A semi-analytical formulation for thermo-mechanical advective-diffusive heat transport in DFN

Silvia de Simone, Benoît Pinier, Olivier Bour, Philippe Davy, Romain Le Goc

A semi-analytical formulation for thermo-mechanical advective-diffusive heat transport in DFN

Conference Paper · June 2021

5 authors, including:

Silvia De Simone
French National Centre for Scientific Research

Benoît Pinier
Université de Rennes 1

Olivier Bour
Université de Rennes 1

Philippe Davy
French National Centre for Scientific Research

DFNE 21–2343

A semi-analytical formulation for thermo-mechanical advective-diffusive heat transport in DFN

De Simone, S., Pinier B., Bour O., Davy P.

Univ Rennes, CNRS, Géosciences Rennes, UMR 6118, Rennes, FRANCE,

Le Goc. R.,

Itasca Consultants SAS, Rennes, France

ABSTRACT: Modeling heat transfer in complex heterogeneous fractured system is key for geothermal energy applications. Discrete fracture network (DFN) modeling is the ideal framework to reproduce the advective part of the transport, which is determined by the fracture connectivity and heterogeneity. This approach in general sacrifices the representation of the rock matrix, disregarding both its diffusive heat exchange with the fractures and the effects of its thermo-mechanical deformation on the fracture aperture. Here we propose a new semi-analytic formulation that can be implemented in a DFN simulator with particle tracking approach. The contribution of the rock matrix in terms of diffusive heat exchange and thermal contraction/expansion is analytically evaluated, which respectively impact the advective heat transfer and the fracture aperture variation. The methodology enables investigating the reservoir behavior and optimizing the geothermal performance while keeping the computational effort within reasonable values. This allows exploring the uncertainty in cases when the characterization is poor, which is the spirit of the DFN modeling.

1. INTRODUCTION

Deep geothermal energy represents a powerful and clean energy prospect, with the potential to generate huge and virtually unlimited energy (Giardini, 2009). Geothermal plants generally involve hot reservoirs located in the crystalline basement, characterized by a very low permeability of the rock mass and a complex system of preexisting natural fractures. A characterization of the rock and fracture properties is in general difficult, which increases the uncertainty on the performance. Thus, numerical modeling is key to forecast the performance of geothermal energy applications under a number of scenarios.

Modeling heat transfer in complex heterogeneous fractured systems is challenging. The flow through the fracture network controls the heat advective transport, while the diffusive thermal exchange with the host rock controls the geothermal performance (Bruel, 2002). The two processes occur on very different length and time scales, which complicates their numerical simulation especially in the case of a large reservoir domain with thousands of fractures. The complexity is further increased when thermal deformations are taken into account. Host rock cooling provokes thermal contraction which tends to increase the fracture aperture. This may result in the variation of the advective transport, slowing down or speeding up the heat production, or opening new fast flow paths that may shortcut the geothermal doublet. Quantifying these processes is crucial for the optimization of heat extraction.

Discrete fracture network (DFN) modeling is the ideal framework to reproduce the advective part of the problem, which is determined by the fracture connectivity and heterogeneity. However, the representation of the diffusive heat exchange with the rock matrix and of the thermo-mechanical (TM) coupling suffers from some limitations (Jing, 2003). To represent the exchange with the rock, the assumption of a diffusive heat flux perpendicular to the fracture plane is widely adopted, which is solved either numerically with explicitly representation of the rock matrix mesh, or analytically. Although the first approach is more accurate, and it may be combined with the numerical simulation of the TM coupling, it incurs high computational cost and therefore it limits the number of fractures that may be represented. On the contrary, the adoption of analytical solutions greatly reduces the computational effort, but it is limited by some underlying assumptions. The first is the one of infinite matrix, which is reasonable only for short time or large fracture spacing. The second is that the trajectories are stationary in time and independent on each other

(Cvetkovic et al., 1999). In addition, the TM coupling is rarely contemplated in this context.

In this paper, we propose a new semi-analytic formulation that can be implemented in a particle tracking approach and it is similar to the method of the characteristics. The contribution of the rock matrix in terms of diffusive heat exchange and thermal contraction/expansion is analytically evaluated, which respectively impact the heat transfer and the fracture aperture variation. This latter in turns affects the velocity field, which is updated in time assuming quasi-steady state conditions. We disregard the effects of the pore-pressure on the fracture aperture (hydro-mechanical (HM) coupling). However, we do take into account the impact of the flow velocity on the heat transfer, as well as the effects of the temperature on the fracture aperture and the impact of the aperture variation on the flow velocity. We therefore describe this approach as a model for simulating T(H)M coupling. The formulation overcomes the assumption of stationary and independent trajectories, to consider the variation of the diffusive exchange flux in the case of different flow paths passing through the same location.

