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Estimation of the seismic moment rate from an incomplete seismicity catalog, in the context of the InSight mission to Mars

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Abstract

We evaluate methods to estimate the global seismic moment rate of a planet from the $k \geq 1$ largest events observed during a limited and possibly short time span, as can be expected e.g. for lander missions to Mars. The feasibility of the approach is demonstrated with a temporary broadband seismometer that was recording in the Mojave Desert, California, for 86 days in 2014, and by application to the Global Centroid Moment Tensor catalog, subsets thereof, and a catalog of stable continental regions seismicity. From the largest event observed at Goldstone alone ($M_W \approx 7.9$) we estimate the Earth’s global moment rate to be $1.03 \times 10^{22} \text{Nm/yr}$, while an estimation based on the 10 strongest events yields a rate of $5.79 \times 10^{21} \text{Nm/yr}$. Summation of 42 years of GCMT solutions result in an average of $7.61 \times 10^{21} \text{Nm/yr}$. In general, a two years interval of GCMT solutions is sufficient to estimate the Earth’s annual moment rate to within a factor 5 or better. A series of numerical experiments with more than 560 million synthetic catalogs based on the Tapered Gutenberg-Richter distribution shows that the estimation is rather insensitive against an unknown slope of the distribution, and that bias and variance of the estimator depend on the ratio between moment rate and corner moment of the size frequency distribution. Moment rates of published Mars models differ by a factor 1000 or more. Tests with simulated catalogs show that it will be possible to reject some of these models with data returned by NASA’s InSight mission after two years of nominal mission life time.


**Introduction**

The InSight mission (Banerdt et al., 2013) to Mars was launched on May 5, 2018 from Vandenberg Air Force Base, California, and touched down softly in the targeted landing ellipse on Elysium Planitia, Mars, on November 26, 2018. A combined very broad band and short period seismometer was deployed on the surface on 19th of December 2018 and is intended to operate until the end of 2020 (Lognonné & Pike, 2015, Lognonné et al., submitted).

Models for the seismicity of planet Mars usually estimate the long-term average annual seismic moment rate (Table 1), and also the average annual event rate. This holds for estimations based on geological evidence (Golombek et al., 1992, Golombek, 2002, Taylor et al., 2013) as well as for models based on thermal evolution and cooling of the Martian lithosphere (Phillips, 1991, Knapmeyer et al., 2006, Plesa et al., 2018). All studies are compatible with the conclusion drawn from the non-observation of any unambiguous seismic event by the Viking missions (Anderson et al., 1977, Goins & Lazarewicz, 1979), i.e. that Martian seismicity ranges somewhere between that of the Moon and that of Earth. However the range of predicted moment rates is wide and observational constraints of moment rate are therefore desirable to reject models and further our understanding of planetary heat engines. Here we investigate to what extent the InSight mission can be expected to contribute to the rejection of seismicity models. The main obstacles are that the detection threshold of a single station will be relatively high compared to a global network, while the expected life time is relatively short compared to the instrumental record available for Earth. Our approach basically relies on most of the seismic moment being released in a few, large events, and employs Monte Carlo methods to assess how likely any given seismicity model results in the moment rate estimation obtained from the recorded marsquakes.

Statistical seismology on celestial bodies other than Earth generally suffers from limitations in the life time and station spacing of seismic networks: On the Moon, the first long period seismometer was de-
ployed in 1969, later augmented by four more stations to form a triangle (Apollo 12 and 14 were relatively close together, compared with Apollo 15 and 16). The network was shut down in late 1977 (Vos-}

trey, 1980). One seismometer (Viking 2) was in operation on Mars from September 1976 to April 1978, i.e. for about 20 months (Toksöz, 1979). The Venera 13 and 14 missions each landed one seismometer on Venus on the 1st of March 1982 (Venera 13) and 3rd of March 1982 (Venera 14), respectively, and with lander life times of 127 and 57 minutes (Siddiqi, 2002). During these times, about 1.5 minutes of seismic waveforms and, additionally, statistical waveform parameters derived on board, were downlinked to Earth (Ksanfomaliti et al., 1982). The Huygens probe contained a suite of accelerometers to monitor its path through the atmosphere of Titan and to record the touchdown shock. Although not intended to conduct a surface seismic experiment, the HASI accelerometer onboard the Huygens probe continued to record data for approximately 30 min after touchdown (Hathi et al., 2009). A network of three triaxial accelerometers was deployed on comet 67P/Churuyomov-Gerasimenko within the landing gear of Rosetta’s lander Philae, resulting in a network aperture of about 2.5 m (Knapmeyer et al., 2016). From this experiment, about 70 s of high-frequency data (sampling rates up to 16 kHz) were returned. With a design life time of two terrestrial years, InSight will significantly surpass these recent experiments.

An instructive comparison to the much better coverage of the Earth arises from the Global Centroid Moment Tensor project (GCMT, Dziewonski et al., 1981, Ekström et al., 2012). At the same time the first seismometers were launched to Mars, the GCMT started producing its catalog of Earthquake source mechanisms, complete to magnitudes of about five and currently based on data from nearly 200 seismic observatories distributed worldwide. The GCMT continuously contributed to the picture of Earth’s moment-frequency distribution for the whole 42 years during which there was not even a single seismometer on Mars.

Besides the relatively short period of time covered by extraterrestrial seismicity catalogs so far, spatial coverage and completeness thresholds result in additional obstacles for the estimation of distribu-
Even within the Apollo network on the Moon, the most extensive network outside Earth so far, distance between stations was around 30°, i.e. the network triangle covered only a small fraction of the lunar surface. Limited spatial coverage results in a distance dependent detection threshold: small events will be detected only in the vicinity of the network or station, if there is but one. In turn, large, globally detectable events will be overrepresented in the catalog such that the moment-frequency distribution appears to be less steep than it actually is. Also, the estimation of event rates must rely on assumptions about source distribution in distant areas in order to extrapolate the detected local or regional seismicity to the entire planet.

The seismic moment of Earthquakes is predominantly released by the few largest events. This suggests that it is possible to obtain a reasonable estimate of the seismic moment rate from evaluating only a small number of large events, or even from the single largest event ever observed. Central questions are how such an estimate can be obtained and how accurate it will be, and also what accuracy is necessary to make a rate estimation a useful constraint to improve our understanding of Mars.

To answer the latter, we consider the moment rate estimates summarized in Table 1. Moment rates given there for Earth and Moon differ by almost exactly ten orders of magnitude, or 4.7 moment magnitude units. The individual estimations for Mars cover a range of 2.1 moment magnitude units, or a factor of approximately 1150 in the seismic moment. It is thus desirable to estimate the moment rate with accuracy better than this in order to reject some or several of the published seismicity models.

In the following, we will first describe our proposed approaches for moment rate estimation, and how we generate synthetic seismicity catalogs for numerical testing (section "Methods"). The available catalogs are described in section "Data". Since no event catalog is currently available for Mars, we briefly summarize the seismotectonic background information available for Mars. To test if reasonable results can be obtained, we evaluate several seismicity catalogs for the Earth, discuss the applicability to the Moon, and evaluate synthetic catalogs in a generic parameter study and for Mars-specific models (sec-
tion “Results”). Section "Single Station Completeness Threshold" specifically demonstrates that a single broadband station is sufficient to obtain a meaningful moment rate estimate for an entire planet within a relatively short time.

**Methods**

As Kagan & Schoenberg (2001) point out, a statistical consideration of magnitudes rather than moments is advantageous by transforming multiplicative uncertainties into additive ones. Besides, taking the logarithm of the moment transforms the numerical values into a more convenient range.

Throughout this work, we will frequently convert seismic moments $M$ into equivalent moment magnitudes $m$ via

$$m = \frac{2}{3} (\log_{10} M - 9.1) \quad (1)$$

(after Bormann et al., 2002). Using this formula, we also express moment rates as if the entire moment was released in a single event. All magnitudes used throughout this paper are moment magnitudes following from the above equation, unless indicated otherwise. The coefficient $2/3$ implies that a factor of 1000 in the moment $M$ corresponds to a difference of exactly 2 moment magnitude units. We denote seismic moments by uppercase letters and magnitudes by lowercase letters to avoid the excessive indexing that would result from the conventional lettering, and two indexed quantities $M_x$ and $m_x$ refer to moments and magnitudes connected by the above equation.

The coefficients of eq. 1 originate in a calibration of the seismic moment scale to other magnitude scales and thus carry a certain heritage of Earth’s seismological properties, e.g. via the decay of surface waves over distances not available e.g. on Mars. It must be kept in mind that relations to seismogram
amplitudes and, moreover, intensities will differ between Earth, Mars, and other bodies. Appropriate magnitude scales for Mars have been developed by Böse et al. (2018).

Seismic Moment Rate Estimation

Following the seminal works by Gutenberg & Richter (1941, 1944), the size-frequency distribution of earthquakes is often described by what we now call a power law, although Gutenberg & Richter were aware that the frequency of very large events cannot follow the same law as the frequencies of small and medium ones for physical reasons. Assuming a fault plane as large as the Earth’s surface, and with a slip proportional to the square root of the fault area as in the relations given in Knapmeyer et al. (2006), we find a resulting seismic moment of \(7 \times 10^{28} \text{Nm} \) (corresponding to \(m = 13.2\)) being a hard upper limit for quake size on Earth, as any geologically plausible fault would have a smaller rupture area. The extrapolation of the linear part of the size-frequency distribution of the GCMT predicts that events of this size occur every 10000 to 100000 years: The classical Gutenberg-Richter law, when interpreted naively, predicts that ridiculously large events recur on time scales that are short compared to e.g. the Wilson cycle. Given that there are no traces at all of such events, we take it as an observational fact that this prediction is not true and the size-frequency distribution must bend down significantly somewhere between \(5.3 \times 10^{22} \text{Nm} \) (the Tohoku event of March 2011) and \(10^{29} \text{Nm}\).

We assume that the moment frequency distribution follows a tapered Pareto distribution (a.k.a. tapered Gutenberg-Richter distribution), which basically is a Gutenberg-Richter law tapered with an exponential function for the largest events (Figure 1).

In this distribution, the cumulative relative number of events with a seismic moment larger than \(M\) is given by (Kagan, 2002a, b, and references therein)

\[
\Phi(M) = \left( \frac{M_t}{M} \right)^{-\beta} e^{\beta \left( \frac{M_t-M}{M_{c}} \right)} \quad \text{for} \quad M_t \leq M < \infty \quad (2)
\]
Here $M_c$ is the “corner moment” where the exponential taper gains dominance over the classical Gutenberg-Richter power law relation, $M_t$ is the threshold moment above which the catalog is complete (not to be confused with the detection threshold $M_d$, i.e. the smallest moment for which the catalog contains some, but not all, events), and $\beta$ is a slope parameter. The part of the graph of Figure 1 for $M_d \leq M \leq M_t$, i.e. the incomplete detection of events between detection threshold $M_d$ and completeness threshold $M_t$, is not described by eq. 2 as it is at least partially controlled by effects and properties not related to the quake production, like background noise, anelasticity, and instrument sensitivity.

