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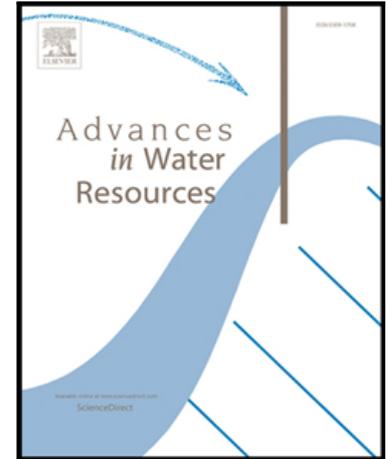
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Highlights

- Integrating seepage dynamic in subsurface flow modeling at the hillslope scale.
- Coupling storage variation and seepage through a robust partition formulation.
- Smooth convex regularization of seepage onset, sharp seepage retreat.
- Numerical application and demonstration of hillslope storage Boussinesq equations.

1 **DYNAMIC COUPLING OF SUBSURFACE AND SEEPAGE FLOWS SOLVED WITHIN A**
2 **REGULARIZED PARTITION FORMULATION**

3 J. Marçais^{1a,b}, J.-R. de Dreuzy^b, J. Erhel^c,

4 ^a*Agroparistech, 16 rue Claude Bernard, 75005 Paris, France*

5 ^b*Géosciences Rennes (UMR 6118 CNRS), Université de Rennes 1, Campus de Beaulieu,*
6 *35042 Rennes Cedex, France*

7 ^c*INRIA Rennes-Bretagne Atlantique, campus de Beaulieu, 35042 Rennes, France*

8 **Abstract**

9 Hillslope response to precipitations is characterized by sharp transitions from
10 purely subsurface flow dynamics to simultaneous surface and subsurface flows.
11 Locally, the transition between these two regimes is triggered by soil saturation.
12 Here we develop an integrative approach to simultaneously solve the subsurface
13 flow, locate the potential fully saturated areas and deduce the generated sat-
14 uration excess overland flow. This approach combines the different dynamics
15 and transitions in a single partition formulation using discontinuous functions.
16 We propose to regularize the system of partial differential equations and to use
17 classic spatial and temporal discretization schemes. We illustrate our method-
18 ology on the 1D hillslope storage Boussinesq equations (Troch et al., 2003). We
19 first validate the numerical scheme on previous numerical experiments without
20 saturation excess overland flow. Then we apply our model to a test case with
21 dynamic transitions from purely subsurface flow dynamics to simultaneous sur-
22 face and subsurface flows. Our results show that discretization respects mass
23 balance both locally and globally, converges when the mesh or time step are
24 refined. Moreover the regularization parameter can be taken small enough to
25 ensure accuracy without suffering of numerical artifacts. Applied to some hun-
26 dreds of realistic hillslope cases taken from Western side of France (Brittany),
27 the developed method appears to be robust and efficient.

¹Corresponding author: jean.marcais@polytechnique.edu

28 *Keywords:* Subsurface flow, Saturation Excess, Seepage, ODE with
29 discontinuous right-hand side, Regularization, Boussinesq equations

30 **1. Introduction**

31 Under the same term, runoff gathers several processes of different origins
32 including infiltration excess overland flow and saturation excess overland flow
33 (Kirkby, 1978; Bonell, 1998; Horton, 1933; McGlynn et al., 2002; Freeze & Har-
34 lan, 1969). While infiltration excess overland flow is controlled by rainfall in-
35 tensity and surface properties like roughness (Horton, 1933; Smith & Goodrich,
36 2006; Darboux et al., 2002), saturation excess overland flow is generated by
37 subsurface flow and saturation dynamics (Dunne & Black, 1970; Sophocleous,
38 2002). It occurs locally in so-called saturated source areas (Ogden & Watts,
39 2000) when the soil column is saturated up to the surface (Dunne, 1978; Musy
40 & Higy, 2004). It comes from precipitation that cannot infiltrate (sometimes
41 called direct precipitations onto saturated areas) and from subsurface flows that
42 exfiltrate and return to the surface, i.e. seepage flow or return flow (Dunne &
43 Black, 1970). Once the soil column remains fully saturated, the subsurface flux
44 stabilizes, remaining equal to the product of the soil transmissivity by the local
45 hydraulic gradient. At the surface, fluxes can be highly variable depending on
46 exfiltration and precipitations dynamics. Saturated source areas are found at
47 the bottom of slopes, in the vicinity of rivers, in wetlands where water table
48 can quickly rise up to the surface as well as upslope in slope hollows that force