In the following, we present the details of the method, its implementation and validation. Then we show results for the preliminary case of a single fracture and we discuss the advantages with respect to other methods.

2. GOVERNING EQUATIONS FOR HEAT TRANSPORT AND THERMO-MECHANICAL DEFORMATION ON A SINGLE FRACTURE FLOW PATH

We consider planar and perfectly symmetric fractures with semi-aperture b, variable in time and space. The variability in space is smooth such that we can keep the assumption of planar fracture as valid. Fluid flow occurs exclusively in the fracture network, while the rock matrix is impermeable. We assume that the fracture aperture variation is slow in time, which allows for considering quasi-steady state conditions for flow, i.e., the velocity field does not change over a certain time interval. Cubic law applies. Heat transfer occurs by advection within the fracture system and by orthogonal diffusion from the fractures to the rock matrix. Lateral diffusion is disregarded both in the fracture and in the matrix. We also assume infinite diffusive matrix and flat temperature profile within the fracture thickness. We solve the problem of constant temperature imposed at one boundary of the domain. We start by considering one trajectory, with longitudinal coordinate ℓ, and we describe the heat transport problem in the fracture, which is strictly coupled with the heat transport in the matrix.

Adopting a Lagrangian approach, the heat transport in the fracture is described by the energy balance (Carrera et al. 1998)

$$C_f \frac{\partial T_f(\ell,t)}{\partial t} + C_f \frac{\partial T_f(\ell,t)}{\partial \tau} = -\frac{J(\ell,t)}{b(\ell)}\ , \qquad (1)$$

where C_f is the fluid heat capacity, T_f is the fluid temperature, τ is the advective travel time to reach the position ℓ, i.e., $\tau = \int_0^\ell d\ell/u(\ell)$, u is the fluid velocity, and t is time. J represents the diffusive heat exchange between the fluid in the fracture and the rock matrix per unit surface, which is expressed as the gradient of the temperature in the matrix at the fracture/matrix interface

$$J(\ell,t) = -\ \kappa \left.\frac{\partial T_m(\ell,z,t)}{\partial z}\right|_{z=b}, \qquad (2)$$

where k is the rock thermal conductivity, T_m is the temperature in the rock matrix, and z is the coordinate in the direction perpendicular to the fracture with origin in the fracture center. Heat in the matrix is governed by a linear diffusion problem with the boundary condition such that $T_m(\ell,0,t) = T_f(\ell,t)$, which solution is given by the convolution integral of $T_f(\ell,t)$ and the diffusive one-dimensional response to the pulse injection $\psi(z,t)$, i.e., $T_m(\ell,z,t) = T_{f(\ell,t)} * \psi(z,t)$, with

$$\psi(z,t) = \frac{z-b}{2\sqrt{\pi D}\,t^{-\frac{3}{2}}} \exp\left(-\frac{(z-b)^2}{4Dt}\right), \qquad (3)$$

where $D = \kappa/C_m$ is the rock thermal diffusivity, and C_m is the rock heat capacity.

Substitution of Eq.(3) into Eq.(2) and differentiation under the integral sign returns

$$J(\ell,t) = -\kappa \int_0^t \frac{T_f(\ell,t\,)}{2\sqrt{\pi D (t-t')^{\frac{3}{2}}}} \times$$

$$\left. \exp\left(-\frac{(z-b)^2}{4D(t-t')}\right)\left(1-\frac{(z-b)^2}{2D(t-t')}\right) \mathrm{d}t' \right|_{z=b} \qquad (4)$$

Note that the thermal problem in the rock matrix is linear, and therefore superposition applies.

For the mechanical problem, the rock matrix is assumed as a linear elastic material whose thermal deformations are proportional to the local temperature through a thermal expansion coefficient, α_T. We assume one-dimensional thermal deformation in the direction transverse to the fracture into the matrix, z, considering that the temperature profile is such that plane strain conditions in the y-direction applies, and such that the deformations in the direction longitudinal to the fracture are small. Therefore, the variation of the semi-aperture with respect to the initial conditions, $\Delta b(\ell,t)$, is equal to the displacement undergone by the fracture/rock interface in the direction perpendicular to the fracture, which in turn is the integral over z of the rock matrix deformation because we assume that the displacement is 0 at $z = \infty$, where the temperature variation is zero,

$$\Delta b(\ell,t) = \int_0^\infty \frac{1+\nu}{1-\nu}\alpha_T[T_m(\ell,z\,t) - T_m(\ell,z,0)]\,dz\,, \quad (5)$$

where ν is the Poisson ratio.