To obtain absolute numbers, $\Phi(M)$ has to be multiplied with a calibration factor following Kagan (2002b)

$$\alpha_0 = \frac{\dot{M}_S M_c^{\beta(1-\beta)}}{M_t^{\beta} M_c \Gamma(2-\beta) \exp(M_t/M_c)}$$  \hspace{1cm} (3)

where $\dot{M}_S$ is the moment release rate per unit time of events exceeding $M_t$, and $\Gamma(x)$ is the Gamma function with $\sqrt{\pi/4} \leq \Gamma(2-\beta) \leq 1$ for $0.5 \leq \beta \leq 1$ (e.g. Weisstein, undated). The absolute rate $\dot{N}(M)$ of events with a seismic moment larger than $M$ is thus obtained from combining eqs. 2 and 3

$$\dot{N}(M) = \alpha_0 \Phi(M)$$ \hspace{1cm} (4)

In addition to the rate $\dot{M}_S$ we introduce the cumulative moment $M_S$ such that one event of size $M_S$ per unit time would account for the entire moment rate.

If the completeness threshold can be made small enough, $M_t \ll M_c$, the exponential term approaches one and $\dot{N}(M) \propto \dot{M}_S M_c^{\beta-1}$, which plots as a straight line in log-log coordinates, such that the observed event rate allows inferences on $\dot{M}_S M_c^{\beta-1}$.
For any catalog of observed events, one can estimate $\dot{N}(M)$ via the number $N$ of events listed in the catalog (not taking into account those below $M_t$), and the time span covered by it. The moment rate can be estimated by summing up all events and dividing by the number of unit time intervals covered by the catalog, as was done with the GCMT in Table 1.

The total moment released can be computed by integration over the distribution, but since the moment rate is a factor in $\dot{N}(M)$ we can save the integration and solve $\dot{N}(M)$ for $\dot{M}_S$

$$\dot{M}_S = \dot{N}(M) \frac{\Gamma(2-\beta)}{1-\beta} \frac{M^\beta}{M_c^{\beta-1}} \exp \left( \frac{M}{M_c} \right)$$

Although $M_t$ cancelled out, $\dot{M}_S$ is still only the moment rate resulting from events exceeding $M_t$ by implication via $\dot{N}$. If $M_t \ll M_c$ and $\beta < 1$, the difference to the moment rate resulting from all events is however negligible.

Now we consider only the largest event that ever occurred. By definition, the largest event ever (or LVR, for short, from Largest eVent eveR) is contained in the catalog only once. Thus, if the catalog covers a time interval of $n$ unit intervals, we identify $M = M_{LVR}$ and obtain $\dot{N}(M_{LVR}) = 1/n$, and eq. 5 allows estimating the global moment rate from a single event, provided the corner moment $M_c$ of the distribution and its slope parameter, or reasonable estimates, are given.

Using eq. 2 we find the relative frequencies of events exceeding $M_c$ by a factor $x$ is

$$\frac{\Phi(xM_c)}{\Phi(M_c)} = \left( \frac{1}{x} \right)^\beta \exp(1-x)$$

With a convenient (and not entirely unrealistic) value of $\beta = 2/3$, this results in a drop of frequency by a factor $5 \times 10^{-15}$ for events one magnitude unit larger than $m_c$, while the frequency of events two magnitude units larger is reduced by $10^{436}$ with respect to the corner moment. Thus the former are
extremely rare, while the latter do practically not occur. This holds for slightly different choices of $\beta$ as well.

The largest event ever hence poses a (statistical) lower limit $M_{LVR} \leq M_c$ simply by its existence. Moreover, it will approach the corner moment with time, such that $M_{LVR} \approx M_c$ becomes a reasonable guess. Inserting this into eq. 5 yields

$$M_S \approx \frac{1}{n} \frac{\Gamma(2-\beta)}{1-\beta} M_{LVR} \quad (7)$$

and we call this the normalized largest event ever, NLVR for short, estimation of the moment rate.

The middle term approaches infinity for $\beta \to 1$, but in reality $1/2 \leq \beta \leq 2/3$ appears to hold, from which $\sqrt{\pi} \leq \Gamma(2-\beta)/(1-\beta) \leq 2.68$ follows.

We define $n$ based on the duration of the observation: if the unit time interval has a duration $\Delta T$, seismicity is recorded between $t_{\text{start}}$ and $t_{\text{end}}$ (where $t_{\text{start}}$ is not necessarily the time of the first event, and $t_{\text{end}}$ is continuously growing in an ongoing experiment), then the normalization factor for the largest event ever which occurred at a time $t_{LVR}$ between start and end is $n = (t_{\text{end}} - t_{\text{start}} + 1)/\Delta T$, and also continuously grows in an ongoing experiment.

With this definition, $1/n$ and thus the estimated value of $\dot{M}_S$, will approach zero for $t_{\text{end}} \to \infty$. Mathematically, this appears to be a flaw, but in the long run the observer can take the onset of a continuous decay as indication that the largest event ever has exceeded the corner moment of the distribution.

A more sophisticated estimation of $M_c$ is given by Kagan & Schoenberg (2001). As they found that a maximum likelihood estimation of $M_c$ is severely biased for small catalogs, they derive an alternate estimator based on expressions for the first and second statistical moment of the tapered Gutenberg-Richter distribution. With the seismic moment of the $i$-th event in the catalog termed $M_i$, a total of $k$ cataloged events, the standard deviation $\sigma$ of the $M_i$, and the mean moment...
they obtain the estimator

$$\tilde{M}_c = \frac{\sum(M_i^2 / k) - M_{t}^2}{2[M_t^2 \beta + (1 - \beta)\bar{M}]} \quad (9)$$

from which a bias correction

$$\tilde{M}_{bias} = \frac{(\beta - 1)\left[2M_t^2 + 3M_t^2 \bar{M} \beta + (\sigma^2 + s^2)(6M_t^2 - 3\bar{M} \beta - 2M)\right]}{4k[M_t^2 \beta + (1 - \beta)\bar{M}]^2} \quad (10)$$

has to be subtracted. The difference $\tilde{M}_c - \tilde{M}_{bias}$ is then substituted for $M_c$ in eq. 5.

The estimator $\tilde{M}_c - \tilde{M}_{bias}$ holds for i.i.d. events, i.e. the parameters of the earthquake-generating distribution must be constant in time, and the quakes must not trigger each other. Therefore the catalog has to be declustered before estimating the corner moment. We call the combination of equations 5, 9, and 10 the k-events Kagan-Schoenberg, or KS$_k$ for short, estimator of the moment rate.

The KS$_k$ estimator requires a catalog containing at least $k$ events and a reasonable estimate about the completeness threshold $M_t$ of that catalog to be computable and useful. If only a single event is available, one can either equate $M_t$ with the size of that event, resulting in $\tilde{M}_c = 0$, or assume that $M_t$ is smaller than the event, resulting in $\tilde{M}_c$ being dependent only on that assumption but not on the observed event. Moreover, $\sigma$ and hence $\tilde{M}_{bias}$ are undefined for a single event. Therefore the KS$_k$ estimator cannot be used with a single event.

The event magnitudes or seismic moments in a real marsquake catalog will carry an uncertainty as discussed by Böse et al. (2018). This uncertainty for individual events is propagated into the uncertainty of NLVR and KS$_k$ estimations in a Monte Carlo fashion by evaluating a number of modified catalogs, where each event's seismic moment is perturbed according to a normal distribution with the standard
deviation cataloged for this individual event, and taking care for error propagation through conversion between different types of magnitudes or magnitudes and seismic moments.

**Emission Probabilities**

Due to the stochastic nature of the tectonic process generating the real quakes, as well as of any catalog simulation process, simulated catalogs will never match observed quake sequences on an event-by-event basis even if the model underlying the simulation matches the real process perfectly, and all parameters are known exactly.

Due to the limitations of the data set we expect to obtain for Mars, we do not consider a maximum-likelihood fit of an assumed type of distribution in order to estimate its parameters feasible.

The moment rate estimation described above is intended to be a direct estimation of one of the parameters of the size-frequency distribution, but is likely to be biased and carry some variance, not the least due to the uncertainty of the seismic moments of observed events. Bias and variance will depend on several factors, such as the parameters of the Tapered Gutenberg-Richter distribution themselves, and the period of time covered by the simulated catalog.

We borrow (without intending to over-stress the analogy) from the field of hidden layer Markov chains the term "emission probability" for the probability that a stochastic process produces a statistical diagnostic with the same value as obtained from another, inaccessible process: In our case, a quake catalog is a realization of a stochastic process which is transformed into a single-valued diagnostic using one of our estimators. This transformation is a secondary stochastic process of which the resulting diagnostic is a single realization. We aim to estimate the probability that a numerical simulation of seismicity catalogs results in the same moment rate estimations as the observed catalog.

The catalog simulation we describe in the next section is a random number generator with three parameters: moment rate, corner moment, and distribution slope. These parameters form a state space,
which we scan on a search grid with a regular node spacing when moment rate and corner moment are expressed as magnitudes following equation 1. For each node, we produce a large number of synthetic catalogs and evaluate the NLVR and KS estimators for all these catalogs, resulting in a distribution of estimations. Comparing these distributions with the respective estimations of the real catalog, we can determine the probability that a certain combination of state parameters makes the catalog simulation emit a moment rate estimation within the uncertainty limits of the moment rate estimation derived from the real catalog.

This finally yields a map of emission probabilities for the scanned part of the state space, which can then be compared with predicted parameters e.g. from Knapmeyer et al. (2006), or with values for Earth and Moon.

Examples will be shown below in sections dealing with seismicity catalogs for Earth and Moon, and synthetic catalogs for Mars. These are computed on a search grid with node spacing of 0.1 moment magnitudes in both moment rate and corner moment dimensions. For each grid node, we evaluate 1000 catalogs, resulting in 3.1 million catalogs for the depicted grids. We evaluate for each node the probability that a given state emits a value within 0.2 magnitude units (corresponding to a factor 2 in seismic moment) from the respective target values as given in figure titles (the GCMT comes without uncertainty ranges for the scalar moment, therefore we use a fixed interval width here). Computation time on a desktop computer fits into a lunch break and thus leaves room for refining the search grid or evaluating more catalogs per grid node.

