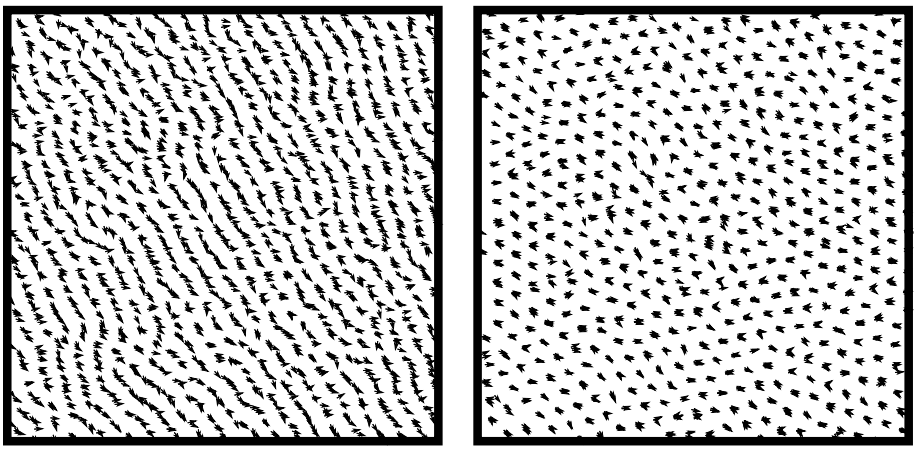


Figure A.1. To benchmark our code, we show here a comparison of our simulation results for the order parameter at vanishing disalignment with data published in [10, figure 1]. Other simulation parameters are $N = 2048$, $\rho = 2.0$, $\alpha = 0^\circ$ and $v = 0.5$.



(a) Medium Duration (b) Long Duration

Figure B.1. To demonstrate two peculiarities of the disalignment model at high densities, we show it here in snapshots at a medium duration, as well as at a very long duration after randomizing initial positions with simulation parameters: $\alpha = 20^\circ$, $N = 2048$, $\rho = 4.0$, $\eta = 25^\circ$, $v = 0.1$. Instead of an even grid, particles first form street-like structures in (a). This is due to the nature of our SPP model: because particles cannot slow down or accelerate, the distances between particles along the movement direction cannot be adjusted by the repulsive interaction as fast as distances to the side, such that streets form. This happens especially at high density, where particles get 'squeezed' into streets. After running the simulation for a long enough time (b), the system ultimately settles into a grid where several particles occupy a single grid site.

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Appendix A. Benchmark

A comparison of our simulation results for the order parameter at vanishing disalignment with data published in [10] is given in figure A.1.

Appendix B. The high-density structure

The demonstration of two peculiarities of the disalignment model at high-density structure is given in figure B.1.

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