3. SEMI-ANALYTICAL FORMULATION FOR THERMO-(HYDRO)-MECHANICAL COUPLING IN HETEROGENEOUS SYSTEMS

The governing equations detailed above are now extended to the case of multiple trajectories in a heterogeneous fractured system subject to continuous heat injection at one boundary. Since we want to solve the non-linear problem with variable in time b and u, we adopt a time and space discretization and we derive a semi-analytic formulation where the advective part of the problem is solved by a particle tracking approach. The condition of fixed imposed temperature T_0 corresponds to an imposed heat energy $E_0 = C_f V_0 T_0$ over a volume V_0. We represent this energy as an ensemble of N particles, which move into the fracture system by advection along different trajectories. We consider the particles as fluid cells with equal volumes, whose sums equates V_0. All particles are initially charged with the same energy, and the total energy is given by the sum of the individual particle energies. Therefore, particle volumes are initially charged with the injection temperature, i.e., $T_p(0;\mathrm{x}_0,t_0) = T_0, \forall p.$ Fluid particles change their energy because of their diffusive heat exchange with the rock matrix. Since we assume that both particle volume and heat capacity are constant, particle energy and temperature are related by a constant proportion. Therefore, we can arbitrarily adopt one or the other as working variable. We adopt the temperature for our convenience.

Particles move along flow paths that may change from one time-step to another, due to the temporal variation of the velocity in the system. In the aim of solving the continuous injection problem, it is convenient to express time as the time of the injection plus the travel time, such as $t = t_0 + \tau$. The duration of the injection is discretized into time steps Δt_0, whereas $\Delta\tau$ is the jump time over each mesh element along the trajectory. At each t_0 we release N particles with assigned temperature at the inlet section and we consider continuous injection during the time step. Within each time step we assume that the fracture aperture is constant and that the fluid flow is steady state. The time step is assumed as much larger than the advective time to displace over a mesh element. At each time step we estimate the fracture aperture variation and the consequent new velocity field.

According with this framework and operating a substitution of variable such as $t = t_0 + \tau$, we may adopt the total derivative in Eq. (1), which is expressed at each mesh element and in terms of particle temperature as

$$\frac{\mathrm{d}\,T_p\left(\xi_k;\mathbf{x}_{\mathbf{0}_\mathbf{p}},t_{0_i}\right)}{\mathrm{d}\tau} = -\frac{J_p\left(\xi_k;\mathbf{x}_{\mathbf{0}_\mathbf{p}},t_{0_i}\right)}{C_f b}, \quad (6)$$

where $T_p\left(\xi_k;\mathbf{x}_{\mathbf{0}_\mathbf{p}},t_{0_i}\right)$ and $J_p\left(\xi_k;\mathbf{x}_{\mathbf{0}_\mathbf{p}},t_{0_i}\right)$ respectively express the temperature and the heat exchange that the particle p released at $\mathbf{x}_{\mathbf{0}_\mathbf{p}}$ at the time t_{0_i} has at the position ξ_k.

We differentiate the temperature at the fracture-rock interface, defined in the Eulerian field, from the particle temperature, defined in the Lagrangian field. Particles move along their trajectory and pass each cross section at a certain time and with a certain temperature. We assume that there is no direct heat exchange between different fluid particle volumes because the differences in arrival times is small compared to the diffusive time to cross the distance between different trajectories. Therefore, particle trajectories do not directly interact during their travel. However, particles have indirect effects on future particles. In fact, the heat that each particle exchanges with the rock matrix by diffusion is a function of the local Eulerian temperature history, which we evaluate at the mesh scale. We consider that the time step is sufficiently large for a complete homogenization of each particle temperature to occur over the mesh cross section thanks to diffusion processes. Thus, at each time step we operate an average over the passage times and over the temperatures observed at each mesh node, estimating $\overline{T}$ and $\overline{t}$. This dichotomy between the particle Lagrangian temperature and the local Eulerian temperature enables dealing not only with apertures that change in time because of the mechanical deformation, but also with trajectories that converge to the same location at different times.