**Simulation of Seismicity Catalogs**

Equation 2 only predicts the relative frequencies of events. In order to reproduce the seismicity observed during a time interval of duration $\Delta t$, it needs to be normalized in such a way that also the absolute number of events $N(M)$ and the observed cumulative moment $M_{\text{cum}} = \dot{M} \Delta t$ released by all events

""
in the catalog are reproduced. Following Kagan (2002b), the long term average $\alpha_0$ is computed by evaluating eq. 3.

We adjust the – in our context entirely fictitious – completeness threshold $M_t$ such that we obtain catalogs of approximately 10000 events.

The number $N_{\Delta t}$ of events in an actual time interval, e.g. a given calendar year, follows a Poisson distribution describing the probability that the value of $N_{\Delta t}$ is $n$ (e.g. Zhuang et al., 2012)

$$P\{N_{\Delta t} = n\} = \frac{\lambda^n}{n!} e^{-\lambda}$$  \hspace{1cm} (11)

where $\lambda$ is the rate parameter of the distribution.

For a rate parameter exceeding about $10$, the Poisson distribution can be approximated by a normal distribution with mean $\bar{x} = \lambda$ and standard deviation $\sigma = \sqrt{\lambda}$. For time intervals of the order of years and a sufficiently low detection threshold, one can expect hundreds or thousands of quakes, such that this condition is met. We thus draw the number of events per time interval from a normal distribution with $\lambda = \alpha_0$.

The distribution of earthquakes in time (when disregarding aftershocks) also follows a Poisson distribution, as was first shown for California by Gardner & Knopoff (1974). With the number $n$ of events occurring during interval $\Delta T$ already being given by eqs. 3 and 11, is it thus sufficient to distribute the source times uniformly over the time interval (see Zhuang & Touati, 2015, algorithm 10).

Finally, for each source time generated thus, a seismic moment is drawn from the tapered Pareto distribution, following the method described by Vere-Jones et al. (2001), which we reproduce here for self-containedness. We first generate a first set $U_1$ of $n$ uniformly distributed random numbers from the interval $]0,1[$. From these we compute
In addition, we employ the inverse method to produce \( n \) exponentially distributed random numbers from a second set \( U_2 \) of \( n \) uniformly distributed random numbers which are then filtered through the inverse of the exponential distribution

\[
X = -\frac{1}{\lambda} \log(1 - U_2) \quad (13)
\]

to obtain a set \( X \) of exponentially distributed random numbers, where

\[
\lambda = \frac{M + M_t}{M_c} \quad (14)
\]
is the rate parameter of the exponential distribution \( f(\tau) = \lambda \exp(-\lambda \tau) \). Finally,

\[
K = (M + M_t) \cdot \min(y, 1 + X) - M_t \quad (15)
\]
is a tapered Pareto distributed seismic moment value following the distribution given in eq. 2.

It should be noted that assuming a Poisson process is equivalent to the assumption that the individual events are entirely independent of each other, i.e. the process has no memory of the past. This assumption does not apply to foreshock/aftershock sequences, therefore a comparison of real catalogs with synthetic ones generated with the above procedure requires the declustering of the real catalog. Moreover, this kind of catalog simulation might be unsuitable for sources with a periodic driving force, such as deep moonquakes (DMQ).

It has been argued that the slope parameter \( \beta \) is actually not a free parameter but a universal constant with a value \( \beta = 1/2 \) (initially apparently by Vere-Jones, 1976, recently e.g. by Kagan, 2014). Analyses of different tectonic settings (Kagan, 1997), Flinn-Engdahl regions (Kagan, 1999) and rectangular regions defined by parallels and meridians (Godano & Pingue, 2000) support that \( \beta \) is a global constant
for quakes on Earth (except mid-oceanic ridges), however with a value $0.6 \leq \beta \leq 2/3$. Kagan (2014) argues that the deviation from the prediction is a combination of several observational biases.

Thermal moonquakes appear to follow a distribution where most of the moment is released by the small events (implying $\beta > 1$, Cooper & Kovach, 1975), and it is questionable that deep moonquakes, with a stress source that reverts direction on a monthly scale, fall under the same universal relation as subduction zone events resulting from a uniformly progressing slab. So although we follow Knapmeyer et al. (2006) that $\beta = 0.625$ is a good initial choice for Mars, lacking a better one for now, we are aware that this needs to be established experimentally, and it must not be taken for granted.

Data

The following subsections describe three seismicity catalogs for the Earth and one catalog for the Moon that were used as test cases for moment rate estimation. In the absence of a Mars seismicity catalog we give a brief summary of Martian seismotectonic properties that make us conclude that earthquake statistics methods based on terrestrial circumstances are applicable to Mars as well.

GCMT catalog

We downloaded the complete catalog of the Global Centroid Moment Tensor project (GCMT, Dziewonski et al., 1981, Ekström et al., 2012) from IRIS (see "Data and Resources"). This catalog covers the 15456 days from 05. January 1976 (date of the first event in the GCMT) to 17. May 2018, or 42.32 years of 365.25 days, and contains 49178 events with seismic moments between $3.49 \times 10^{15} Nm$ and $5.32 \times 10^{22} Nm$. The completeness threshold for the catalog as a whole is $M_t \approx 2 \times 10^{17} Nm$, but de-
creases with time; especially the year 1976 is significantly less complete, while $M_t$ is lower for the more recent years.

The cataloged events released a cumulative seismic moment of $\sum M_0 = 3.23 \times 10^{23} \text{Nm}$. The ten largest events in this catalog released 47.5 % of $\sum M_0$, and the 615 largest events (i.e. 1.25 % of all events) released 90 % of $\sum M_0$.

**Stable Continental Regions catalog**

The Stable Continental Regions catalog of Schulte and Mooney (2005, see also "Data and Resources") represents a compilation for several sources of historically witnessed events and of events for which instrumental recordings exist, and covers the entire Earth for the time from 495 AD to 2003. Schulte and Mooney have estimated seismic moments for most of the 1373 events in the catalog. Cataloged moments range from $5.3 \times 10^{15} \text{Nm}$ to $7.5 \times 10^{23} \text{Nm}$, the latter however is assigned to a reservoir-associated event in India in September 2000 and is most likely erroneous. We exclude this and another reservoir-associated event for which a moment of $> 10^{23} \text{Nm}$ is given from our analysis, leaving as the largest listed event an earthquake that occurred in July 1819, also in India, with a seismic moment of $6.6 \times 10^{20} \text{Nm} (M_w = 7.8)$.

For each event, the catalog indicates if instrumental records, historical descriptions, or both are available. For events after 1976, the catalog is informed by the GCMT.

**Temporary Installation at Goldstone, CA**

A single Nanometrics Trillium Compact seismometer (120 s eigenperiod) was installed temporarily during summer 2014, in a dry lake bed in the Mojave desert, approximately 60 km NNE of Barstow, California, and approximately 1 km southwest of the 70 m “Mars” antenna dish of NASA’s Deep Space Network (Lorenz et al., 2015). The purpose of this experiment was to gather seismic and microbarometric
recordings of dust devils, to support the interpretation of dust devil recordings expected from Mars. The experiment was not intended to record earthquakes, but these are of course hard to avoid.

The surface of the lake bed is composed of fine mud that grades into denser sediment overlying a rock with increasing seismic velocities with depth, which is likely analogous to the InSight landing site (see Lorenz et al., 2015, Golombek et al., 2017, and Knapmeyer-Endrun et al., 2017). The seismometer was shallowly buried and accompanied by a microbarometer and an anemometer on the surface. The seismic sampling frequency was 100 Hz, but we resampled to 2 Hz to simulate the continuous data stream that will be available from InSight (Lognonné et al., submitted). The seismometer operated from 05. June 2014 to 29. Aug. 2014. Several downtime periods add up to 18 days (24h periods, not to be confused with the 24.6h of the martian rotation period) of lost data, such that a total of 68 days of seismic recordings remains.

During this experiment, the USGS National Earthquake Information Center (NEIC) catalog lists 304 global events with magnitudes of 5 and larger.

We convert the 7 different types of magnitudes found in the NEIC catalog (Ms, mb, ML, and several versions of MW) into seismic moments to quantify source strength, using relations from Wason et al. (2012), and from Bormann et al. (2002).

By visual inspection, we categorize all events according to their signal quality with categories given in Table 2. We expect that magnitudes or seismic moments could be determined from InSight data for events comparable to categories A through E, and sufficiently precise locations to reduce the distance effect in magnitude uncertainty to below 0.3 magnitude units for categories A, B, and E (Böse et al., 2018).
Moonquakes, deep and shallow

Seismic observation of Apollo has detected more than 13000 seismic events (Nakamura, 2003; see "Data and Resources"), encompassing endogenous moonquakes, meteorite impacts, and artificial sources.

The most frequent type of events is the deep moonquakes (DMQ) at depths between 700 km and 1100 km (Nakamura, 2005). They occur in clusters which are activated and deactivated in cycles corresponding to the orbital periods of the Moon-Earth-Sun system. Time lags between individual events of individual clusters are also not random (Bulow et al., 2007, Weber et al., 2010, and references therein).

Kawamura et al. (2017) show that stress drops associated with DMQ are of the order of magnitude of the tidal stresses in their source regions. The DMQ are thus not only triggered but also caused by tidal stresses, i.e. the orbital motions in the Moon-Earth-Sun system can be considered as a part of the DMQ source mechanism. Since orbital motion is described by differential equations that allow evaluation forward and backward in time, the DMQ mechanism has a memory of the past and therefore cannot be i.i.d. Poissonian. The situation appears to be different for the (poorly understood) shallow moonquakes, which we discuss in section "Moon: High Frequency Teleseisms".

Therefore, we focus on shallow moonquakes in the following. The source mechanism of shallow moonquakes is not yet clear but they are likely to be caused by thermal contraction of the Moon (e.g. Binder & Lange, 1980), i.e. the same mechanism expected for marsquakes (Phillips, 1991, Knapmeyer et al., 2006, Plesa et al., 2018). If this is correct, shallow moonquakes can be regarded as lunar analogs of marsquakes.

Martian Seismotectonics

The seismic data available from Mars so far (up to late November 2018) contains only one candidate marsquake, which was recorded by the Viking 2 seismometer (Anderson et al., 1977), but could not iden-
Seismic Moment Rate Estimation, v 2.0.3, last saved 08/01/2019 10:11:00

tified as such unambiguously. At the time of writing, we are thus not in the possession of a seismic data set that allows for statistical evaluation. In this section we describe in more detail what is known about the seismotectonics of Mars, in order to support that our statistical approach is meaningful for Mars as well as for Earth.

After the successful deployment on December 19, 2018, broad band and short period seismic data from the InSight lander is expected to be received from January 2019 onwards. Routine evaluation of events, encompassing the identification of seismic phases, epicenter location based on multi-orbit surface waves and polarization analysis (Böse et al., 2017, Panning et al., 2015), will be conducted by a team at the Swiss Seismological Service located at ETH Zürich (Clinton et al., 2018), the Marsquake Service (MQS). Mars-specific magnitude scales were developed (Böse et al., 2018) and will be used by the MQS to quantify source strength on Mars. The event catalog produced by the MQS will also discriminate between different source types (tectonic, impacts, landslides, and atmospheric effects) based on the experience with Apollo data and physical differences in the source mechanism (see Daubar et al., 2018, concerning impacts, and Clinton et al., 2018, concerning the MQS).

Mars is a one-plate planet which shows no unambiguous indications of subduction zones or mid-ocean ridges, neither in topography nor in free air gravity anomalies or lithospheric magnetic field (e.g. Zuber et al., 2000, Hirt et al., 2012, Morschhauser et al., 2014). Nevertheless, it must have witnessed much seismic activity in the past, as its surface is crisscrossed with complex patterns of normal and thrust faulting. Knapmeyer et al. (2006) mapped more than 8500 normal and thrust faults with a cumulative length of about 680000 km, updated later on to approx. 14800 faults with a cumulative length of 940000 km (Knapmeyer et al., 2008). The shortest of these has a length of approx. 600 meters, while the longest graben that dissects the Tharsis bulge is almost 1450 km long. The Knapmeyer et al. (2006) fault catalog appears to be complete for faults longer than 80 km. The overall size-frequency distribution of Martian faults can be described by a power law with exponent well in the range found for fault popula-
tions on Earth (see Knapmeyer et al., 2006, and references therein). If one assumes that all fault seg-
ments are equally likely to break, and that large marsquakes require long breaking segments, then the
length-frequency distribution of Martian faults is in favor for a power-law-like distribution of Marsquake
sizes.

Taking the age of the youngest surface crossed by a fault as indicator of its age, no age preference for
shorter or longer faults is found, i.e. the size frequency distribution appears to be independent of time. It
must be noted, however, that 75 % or more of the total fault length was produced more than 3.5 Ga ago.
The youngest faults are estimated to have an age of approx. 300 Ma.

Fault populations on Mars show features like fault linkage or relay ramps well known from Earth (see
Schulz et al., 2010, and references therein, for a general treatment). The mechanisms of fault growth are
considered to be the same on Earth and Mars (see Wyrick et al., 2018, for a recent analog study), alt-
ough faults on Mars accumulate less displacement per length than faults on Earth. This can however be
attributed to properties that all scale with gravity (Schulz et al., 2006).

Stresses resulting from differential cooling of the planetary interior are widely accepted as the main
stresses resulting from mantle drag due to convection and found that these might under certain circum-
stances contribute significantly to the moment rate, and especially modify the regional distribution of
effective stresses. Knapmeyer et al. (2006) defined end-member and “typical” models with “strong”,
“weak” and “medium” moment rates of $4.78 \times 10^{18} \, \text{Nm/yr}$, $3.42 \times 10^{16} \, \text{Nm/yr}$, and $5.99 \times
10^{16} \, \text{Nm/yr}$, respectively. Due to different corner moments, their models StrongFew, StrongMany,
WeakFew, WeakMany, and Medium produce $m = 4$ events with recurrence rates between 13 hours and
6.7 years. Based on these and other estimations of martian seismicity, Panning et al. (2017) expect about
100 events per year that are detectable at distances beyond 60°.
While the Moon's seismic event rate is dominated by the deep moonquakes that are tied to Earth tides, Phillips (1991) found that although tidal loading by the sun produces high strain rates, the associated strains are small, and concluded that tidal loading of Mars is unimportant from the seismicity point of view. We expect that Mars, unlike the Moon, has no significant periodic seismicity.

Mars is well known for its giant volcanos. These have been active through much of the Martian history. Although some of the youngest volcanic surfaces might be only a few million years old (Neukum et al., 2004), no actual volcanic activity has ever been reported. The wristwatch pattern of faults around the Alba Patera volcano actually shows that its magma chamber retreated (Cailleau et al., 2003). Volcanic contributions to seismic activity are thus considered unlikely.

Fresh impacts have been found on Mars in images taken repeatedly from the same area (Malin et al., 2006), and might provide semi-controlled seismic sources when the location of an impact detected by a seismometer can be identified in imagery. Quantitative analysis shows that the recurrence intervals of globally detectable impacts is between one year and 10 years (Teanby & Wookey, 2011), where the longer intervals are expected to be more likely due to expected noise conditions. Contrary to an intuition one might have based on the cratered surface of especially the southern hemisphere of Mars, impacts are also not expected to dominate the seismic record.

Results

To assess the reliability of moment rate estimations we conduct numerical tests where we compare estimated rates with the GCMT catalog of earthquakes from the time interval 1976 to 2018 (section "Earth"), and also with the known true values of synthetic catalogs (section "Generic Synthetic Cata-
logs"). Finally, we generate synthetic catalogs for several models of Martian seismicity and investigate if and how we can distinguish between them (section "Mars-specific synthetic catalogs").

**Earth**

Since the GCMT, as the most comprehensive and homogenous long-term catalog of moment tensor solutions, relies on a global network of about 200 stations, we first need to establish that a single station can provide data representative for Earth's global moment rate. The question here is what the global completeness threshold for a single (broadband) seismometer is, and how long it takes until the largest recorded event is above this completeness threshold.

We then compare how the output of different estimators evolves in time, i.e. re-evaluate all estimators for each new event in the catalog throughout the time covered. Finally, we compare estimations for different tectonic settings (subduction zones, mid-ocean ridges, and stable continental regions) by subdividing the GCMT catalog according to Flinn-Engdahl-Zones and by evaluation of an additional catalog dedicated to stable continental regions (as an analog for the single plate regime on Mars) which also contains events predating the instrumental recording of earthquakes.

Since we will likely not be able to determine reliably the slope parameter of the martian seismicity distribution, we use a fixed value of $\beta = 2/3$ throughout all these tests.

**Single Station Completeness Threshold**

We use as single station the temporary installation at Goldstone described in section "Temporary Installation at Goldstone, CA". To find its completeness threshold, we compare the size-frequency (survivor) distribution of events detected at Goldstone with the size-frequency distribution of the GCMT
(Figure 2). Distributions from short recordings are less smooth than that of the whole GCMT data base, and also differ between independent time windows of the same duration. In addition, the magnitude-to-moment conversion is based on empirical relations between uncertain quantities and may thus provide an uncertain moment value only, thus adding to the roughness of the curve. We therefore do not attempt to pin down the completeness threshold very precisely, but only estimate that it is somewhere between $10^{18} \text{Nm}$ and $10^{19} \text{Nm}$ for events of grade A alone, and possibly lower when taking A and B, or A, B, and E together. The GCMT lists 1273 events exceeding $10^{19} \text{Nm}$, and 5891 events exceeding $10^{18} \text{Nm}$, resulting in recurrence times of 12.2 and 2.7 days, respectively.

The strongest events in the grade A, $A \cup B$, and $A \cup B \cup E$ subsets are the same, so it is sufficient to use one of them for moment rate estimation that relies on the strongest events. From the grade A events alone, we obtain a NLVR moment rate estimate of $1.03 \times 10^{22} \text{Nm/yr} \ (m_S = 8.6)$, while the $K_{10}$ estimator returns $5.79 \times 10^{21} \text{Nm/yr} \ (m_S = 8.4)$.

For comparison, we subdivide the GCMT catalog into 180 non-overlapping time windows, each of which has the same 86 days duration as the Goldstone experiment. Using the NLVR estimator we obtain a moment rate equivalent to magnitude $8.28 \pm 0.42$ from these time windows, while the $K_{10}$ estimator results in an equivalent magnitude of $8.25 \pm 0.24$.

Compared with the $7.61 \times 10^{21} \text{Nm/yr} \ (m_S = 8.52)$ obtained from the complete GCMT catalog, and comparing with Figure 3 and the $K_{10}$ emission probabilities in Figure 4, we conclude that the Goldstone experiment suffices to estimate the Earth's global seismic moment rate with acceptable accuracy.

One can thus expect that a seismometer installation comparable to the one at Goldstone produces a globally representative “largest event ever” time series three months or less after deployment, meaning that all comparable installations worldwide would produce the same such time series. For any longer recording time it is thus justified to evaluate the GCMT catalog instead of a single station catalog.
Moment Rate Estimations: Entire Earth

We compare the evolution in time of several different moment rate estimators. For this exercise we use only events from 1977 onwards since the GCMT completeness threshold for 1976 is significantly higher than for later years. We expect that the $NLVR$ and especially the $KS_k$ estimators have a certain burn-in time: one cannot evaluate $KS_k$ before having cataloged $k$ events. Moreover, if the catalog contains events below the threshold for global detectability, the first listed events will not be representative for the global moment rate. By excluding 1976 from consideration we avoid mixing the burn-in time of the GCMT project with that of the rate estimation.

As a baseline we consider the summation of all events contained in the catalog between a time $T_{start}$ and up to a time $t$, and divide by the number of years (assuming 365.25 days per year) elapsed between $T_{start}$ and $t$ to obtain an annual rate. The resulting time series $M_{s,base}(t)$ is thus based on all data available at time $t$.

We compare this baseline with estimations based on the single strongest event ($M_{s,NLVR}(t)$, based on the $NLVR$ estimator) and on the 2, 5, 10, and 50 strongest events ($M_{s,KS_k}(t)$, based on the $KS_k$ estimator). The latter are computed only after a sufficient number of events is available, thus the respective time series start somewhat later than $M_{s,base}(t)$ and $M_{s,NLVR}(t)$. All time series are compiled in Figure 3.

A common feature of all time series is that they decay like $1/t$ or shallower as long as no large new contribution to the moment release occurs. This is of course most pronounced in $M_{s,NLVR}(t)$, which can have a new peak only if an event occurs that is larger than any event before. But the effect is also clear in the $M_{s,KS_k}(t)$ time series, where smaller peaks indicate that new events entered the top-$k$ list. Since new events can enter this list at any position, the value of the $KS_k$ estimator will change more often with increasing $k$, hence the time resolution of $M_{s,KS_k}(t)$ increases with $k$.