Under this framework, the heat flux that a particle released at time t_0 exchanges at a given position, ξ, is a function of the temperature recorded in the current time step (which is equal to the particle one at that position) and of the average temperatures recorded at the position in the past time steps. That allows for deriving a discretized version of J_p (Eq.(4)) considering that each temperature is constant over the time step. Assuming that J_p is constant over the jump length, Eq. (6) reads

$$\left.\frac{\Delta T_p}{\Delta\tau}\right|_{\tau_{k,i,p}} = -\frac{T_i\left(\mathbf{x},t_{k,i,p}\right)}{\sqrt{t_D\Delta t_0}} + \quad (7)$$

$$\sum_{j=1}^{i-1}\frac{\overline{T}\left(\mathbf{x},\overline{t_j}\right)}{\sqrt{t_D}}\left(\frac{1}{\sqrt{t_{k,i,p}-\overline{t_j}}} - \frac{1}{\sqrt{t_{k,i,p}-\left(\overline{t_j}-\Delta t_0\right)}}\right),$$

where $T_i(\mathbf{x}, t_i) = T_p\left(\xi_k; \mathbf{x}_{\mathbf{0_p}}, t_{0_i}\right)$, $t_{k,i,p} = \tau_{k,i,p} + t_{0_i}$, $t_D = \pi b^2/D$ is the diffusive characteristic time, and where we have assumed that $C_f = C_m$.

Fracture aperture variation is estimated at the average time and according with the average values of temperature and time, such that Eq.(5) translates into

$$\Delta b(\mathbf{x}, \overline{t_i}) = \frac{1+\nu}{1-\nu}\alpha_T\sqrt{\frac{D}{\pi}} \times \sum_{j=1}^{i}\overline{T}(\mathbf{x}, \overline{t_j})\left(\sqrt{\overline{t_i} - \overline{t_j}} - \sqrt{\overline{t_i} - \overline{t_j} + \Delta t_0}\right). \quad (8)$$

At each time step the fracture aperture field is updated according with the more recently recorded aperture variations, so the velocity field is associated with the most recent fracture aperture field.

4. VALIDATION OF THE SEMI-ANALYTIC SOLUTION

In this section we test the validity of the above-described formulation by considering a single trajectory. We first consider the case of a single trajectory with constant in time aperture and velocity, for which an analytic solution can be derived (e.g., Tang et al., 1981). We consider a fracture of 10 m length with uniform semi-aperture $b = 10^{-4}$ m. The fluid velocity u is constant and uniform and it is equal to 8×10^{-3} m/s. We impose a constant temperature T_0 at $x = 0$. The rock matrix thermal diffusivity is $D = 1 \times 10^{-6}$ m²/s. For the semi-analytic formulation, we adopt a space discretization such that Δx=1 m, while for the time discretization we consider different time intervals Δt_0 from $0.1t_c$ to $100t_c$. The characteristic time $t_c = D\Delta\tau^2/(2b^2)$, defined as the ratio of the square of the advective characteristic time for the unitary jump, $\Delta\tau = \Delta x/u$, to the diffusive characteristic time, t_D, governs the process.

Figure 1 shows the temperature variation in time for three different control planes. The semi-analytic solution is most accurate for $\Delta t_0 = t_c$, while it incurs a small underestimation of temperature for $\Delta t_0 < t_c$ and a small overestimation for $\Delta t_0 > t_c$. We also observe some instability for short times. This is a long-standing stability problem in Eulerian methods and it is related with the way the temperature variation is estimated. Although results are sufficiently accurate under a time step spanning four orders of magnitude, the evaluation of an optimum value of time step is recommended, especially in the case of heterogeneous media with large variability of velocity, fracture aperture and mesh size.