In analogy to taking the absolute value of the difference between two values we compute
\[
\exp \left( \left| \log \left( \hat{M}_s(\text{est}, t) \right) - \log \left( \hat{M}_s(\text{base}, t) \right) \right| \right) \quad (16)
\]

to evaluate the deviation between an estimator and the baseline value, as it is sufficient to know that one of the two values has to multiplied by some factor to obtain the other one.

All estimators except \( KS_2 \), which sometimes overestimates by three orders of magnitude, follow the full-catalog baseline closely. Even the single-event \( NLVR \) estimation never deviates by more than a factor of nine, as does \( KS_5 \). Both \( KS_{10} \) and \( KS_{50} \) are always within a factor 2.5 or less from the baseline, most of the time the factor is smaller than 2. When subdividing the GCMT into 21 consecutive, non-overlapping intervals of 2 years, the \( KS_{10} \) value is always within a factor 5 of the 42-years average, and within a factor 2 in half of all cases. All estimators tend to underestimate. We however do not think that the evaluation of a single catalog is sufficient to declare this a rule.

**Moment Rate Estimations: Specific Tectonic Settings**

Earth’s seismicity is mainly due to plate tectonics, and by far most of its seismicity (approximately 75% of all events) occurs at active plate boundaries. Subduction zones alone account for approximately 60% of all earthquakes. Mars, on the other hand, is a one-plate planet. It is thus desirable to investigate if reasonable moment rate estimations can be made for selected, either Earth specific or Mars specific tectonic settings rather than for the entire Earth.

Based on the Flinn-Engdahl regionalization scheme of seismic regions (Young et al., 1995), we extract events belonging to mid-ocean ridges and subduction zones from the GCMT according to the region IDs listed in the appendix 0. Since the F-E-Regions define regions on the Earth’s surface but not in its interior, we further subdivide the events classified as “subduction zone” into two depth layers, above and below 100 km, in order to clearly separate subducting slabs from crustal events. The number of events sorted into each of the four resulting groups of events is given in Table 3.
As reported earlier (e.g. Bird & Kagan, 2004), the distribution for mid-ocean ridge events appears to be significantly steeper than for the other groups (Figure 5), the slopes of which appear to be essentially identical with that obtained for the complete catalog. Especially a test using mid-ocean ridge events is thus a scenario different from the other groups, or the GCMT taken as a whole.

For each group we evaluate (a) the sum of all events divided by the number of years covered by the catalog (42.3 years), (b) the NLVR estimator, and (c) the KS10 estimator, thus obtaining three values for each group. The resulting moment rates, expressed both in Nm/yr and as moment magnitude of a single event releasing the same amount of seismic moment, are also summarized in Table 3.

In most cases, both the NLVR and the KS10 estimator underestimate the value obtained by summing up all events of a subset. The ratios between these estimators and the complete summation are well in the range found for the entire Earth. This holds for all four subsets.

Thus the results obtained for the GCMT are encouraging, but we note that an evaluation of a station installed e.g. in early 2004 would look different, since its event catalog would report two very strong events (Banda Aceh and Tohoku) within only 6 years.

Neither subduction zones nor mid ocean ridges exist on Mars. We thus use the Earthquake Catalog for Stable Continental Regions of Schulte & Mooney (2005, see also "Data and Resources") as a better analog of martian seismicity.

We subdivide the catalog into two groups: the first contains events for which no instrumental record is available, while the second contains events that were interpreted solely based on instrumental records (a third group of events, where historical information and instrumental data is interpreted together, was excluded from our analysis). The goal is to compare moment rate estimations resulting from the historical events with independent estimations for the instrumental epoch.
We do not consider the 21 events listed for the time between 495 AD and 1500 AD (only three of which occurred before 1000 AD). This may look like a somewhat arbitrary modification of the catalog, but we think it is justified since an extension of the covered period by 1000 years biases the duration of observation in a more severe way than the omission of 21 events with moment magnitudes below 6.1 biases the released seismic moment. We also delete two reservoir-associated events that occurred in India in 2000, since the catalog assigns unlikely seismic moments beyond $10^{23} \text{Nm}$ (corresponding to $M_W > 9$) to both of them, which are also in clear contradiction with the listed magnitude values (around 5). We thus obtain 226 events from 1500 AD to 1949 and 879 events from 1950 to 2003. Their size-frequency distributions are shown in Figure 5. Here the group of historical events shows a much lower slope than all other distributions shown. This is a property expected for incomplete catalogs, as this historical events list certainly is. The slope of the distribution of the instrumentally evaluated events is very similar to that of the GCMT, but this is not surprising since many of the corresponding catalog entries are based on the GCMT.

The resulting moment rate estimates are consistent within each of the two groups (see Table 3), but the values obtained for the events from the instrumental epoch are also consistently higher than those for the historical events, by factors between 1.7 and 4.1, with the NLVR estimator producing the largest deviation. Although these deviations are still in the range of variability seen for the different estimators in Figure 3, they exhibit a certain trend. If expressed as magnitudes, we find that the differences between the individual estimates is no larger than the uncertainty attributed to the individual magnitude estimations as listed in the catalog, which is typically 1 magnitude unit, and never below 0.4 magnitude units.

In summary we find that the evaluation of the GCMT is encouraging, although it was purely accidental that its largest events occurred late. The deviations obtained with the Stable Continental Regions catalog might be attributed to the very circumstantial nature and general uncertainty of historic observations,
but might also point to a systematic shortcoming of our approach. To further investigate bias and variance of our estimators, we conduct an extensive series of tests with synthetic catalogs.

**Moon: High Frequency Teleseisms**

For the shallow moonquakes, we use here the seismic moments estimated by Oberst (1987). In total, 28 shallow moonquakes were detected by the Apollo network. We evaluated the annual seismic moment release of the following subsets: 1) All shallow moonquake events; 2) Events during 1972-1973; 3) Events during 1974-1975; 4) Events during 1976-1977; 5) Events during 1973-1974; 6) Events during 1975-1976. We tested if the seismic moment release evaluated from two-years subsets differ or agrees with that obtained from the complete observation period. We also tested how fast the obtained seismic moment converges to the reference value obtained with the full dataset. We reevaluate the seismic moment release every time a new shallow moonquake is detected and studied the evolution of obtained seismic moment rates like with the GCMT. The results are summarized in Figure 6. KS$_{10}$ Emission probabilities for the moment rate resulting from all shallow moonquakes together are shown in Figure 7. The maximum emission probability coincides with the rate obtained from all events. In the case of the Moon, the scattering of obtained seismic moment rate is rather small and the difference is about 50% in maximum. The small deviation is likely to be due to the smaller range of magnitude (from $4.7 \times 10^{12}$ $Nm$ to $1.6 \times 10^{15}$ $Nm$) and the small number of events for shallow moonquakes. Likewise, the seismic moment rate converged relatively quickly after 4-5 events. Unlike on the Earth, the possible source of excitation is limited and result in smaller variety of events. This may be the case on Mars as well, allowing to obtain a representative seismic moment rate rapidly.

**Generic Synthetic Catalogs**

We set up a logarithmically equidistant grid of $M_S$ and $M_C$ values via an equidistant grid of equivalent magnitudes. We vary $m_S$ from 3 to 10 in steps of 0.1 magnitude units. The equivalent magnitude of the
corner moment is varied relative to \( m_\text{s} \) from \( m_\text{s} - 4 \) to \( m_\text{s} + 4 \), also in steps of 0.1 magnitude units. This results in a \( 81 \times 71 = 5751 \) elements matrix of combinations of moment rates and corner moments.

The resulting event rates, computed for an assumed completeness threshold magnitude \( m_\text{t} = 2 \), range from \( 10^{-216} \) to \( 10^9 \) events per year (some parameter combinations result in even smaller rates that were not numerically resolvable, these cases were excluded from the simulation process). Neither of these extremal values is suitable to produce meaningful catalogs. We therefore adjust the completeness threshold individually for each combination of moment rate and corner moment in order to obtain catalogs of uniform sizes for all cases. A simple iterative search scheme is used to solve eq. 3 for \( M_\text{t} \) numerically for this purpose. In analogy to the planned life time of the InSight mission, we generate catalogs covering 2 terrestrial years (or 730 terrestrial days), each containing approximately 10000 events (in a few cases, where this requires \( M_\text{t} > m_\text{c} \), we force \( M_\text{t} < m_\text{c} \) and obtain catalogs with about \( 10^5 \) events).

The result of any moment rate estimation will depend on the individual realization of the catalog generator, depending on what the largest cataloged event actually is. To analyze the resulting variability of the estimated moment rate, we thus generate a total of 1410 catalogs (a number chosen with respect to the overall computation time) for each of the 5751 parameter combinations, resulting in approximately eight million usable catalogs. The complete simulation can be run overnight on a desktop PC.

For each catalog, we evaluate the NLVR and \( \text{KS}_{10} \) estimators and record the resulting bias between the moment rate’s equivalent magnitude \( m_\text{s} \) used during catalog generation and the estimated value’s equivalent magnitude \( m_{\text{est}} \), defined as

\[
m_{\text{bias}} = m_\text{s} - m_{\text{est}} \quad (17)
\]
i.e. the bias is positive for underestimation. From the biases obtained for catalogs resulting from the same value of the ratio $M_S/M_c$ (or from the same difference $m_S - m_c$) we construct distribution densities, median values and the boundaries of 95% quantiles centered around the median, i.e. intervals which contain 95% of all bias values.

**Estimation with correct slope**

In a first test, we use the same slope $\beta \in \{0.4, 0.5, 0.6, 0.625, 2/3\}$ during catalog generation as well as during the evaluation. We generate independent sets of approximately 8 million catalogs for the evaluation using either the NVLR estimator or the $KS_{10}$ estimator. Since we already established that the $KS_k$ estimator gets better with increasing $k$, we now use a fixed value of $k = 10$ which is realistic for Mars and expected to perform well in the light of the evaluation of the GCMT.

If the corner moment is known (which can be expected only in synthetic cases) a two-dimensional histogram relating the bias between true and estimated moment rate to the difference $m_S - m_c$ can be constructed (left panels in Figure 8 (NLVR) and Figure 9 ($KS_{10}$)). Individual moment rate estimates may vary strongly between individual catalogs, by as much as three or more units of equivalent magnitude.

The ratio $M_S/M_c$ controls systematic variations of distribution median and width, and NLVR and $KS_{10}$ show different behavior: While the NLVR estimator shows the greater variance for $m_S - m_c < 0$, the variance of the $KS_{10}$ estimator is larger for $m_S - m_c > 0$. Both estimators are near unbiased if $m_S = m_c$.