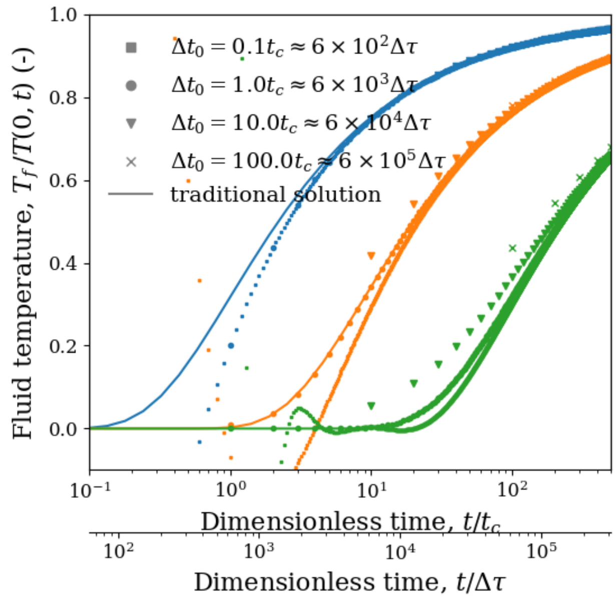

Figure 1. Temporal evolution of temperature in the fracture in response to continuous temperature variation T_0 imposed at x=0 and in the case of constant fracture aperture and velocity. Comparison of the results calculated by means of the proposed semi-analytic approach (markers), and by a full analytical solution (lines), as in Tang et al., (1981). The different colors correspond to different control planes along the fracture, at $x = \Delta x$ (blue), $x = 3\Delta x$ (orange), and $x = 10\Delta x$ (green). The different markers refer to different values of time steps for the semi-analytic solution.

Successively, we prove the accuracy of the analytic evaluation of the thermo-mechanical fracture aperture (Eq.(8)) against results from a fully coupled THM numerical simulation performed with the Finite Element Method simulator CodeBright (Olivella et al., 1996). We consider a single fracture embedded in a rock matrix. Both the fracture and the rock are represented as porous media in the finite element simulator. Therefore, we build the model in order to have it consistent with our assumptions, i.e., negligible longitudinal thermal conductivity in the fracture, impermeable rock matrix, negligible hydro-mechanical deformations. We assume constant velocity, i.e., the fracture aperture variation does not affect the velocity field or the transport behavior. Thus, we only evaluate the aperture variation in response to the temperature field generated by a continuous injection for a certain time. The fracture is 100 m long with a steady state flow rate of 5×10^{-5}m²/s. The rock thermal diffusivity D is equal to 1.6$\times 10^{-6}$ m²/s. We simulate 20 days of continuous injection of the temperature T_0 at $x = 0$. Nevertheless, we show the results in terms of dimensionless time, adopting the same characteristic time as above, $t_c = D\Delta\tau^2/(2b^2)$. For the mechanical parameters of the rock, the linear thermal expansion α_T is equal to $1 \times 10^{-5}\,°C^{-1}$, while the Poisson ratio ν equates 0.3.

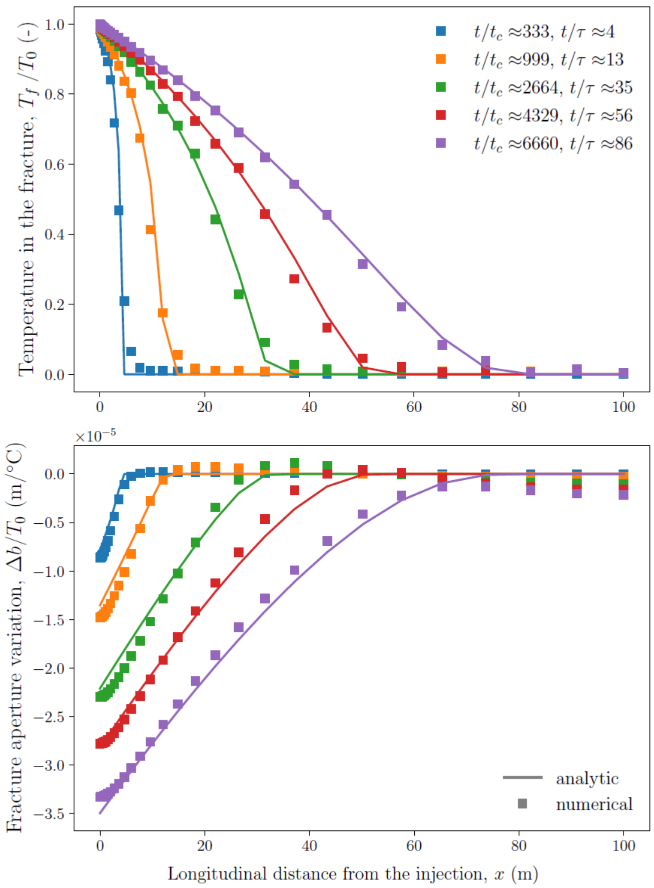

Figure 2. Comparison of the thermo-mechanical behavior of a fracture subject to continuous temperature T_0 imposed at $x = 0$, evaluated by means of the semi-analytic solution proposed in this paper (lines) and by a fully coupled thermo-hydro-mechanical numerical simulation (markers). Colors correspond to different times, which are normalized with respect to the characteristic time t_c. Results are normalized with respect to the injected temperature.