If the correct value of $m_S - m_c$ is unknown, the 2D distribution densities depicted in the left panels of Figure 8 and Figure 9 reduces to the marginal densities depicted in the right panels. These show that, for $\beta = 2/3$, the probability of overestimating the moment rate is approximately 10% for the NLVR estimator, and 41% for the $KS_{10}$ estimator. The probability that the estimated value is within ±1 magnitude
units of the true value is 49% for the NLVR estimator, and 72% for the $KS_{10}$ estimator for the given range of possible $m_S - m_c$ differences.

These percentages depend on the value of the slope parameter $\beta$ and increase as $\beta$ increases. As Figure 10 (NLVR) and Figure 11 ($KS_{10}$) show, the distribution densities get narrower and more symmetric with increasing $\beta$: The probability that the estimated moment rate is within $\pm 1$ magnitude units from the true value increases as the distribution narrows, and the probability of an overestimation increases as the distribution become more symmetric about the zero bias.

**Estimation with incorrect slope**

As mentioned in the introduction, estimating the slope parameter from a single station catalog is likely biased towards small slopes, since only a fraction of the smaller events will be observed. Since both of our moment rate estimators make use of the slope, we must ask how sensitive the outcome is to an incorrectly guessed slope.

We again generate catalogs with a true slope of $\beta \in \{0.4, 0.5, 0.6, 0.625, 2/3\}$, but use the same set of values used for the estimation from our synthetic catalogs. We thus obtain a matrix of true and assumed slopes and evaluate the fraction of moment rate estimates that falls within $\pm 1$ or $\pm 2$ magnitudes from the true value, and also how likely we overestimate the actual moment rate. As before, we evaluate approximately 8 million catalogs for each combination of true and assumed slope parameter.

In Figure 12 we show the resulting percentages, i.e. the width of distribution densities as shown in Figure 10 and Figure 11, for both the NLVR (top) and $KS_{10}$ (bottom) estimator. Although the width parameters depend on both the true slope and the slope assumed by the estimator, the influence of the latter on the number of cases where the estimated moment rate is within $\pm 1$ or $\pm 2$ magnitudes from the true value is much smaller than that of the true slope. It is thus sufficient if the assumed slope is approximately correct within a relatively wide range. This holds for both estimators.
A notable difference is the fraction of cases where the actual moment rate is overestimated: For the NLV estimator, this fraction depends mainly on the assumed slope and not as much on the true slope, whereas it is about equally sensitive on both values for the $KS_{1,0}$ estimator.

**Mars-specific synthetic catalogs**

The tests described so far indicate that variance and bias of both the NLVR and the $KS_{k}$ estimator are such that we cannot expect to solve the question of the Martian moment rate unambiguously, by inverting some estimated value into a best fitting rate. The emission probability approach using a search grid of moment rates and corner moments is more promising. Here we investigate in how far it is possible to distinguish hypothetical states by these emitted rate estimates, and in how far longer experiment life times support this distinction.

We consider the five end-member seismicity models of Knapmeyer et al., (2006), which mark some extreme corners of their parameter space. These models were named *WeakFew, WeakMany, Medium, StrongFew*, and *StrongMany* according to their relative moment rates (weak, medium or strong) and event rates (few, medium, or many). Since Knapmeyer et al. (2006) use the truncated rather than the tapered Gutenberg-Richter distribution, we have to convert their maximum seismic moment $M_{Max}$ into corner moments $M_{c}$ in order to keep the moment rate unchanged when switching to the tapered distribution, using

$$M_{C} = M_{Max} \left[ \frac{\beta}{\Gamma(2-\beta)} \right]^{1/(1-\beta)} \quad (18)$$

after Kagan (2002b). The conversion factor amounts to approximately 0.416 for $\beta = 2/3$, or 0.391 for $\beta = 0.625$ as used in the five models. Moment rates and corner moments of the models are summarized in Table 4. It is worth noting that $m_{S} - m_{C}$, which proved crucial for variance and bias of the estimators.
in the previous tests, is negative for the *StrongFew, Medium,* and *WeakFew* models (but close to zero for *Medium,* but positive for the *StrongMany* and *WeakMany* models (rightmost column of Table 4).

To assess the effect of experiment life time, we consider catalogs covering 1, 2, 4, 8, 16, and 32 (earth) years. The exponential increase of durations was chosen in order to compensate for the power law decay of event probabilities with magnitude. Considering even longer times appears to be unnecessary, since no lander with such a life time is known, and since we hope that improved follow-up experiments will make the entire discussion obsolete at some time.

We generate $10^6$ catalogs, with approximately 10000 events per catalog, for each duration, and thus obtain six different probability density functions for estimated moment rates for each of the five models. We repeat this for both the NLVR and the $KS_{10}$ estimator and compare the resulting probabilities graphically (Figure 13 and Figure 14). Independent realizations are used for the two estimators, but given the smooth PDFs that we obtain, we are convinced that this does not affect our results.

For both estimators, doubling the experiment life time moves the maximum of the histograms (i.e. the mode) in Figure 13 and Figure 14 by a constant amount towards larger values (meaning a multiplication by a constant factor on the seismic moment scale). This shows that the choice of an exponential series of life times was appropriate to investigate the long term behavior.

The NLVR and $KS_{10}$ estimators however show a significant difference concerning the width (i.e. standard deviation) of the distributions: While the NLVR distributions become narrower in most cases (not very much in the case of the *WeakFew* model, left hand side of Figure 15), the $KS_{10}$ standard deviation decreases not as much, or even grows (in the case of the *WeakMany* and *StrongMany* models, right hand side of Figure 15).

The changes in mode and standard deviation of NLVR work together in such a way that probabilities left of the maximum decrease quickly with time, while probabilities right of the maximum increase only
slowly with time. This makes the evolution of the curves look like a caterpillar pulling its rear end towards the head before moving its head forward.

The KS₁₀ estimator behaves differently: here the whole curve moves towards the right essentially unchanged like the shell on the back of a crawling snail, with the notable exception of the StrongMany model which behaves like a caterpillar pushing its head forward towards the right with its tail fixed (Figure 14).

The two estimators also behave differently when it comes to the rejection of models.

After two years of registration, NLVR moment rate estimates between $2 \times 10^{16} \text{Nm}$ and $1.3 \times 10^{18} \text{Nm}$ (equivalent magnitudes $m_s$ between 4.8 and 6) would be rather inconclusive, since all five models have significant probabilities to produce estimates in this range. An estimated rate of less than $3.9 \times 10^{16} \text{Nm}$ ($m_s = 5$) would exclude the StrongMany model. A NLVR estimation below $1 \times 10^{16} \text{Nm}$ ($m_s = 4.6$) would additionally exclude the Medium and StrongFew models. A NLVR result larger than $1.6 \times 10^{17} \text{Nm}$ ($m_s = 5.4$) obtained after two years, on the other hand, would exclude the WeakFew, WeakMany and StrongMany models.

With the KS₁₀ estimator, the StrongMany model could be excluded if the rate estimate is below $8.9 \times 10^{17} \text{Nm}$ ($m_s = 5.9$). KS₁₀ estimates below $5.6 \times 10^{16} \text{Nm}$ or above $1.6 \times 10^{17} \text{Nm}$ ($m_s$ between 5.1 and 5.4) are crucial to distinguish between the WeakMany, Medium, and StrongFew models. More such thresholds could be identified.

As a sketch for the application to InSight data, we simulate a single two years catalog for the Knapmeyer et al. (2006) Medium scenario with parameters as given in Table 4, and evaluate the KS₁₀ estimator. This results in an estimated moment rate of $2.72 \times 10^{17} \text{Nm/yr}$. The emission probability map favors moment rates close to the one used for catalog generation, and clearly allows the rejection of a moment rate as low as the Moon’s (Figure 16). Based on the simulated catalog, a moment rate as high as
the Earth’s could only be accepted when assuming that marsquakes larger than the largest known earthquakes are possible.

The following observations are crucial here: Firstly, in principle, it is possible to reject at least some of the Knapmeyer et al. (2006) models using the NLVR estimator alone. Second, the moment rate thresholds to exclude models evolve with time, i.e. with the time span covered by the catalog. And third, thresholds excluding individual models differ between the NLVR and KS\textsubscript{10} estimators. Also, the use of the KS\textsubscript{10} estimator was a decision made here to keep the amount of tests to run in a reasonable range. If more events become available, there is however no reason not to use them for a KS\textsubscript{k>10} estimation. It is thus meaningful to use different estimators in parallel, and to also track the evolution of the estimators with time.

### Discussion

Our tests with terrestrial data demonstrate that the Earth’s moment rate can be estimated from a single, short-lived broadband station, even if it is a temporary installation rather than being placed in a state of the art observatory shelter (the Lorenz et al., (2015) installation at Goldstone serves as an example). The list of the largest $k$ events becomes representative within a few weeks. This minimum time frame of course depends on the actual moment and event rates and is likely longer on Mars. It will be a crucial exercise to determine from the observations what the duration of this “burn in” time on Mars is. Besides the usual methods to determine the completeness threshold of catalogs, a comparison of the actual Mars catalog with modeled detection probabilities might be useful. For the Earth, the moment rate can be estimated to within a factor 2.5 or better after a burn in of less than 2 weeks from the 10 largest events (Figure 3).
A subdivision of Earth’s seismicity into specific tectonic settings (subduction zones, subducting slabs alone, mid-ocean ridges, stable continental regions) still allows estimating the moment rate of these regions within the uncertainty of the seismic moments of the individual events used. For the stable continental settings we however obtain larger deviations, indicating that either the approach is less reliable than the whole-GCMT results suggest, or that the estimators show a significant variance.

To assess more generic properties of our estimators, we conduct a series of Monte Carlo experiments, simulating seismicity catalogs for a variety of parameters, and compare the estimated moment rates with the known values used for the simulation.

The estimation of the uncertainty of the bias estimate critically depends on the assumed range of $M_S/M_C$, while knowledge of the slope $\beta$ is of minor importance, thus allowing to use a generic value like $\beta = 2/3$. To our best knowledge, there is no self-consistent limitation of $M_S/M_C$ in the origins of the tapered Gutenberg-Richter distribution. Maximum likelihood estimation of $M_C$ often produces a lower limit only (Bird & Kagan, 2004), and any estimator will suffer from the fact that the necessary large events are also the most rare ones. Thus giving a range of $M_S/M_C$ is hardly possible in advance and will remain challenging in the course of the experiment, as far as the InSight mission to Mars is concerned.

We limited the difference of equivalent magnitudes to $|m_S - m_C| < 4$, motivated by the range $-2.4 < m_S - m_C < 1.7$ found with the end-member models of Knapmeyer et al. (2006), and adding some margin.

A physical argument creating an upper limit for the corner moment arises from geological considerations on the largest possible event, as discussed in section "Seismic Moment Rate Estimation" for the Earth, assuming an event with a fault plane as large as the planet’s surface.

The estimation for the seismic moment upper bound used there scales with $R^3$, thus the corresponding estimation for Mars is about 15% of that obtained for Earth, or $10^{28}Nm$ (or $M_W = 12.6$). The crater-
ing record and derived surface ages for Mars show that no event involving the entire Martian surface occurred during the recent 3.8 billion years or so (taking the systematic age difference between Hesperian northern lowlands and Noachian southern highlands as a reference for the creation of the dichotomy, whatever caused it), the longest faults of Knapmeyer et al., (2006) actually suffice for $M_W \approx 9$ if the entire fault breaks in a single event. We thus predict that the corner moment of the Martian quake statistic is such that events of $M_W = 12.6$ do practically not occur in the sense of the discussion in section "Seismic Moment Rate Estimation", i.e. that the corner moment is at least one magnitude below $10^{28} Nm$ and does not exceed $10^{25} Nm$ (this is however a weak prediction given that the Knapmeyer et al. (2006) models hardly exceed $10^{20} Nm$ for good reasons).

The InSight mission will have to deal with a physically reasonable subset of all possible parameters. Testable predictions that can be challenged individually by InSight data have been made in the literature, and a determination of the moment rate sufficiently accurate to reject some models would be a significant achievement.

Our simulations reveal that both the NLVR and KS estimations show considerable biases and variances. Nevertheless the NLVR estimator, as the most simple one, will allow constraining the actual moment rate of Mars when comparing its output with simulated distributions of given seismicity models. Depending on experiment life time and the chosen estimators, threshold exists that clearly discriminate between the Knapmeyer et al. (2006) models. The most promising approach for the interpretation of moment rate estimates appears to be to scan the moment-rate-corner-moment state space like in Figure 4, Figure 7, and Figure 16, and determine the probability that a certain state emits the moment rate estimated from observed events.

Our approach relies on several assumptions:

- Marsquakes are i.i.d. events resulting from a Poisson process.
• Their size-frequency distribution can be described by the Tapered Gutenberg-Richter distribution.

• The slope of the distribution is such that the moment rate is governed by the strongest, rather than by the smallest, events i.e. $\beta < 1$.

• A reasonable guess for the slope is $\beta = 2/3$.

• Martian moment rate and corner moment are sufficiently high to allow for the detection of at least a few globally representative events, or, in other words, InSight’s detection threshold will be low enough that some threshold magnitude exists above which the Marsquake catalog will be complete.

It might not be possible to test all these assumptions with InSight data alone. Establishing the validity of the Tapered Gutenberg-Richter distribution, for example, will likely require much longer observation times and more than one station – even the decade-long catalogs available for Earth do not yet provide a clear picture of the distribution of the strongest earthquakes. All these assumptions, however, can be tested in principle. We intend to evaluate several additional diagnostics based on event location and distribution of events in time to assess the validity of our assumptions.

**Conclusion**

Statistics is best when it is based on a large data set. The boundary conditions of interplanetary missions however make it challenging to obtain quake catalogs comparable to the GCMT. We nevertheless have to make the most of whatever data we obtain.

With equations (2) and (3), the tapered Gutenberg-Richter distribution of quakes is described by slope $\beta$, corner moment $M_C$, moment rate $\dot{M}_S$, and an observer-dependent completeness threshold moment $M_t$. While we frequently find statements in the literature that thousands of quakes are neces-
necessary to determine slope and corner moment reliably, we have discussed ways to estimate the moment rate based on sparse observations.

Two distinct approaches are proposed, the NLVR estimator which relies on the single largest event ever observed, and the \( k \) estimator, which evaluates the \( k > 1 \) largest events.

We have evaluated the performance of both estimators numerically. First we apply them to the current GCMT catalog, covering the time from January 1976 to May 2018 and containing nearly 50000 earthquakes. Secondly, we use synthetic catalogs generated from the Tapered Gutenberg-Richter distribution and represented by a total of more than 560 million catalogs worth of almost 2.5 billion years of simulated seismicity.

We find that the proposed estimators will allow rejecting at least some of the published seismicity models by their predicted moment rates. The uncertainty of the estimates depends on \( \dot{M_S}/\dot{M_C} \), but is sufficiently small to discriminate several published seismicity models even after only two years mission life time on Mars.

Data and Resources

The Apollo seismic event catalog can be found at http://www-udc.ig.utexas.edu/external/yosio/PSE/catsrepts/


Waveform data from the Goldstone experiment was obtained from Lorenz et al. (2015), we kindly refer to their Data and Resources section.

The coordinates of Flinn-Engdahl region boundaries was retrieved from USGS, at ftp://hazards.cr.usgs.gov/web/hazdev-geoserve-ws/FE/, last accessed 15.05.2018

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References


Lognonné, P., and 180 co-authors, SEIS: The Seismic Experiment for Internal Structure of InSight, Space Science Reviews, submitted


## Tables

**Table 1 Annual seismic moment rate of Earth, Moon, and Mars.**

<table>
<thead>
<tr>
<th>Planet</th>
<th>Reference</th>
<th>Method</th>
<th>Annual Rate [Nm/yr]</th>
<th>Annual Rate [Equiv. Mag. per yr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>GCMT</td>
<td>observed quakes, 1976-2018</td>
<td>$7.61 \times 10^{21}$</td>
<td>8.5</td>
</tr>
<tr>
<td>Moon</td>
<td>Oberst, 1987</td>
<td>observed HFT quakes</td>
<td>$7.27 \times 10^{14}$</td>
<td>3.8</td>
</tr>
<tr>
<td>Mars</td>
<td>Anderson et al., 1977</td>
<td>Viking non-observation of quakes</td>
<td>$Moon \leq Mars$</td>
<td>$&lt; Earth$</td>
</tr>
<tr>
<td></td>
<td>Goins &amp; Lazarewicz, 1979</td>
<td>Viking non-observation of quakes</td>
<td>$&lt; Earth$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Phillips, 1991</td>
<td>1D lithospheric cooling model</td>
<td>$4.8 \times 10^{18}$</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>Golombek et al., 1992</td>
<td>Observed surface faulting</td>
<td>$10^{18}$</td>
<td>5.9</td>
</tr>
<tr>
<td>Study</td>
<td>Methodology</td>
<td>Moment Range (N m)</td>
<td>Magnitude Range</td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------------------</td>
<td>-----------------</td>
<td></td>
</tr>
<tr>
<td>Golombek, 2002</td>
<td>Observed surface faulting</td>
<td>$10^{18}$ to $10^{19}$</td>
<td>5.9 to 6.6</td>
<td></td>
</tr>
<tr>
<td>Knapmeyer et al., 2006</td>
<td>1D lithospheric cooling model, weak, medium, and strong scenario</td>
<td>$3.42 \times 10^{16}$ to $4.78 \times 10^{18}$</td>
<td>4.96, 5.8, 6.4</td>
<td></td>
</tr>
<tr>
<td>Taylor et al., 2013</td>
<td>Observed surface faulting</td>
<td>$&gt; 5.5 \times 10^{16}$</td>
<td>&gt;5.09</td>
<td></td>
</tr>
<tr>
<td>Plesa et al., 2018</td>
<td>3D whole-mantle thermal evolution model</td>
<td>$5.7 \times 10^{16}$ to $3.9 \times 10^{19}$</td>
<td>5.1 to 7</td>
<td></td>
</tr>
</tbody>
</table>

Equivalent magnitudes are moment magnitudes computed according to eq. 1 assuming that the entire moment release of one year is put into a single event.
### Table 2: Quality levels used to categorize Goldstone seismograms

<table>
<thead>
<tr>
<th>Category</th>
<th>Definition</th>
<th>Event count</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>prominent arrivals of various body and surface wave phases</td>
<td>38</td>
</tr>
<tr>
<td>B</td>
<td>P arrival and Rayleigh waves clearly above noise level</td>
<td>19</td>
</tr>
<tr>
<td>C</td>
<td>P arrival and Rayleigh waves barely above noise level</td>
<td>13</td>
</tr>
<tr>
<td>D</td>
<td>Only body waves visible</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>Only surface waves visible</td>
<td>60</td>
</tr>
<tr>
<td>F</td>
<td>Invisible</td>
<td>170</td>
</tr>
</tbody>
</table>
### Table 3 Annual moment rate estimations for different settings on Earth

<table>
<thead>
<tr>
<th>Subset (No. of events)</th>
<th>$\Sigma$</th>
<th>$K_{S_{10}}$</th>
<th>$NLVR$</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>All events (49178)</td>
<td></td>
<td></td>
<td></td>
<td>$10^{20} \text{ Nm/yr}$</td>
</tr>
<tr>
<td></td>
<td>76.1</td>
<td>54.9</td>
<td>33.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.52</td>
<td>8.43</td>
<td>8.28</td>
<td>Magnitude</td>
</tr>
<tr>
<td>Subduction Zones (32855)</td>
<td></td>
<td></td>
<td></td>
<td>$10^{20} \text{ Nm/yr}$</td>
</tr>
<tr>
<td></td>
<td>66.9</td>
<td>54.9</td>
<td>33.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.48</td>
<td>8.43</td>
<td>8.28</td>
<td>Magnitude</td>
</tr>
<tr>
<td>Mid-Ocean Ridges (7783)</td>
<td></td>
<td></td>
<td></td>
<td>$10^{20} \text{ Nm/yr}$</td>
</tr>
<tr>
<td></td>
<td>1.63</td>
<td>1.11</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.41</td>
<td>7.3</td>
<td>7.29</td>
<td>Magnitude</td>
</tr>
<tr>
<td>Subduction Zones, $z \geq 100 \text{ km}$ (7297)</td>
<td></td>
<td></td>
<td></td>
<td>$10^{20} \text{ Nm/yr}$</td>
</tr>
<tr>
<td></td>
<td>7.38</td>
<td>6.1</td>
<td>2.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.85</td>
<td>7.79</td>
<td>7.53</td>
<td>Magnitude</td>
</tr>
<tr>
<td>Subduction Zones, $z &lt; 100 \text{ km}$ (26003)</td>
<td></td>
<td></td>
<td></td>
<td>$10^{20} \text{ Nm/yr}$</td>
</tr>
<tr>
<td></td>
<td>59.8</td>
<td>52.1</td>
<td>33.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.45</td>
<td>8.41</td>
<td>8.28</td>
<td>Magnitude</td>
</tr>
<tr>
<td>Stable Continental Regions, historic events (226)</td>
<td></td>
<td></td>
<td></td>
<td>$10^{20} \text{ Nm/yr}$</td>
</tr>
<tr>
<td></td>
<td>0.0571</td>
<td>0.0405</td>
<td>0.0415</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.44</td>
<td>6.34</td>
<td>6.35</td>
<td>Magnitude</td>
</tr>
<tr>
<td>Stable Continental Regions, instrumental</td>
<td></td>
<td></td>
<td></td>
<td>$10^{20} \text{ Nm/yr}$</td>
</tr>
<tr>
<td></td>
<td>0.101</td>
<td>0.124</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.6</td>
<td>6.66</td>
<td>6.75</td>
<td>Magnitude</td>
</tr>
</tbody>
</table>
Annual moment rate estimations for several subsets of the GCMT catalog from 1976 to 2018, for historically witnessed events in stable continental regions between 1500 and 1949, and finally instrumentally recorded events in stable continental regions, between 1950 and 2003. The first column is the estimate resulting from summing up all events in the subset, the second is the KS for the 10 strongest events, and the third column is the NLVR estimator resulting from the single strongest event. Each estimate is given in Newton meters per year and as moment magnitude of a single event releasing the same amount of Newton meters.
### Table 4 Annual moment rate models for Mars