Figure 2 shows that the analytic estimation of both the temperature and the aperture variation is very accurate, which confirms the validity of the assumptions described in the previous sections. We observe that considering the mechanical problem as one-dimensional in z do not incur large errors, because the lateral mechanical propagation of the displacement is well reproduced by the smooth heat propagation.

5. DISCUSSION AND CONCLUSIONS

We have proposed a new methodology to simulate T(H)M behavior in heterogeneous fractured systems. Similar to the method of the characteristics, the advective part of the problem is solved by a particle tracking approach, and each particle holds a temperature that changes because of the diffusive exchange with the rock matrix. The effects of this diffusive flux are estimated semi-analytically, in order to account for trajectory variations due to thermo-mechanical deformations. This allows to overcome the limitations of stationary and independent trajectories of the approach proposed by Cvetkovic et al. (1999) for the case of pulse injection, and successively extended to the case of continuous injection by Liu et al. (2007) and Gisladottir et al., (2016).

The methodology has been implemented in the DFN.lab software platform (Le Goc, 2019; Doolaeghe, 2020; Pinier, in prep.) which provides a set of tools for the generation of stochastic and deterministic DFN with thousands of fractures, and for the modeling of flow and transport in the connected network.

Although subject to several simplifications, the methodology enables investigating the reservoir behavior and optimizing the geothermal performance while keeping the computational effort within reasonable values. This allows exploring the uncertainty in cases when the characterization is poor, which is the spirit of the DFN modeling.

REFERENCES

1. Bruel, D. 2002. Impact of Induced Thermal Stresses During Circulation Tests in an Engineered Fractured Geothermal Reservoir. Oil & Gas Science and Technology 57(5): 459–70.
2. Carrera, J., Sánchez-Vila, X., Benet, I., Medina, A., Galarza, G., & Guimerà, J. 1998. On Matrix Diffusion: Formulations, Solution Methods and Qualitative Effects. Hydrogeology Journal 6(1): 178–90.
3. Cvetkovic, V., J. O. Selroos, and H. Cheng. 1999. Transport of Reactive Tracers in Rock Fractures. Journal of Fluid Mechanics 378: 335–56.
4. Doolaeghe, D., Davy, P., Hyman, J. D. & Darcel, C. 2020. Graph-based flow modeling approach adapted to multiscale discrete-fracture-network models. Physical Review E, 102 (5): p. 053312.
5. Giardini, D. 2009. Geothermal Quake Risks Must Be Faced. Nature 462(7275): 848–49..
6. Gisladottir, V. R., D. Roubinet, and D. M. Tartakovsky. 2016. Particle Methods for Heat Transfer in Fractured Media. Transport in Porous Media 115(2): 311–26.
7. Jing, L. 2003. A Review of Techniques, Advances and Outstanding Issues in Numerical Modelling for Rock Mechanics and Rock Engineering. International Journal of Rock Mechanics and Mining Sciences 40(3): 283–353.
8. Le Goc, R., Pinier, B., Darcel, C., Lavoine, E., Doolaeghe, D., De Simone, S., E. Lavoine, Davy, P. 2019. DFN.lab: software platform for Discrete Fracture Network models. AGU Fall Meeting, San Francisco, CA, USA
9. Liu, H. H., Y. Q. Zhang, Q. Zhou, and F. J. Molz. 2007. An Interpretation of Potential Scale Dependence of the Effective Matrix Diffusion Coefficient. Journal of Contaminant Hydrology 90(1–2): 41–57.

10. Olivella, S., a. Gens, J. Carrera, and E.E. Alonso. 1996. Numerical Formulation for a Simulator (CODE_BRIGHT) for the Coupled Analysis of Saline Media. Engineering Computations 13(7): 87–112.

11. Pinier, B., Le Goc, R., Davy, P. in prep. Computation of flow and transport properties on large 3D heterogeneous stochastic and semi-deterministic fractured media with the platform DFN.lab.

12. Tang, D. H., E. O. Frind, and E. a. Sudicky. 1981. Contaminant Transport in Fractured Porous Media: Analytical Solution for a Single Fracture. Water Resources Research 17(3): 555–64.