<table>
<thead>
<tr>
<th>Model</th>
<th>$\dot{M}_S$ [Nm/yr]</th>
<th>$M_{Max}$ [Nm]</th>
<th>$M_C$ [Nm]</th>
<th>$m_S - m_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>StrongFew</td>
<td>$4.78 \times 10^{18}$</td>
<td>$3.36 \times 10^{20}$</td>
<td>$1.31 \times 10^{20}$</td>
<td>-0.96</td>
</tr>
<tr>
<td>StrongMany</td>
<td>$4.78 \times 10^{18}$</td>
<td>$3.42 \times 10^{16}$</td>
<td>$1.33 \times 10^{16}$</td>
<td>+1.70</td>
</tr>
<tr>
<td>Medium</td>
<td>$5.99 \times 10^{17}$</td>
<td>$2.41 \times 10^{18}$</td>
<td>$9.42 \times 10^{17}$</td>
<td>-0.13</td>
</tr>
<tr>
<td>WeakMany</td>
<td>$3.42 \times 10^{16}$</td>
<td>$3.42 \times 10^{16}$</td>
<td>$1.33 \times 10^{16}$</td>
<td>+0.27</td>
</tr>
<tr>
<td>WeakFew</td>
<td>$3.42 \times 10^{16}$</td>
<td>$3.37 \times 10^{20}$</td>
<td>$1.32 \times 10^{20}$</td>
<td>-2.39</td>
</tr>
</tbody>
</table>

Moment rates, maximum moments, and corresponding corner moments of the end-member seismicity models of Knapmeyer et al., 2006. All models use a slope parameter of $\beta = 0.625$. 
Figures

Figure 1 Schematic depiction of the tapered Gutenberg-Richter distribution in survivor form, showing the number $N$ of events exceeding moment $M$. Bullets mark characteristic moment values: $M_d$ - detection threshold, no event smaller than this is contained in a catalog; $M_t$ - completeness threshold, all events above this are cataloged; $M_s$ - size of an event corresponding to the average moment rate; $M_c$ - corner moment. The corner moment may or may not be larger than $M_s$. Slanted dashed line: prediction according to a Gutenberg-Richter law with the same slope. Horizontal dashed lines: (lower) number of events above completeness threshold $M_t$, to be used for event rate estimation; (upper) number of events in catalog. Gray: an arbitrary grid to indicate that the graph is drawn in log-log coordinates (at $M_c$, the tapered distribution is actually below the classical GR distribution by a factor $e$).
Figure 2 Size-frequency survivor distributions for events detected at Goldstone (in groups containing grade A only, or grades A and B, or grades A, B, and E, as indicated in legend. Note that the ABE line is partially covered by the AB line, which in turn gets covered by the A line), the subset of events listed in the GCMT for the uptime of the Goldstone experiment (labeled 5), the whole GCMT scaled down to a duration of 86 days (labeled 4). The gray curves in the background show distributions for consecutive 86-days time windows (i.e. days 1 to 86, 87 to 173, 174 to 260, ... etc.) throughout the entire GCMT catalog.
Figure 3 Evolution in time of moment rate estimations for Earth. The bold black curve is based on summation of all events in the GCMT catalog. The horizontal line marks its value at the end of the cataloged time (01.01.1977 to 18.05.2018). Other lines show time series for the NLVR and $K S_k$ estimators according to the legend.
Figure 4 $K_{S10}$ emission probability for the events detected at Goldstone (grade A events only). Dots mark the moment rates and corner moments of all five Mars seismicity scenarios of Knapmeyer et al. (2006), pixel color encodes the emission probability for matching the target rate within 0.2 magnitudes, as derived from 1000 simulated catalogs per pixel. Coordinate axes are labeled in terms of equivalent moment magnitudes. The vertical black line to the right gives the moment rates of the Earth, as listed in Table 1.
Figure 5 Size-Frequency distributions of several subsets of the GCMT catalog and of the Stable Continental Regions catalog. Subduction zones and mid-ocean ridges are identified by using the Flinn-Engdahl regionalization scheme, and subduction zone events are further subdivided into events occurring above and below 100 km depth. The complete GCMT catalog is shown for comparison. For the Stable Continental Regions catalog, the distributions of events from 1500 to 1949 and from 1950 to 2003 are shown separately.
Figure 6 Evolution in time of moment rate estimations for shallow moonquakes. Lines denote estimator output as in Figure 3, applied to the seismic moments of Oberst (1987). Squares denote the sum of two calendar years of registrations, plotted at the relative time of the first of January of the first year, intervals used are 1972-1973, 1974-1975, 1976-1977, 1973-1974, and 1975-1976.
Figure 7 KS10 emission probability for shallow moonquakes. Dots mark the moment rates and corner moments of Mars seismicity scenarios of Knapmeyer et al. (2006) for comparison, pixel color encodes the emission probability for matching the target rate within 0.2 magnitudes, as derived from 1000 simulated catalogs per pixel. Coordinate axes are labeled in terms of equivalent moment magnitudes. The two vertical black lines give the moment rates of Moon and Earth, as listed in Table 1.
Figure 8 NLVR Cumulative moment bias for synthetic catalogs, using $\beta = 2/3$ in both generation and interpretation of catalogs. Left: Histograms resulting from the 8108910 synthetic catalogs generated for the cumulative moment to corner moment ratio indicated on the x-axis, shading encodes the logarithm of the percentage of realizations in each moment rate bias bin of width 0.1. Positive values on the y-axis indicate underestimation. Solid line: median, dashed lines: upper and lower boundary of the 95%-quantile centered around the median. Right: marginal distribution density resulting from row-wise integration of the figure to the left. Thick line: all catalogs, other: relation between moment rate $M_s$ and corner moment $M_C$ as indicated in legend.
Figure 9 $K_{S10}$ Cumulative moment bias for synthetic catalogs, using $\beta = 2/3$ in both generation and interpretation of catalogs. See Figure 8 for a description of graph elements.
Figure 10 NLVR estimator marginal distribution densities for different slope parameters. Left: the marginal distribution densities for all catalogs (corresponding to the grey curve in Figure 8). Each curve represents approx. 8 million evaluated catalogs. Right: percentage of estimations that are within ±1 (blue) or ±2 magnitudes (black) from the true moment rate, and the fraction of overestimated moment rates (red).
Figure 11 $K_{S10}$ estimator marginal distribution densities for different slope parameters. See Figure 10 for a plot description.
Figure 12 Distribution width parameters for NLVR (top row) and $K_{S_{10}}$ (bottom row) estimators. Each uniformly colored patch indicates the width parameter of distribution densities as shown in Figure 10 and Figure 11. True slope and slope assumed during catalog evaluation were taken from \{0.4, 0.5, 0.6, 0.625, 2/3\} to form a square matrix of test cases. From left to right: percentage of cases with overestimated moment rate, percentage of moment rate estimations within $\pm 1$ magnitude of true value, or within $\pm 2$ magnitudes, respectively. Panels in the same subplot column share the same color range width, although the absolute values differ.
Figure 13 Distribution of NLVR estimators for Mars-specific seismicity models of Knapmeyer et al. (2006) and registration times of 1, 2, 4, 8, 16, and 32 years (envelopes of histograms with bin width 0.01 magnitudes, scaled to percents). Panels from top to bottom show the distributions for seismicity models as indicated in the panel. Note the different y axis ranges.. The seismic moment rate is given as the moment magnitude of a single event that releases the entire moment in one event (the axis range corresponds to $1.3 \times 10^{12} ... 4 \times 10^{22} N m$). Darker hues correspond to shorter registration times, the two-year case also uses a thicker line. Vertical lines from left to right denote moment rates (as equivalent magnitudes) for the Moon, both Weak scenarios, Medium scenario, both Strong scenarios, and Earth.
Figure 14 As Figure 13, but for the $KS_{10}$ estimator.
Figure 15 Standard deviations and modes of the distribution densities shown in Figure 13 and Figure 14. Brighter shades indicate longer registration times, symbols encode the end member models of Knapmeyer et al. (2006). Diamonds: WeakFew, Upward triangles: WeakMany Downward triangles: Medium, Circles: StrongMany, Squares: StrongFew
Figure 16 KS10 emission probability for a synthetic Mars Medium catalog. Dots mark the moment rates and corner moments of Mars seismicity scenarios of Knapmeyer et al. (2006) for comparison, pixel color encodes the emission probability for matching the target rate within 0.2 magnitudes, as derived from 1000 simulated catalogs per pixel. Coordinate axes are labeled in terms of equivalent moment magnitudes. The two vertical black lines give the moment rates of Moon and Earth, as listed in Table 1.
Appendix

Flinn-Engdahl zones

In the analysis of the GCMT catalog, we used the Flinn-Engdahl zonation scheme, in the revision of Young et al. (1995), to classify events as belonging to either subduction zones or mid-ocean ridges.

We consider as mid-ocean ridge events all events in zones

143 149 150 401 401 402 403 404 405 406 408 409 410 411 413 419 424 425 426 427 428
429 433 634 635 636 637 638 641 648 681 682 683 688 691 692 693 694 698 699 727 729 735
737 738 739 753 754

We consider as subducting slabs all events in the subduction zones as listed above, and deeper than 100 km.

1285 387 388 396 654 657 658 659 660 685 700 701 702 703

We consider as subducting slabs all events in the subduction zones as listed above, and deeper than 100 km.