49 flowpaths to converge and exfiltrate (Fan & Bras, 1998; Troch et al., 2002; Brut-
50 saert, 2005; Birkel et al., 2015). Subsurface and saturation excess overland flow
51 have been proven to be important for providing water for evapotranspiration
52 (Maxwell & Condon, 2016), enhancing erosion (Fox & Wilson, 2010), increasing
53 groundwater flooding risks (Holman et al., 2009; Kreibich & Thieken, 2008; Ha-
54 bets et al., 2010; Bauer et al., 2006; Miguez-Macho & Fan, 2012), shaping the
55 residence time distribution (Tetzlaff et al., 2014; Rinaldo et al., 2015; Harman,
56 2015) maintaining anoxic conditions in the soil and promoting denitrification
57 hotspots (Pinay et al., 2015).

58 There is a sharp transition when infiltration and subsurface flows do no longer
59 sustain the full saturation of the soil column (Vivoni et al., 2007). Overland
60 flow vanishes, a partially saturated area develops in lieu of the saturated source
61 area, the water table falls down below the surface and subsurface flow becomes
62 dynamic and non-linearly dependent on saturation. Such transitions between so-
63 called fully saturated and partially saturated regimes are spatially variable and
64 dynamic. They have been modeled by coupling shallow water and groundwater
65 equations (LaBolle et al., 2003; Barthel & Banzhaf, 2016; Camporese et al.,
66 2010) either by an exchange of fluxes (Govindaraju & Kavvas, 1991; Panday
67 & Huyakorn, 2004; Markstrom et al., 2008) or by assigning the shallow water
68 equations as a boundary to the groundwater equations (Kollet & Maxwell, 2006).
69 Another method is to solve the groundwater equations for a prescribed position

70 of the saturated source areas (Bresciani et al., 2014, 2016; Beaugendre et al.,
71 2006) and to iterate until convergence is met (Diersch, 2013; Harbaugh, 2005).
72 The seepage front (defined as the intersection of the subsurface water table
73 with the land surface) is deduced accordingly (Anderson et al., 2015; Batelaan
74 & De Smedt, 2004) and saturation excess overland flow is eventually derived
75 from mass balance computations.

76 While these integrated hydrological surface-subsurface models (for a com-
77 prehensive review, see Fatichi et al. (2016)) are adapted to well instrumented
78 catchments or critical zone observatories, they require high computation capac-
79 ities (Putti & Paniconi, 2004) and a lot of data to be calibrated with (Reggiani
80 et al., 1998) making them difficult to parametrize because of their long run time.
81 They are also prone to equifinality issues (Beven, 2006) and are sometimes sub-
82 ject to numerical instabilities (Doherty & Christensen, 2011). For applications
83 where the only available information are a DEM, the rainfall time series and a
84 discharge time series, non intensive process-based models have been proposed
85 (Troch et al., 2003; Broda et al., 2012) but, to the best of our knowledge, they
86 do not take into account the non linearity, described above, coming from the
87 dynamic interactions between the water table and the land surface.

88 Here, we propose an integrated approach to simultaneously solve the sub-
89 surface flow, locate the potential saturated source areas and deduce the gener-
90 ated saturation excess overland flow. This approach is well suited to the sharp

91 transitions presented previously. We hypothesize that surface processes can be
92 simplified as follows. We assume instantaneous flood routing as surface flows
93 are some orders of magnitude faster than subsurface flows (Dunne & Black,
94 1970; Fan & Bras, 1998). We consider infiltration processes as negligible. In
95 temperate climates, most of the saturation excess overland flow occurs downhill
96 and only a small amount of this flow is likely to infiltrate (Musy & Higy,
97 2004).

98 We model subsurface flow with the Boussinesq hydraulic groundwater theory
99 based on Darcy's law and Dupuit-Forchheimer assumption (Boussinesq, 1877;
100 Brutsaert, 2005; Troch et al., 2013). It expresses that hydraulic head responds
101 to flow through transmissivity feedbacks without inertial effects (Rodhe, 1987;
102 Bishop et al., 2011). We work at the hillslope scale with the hillslope storage
103 Boussinesq equations (Troch et al., 2003), which include the geologic, pedologic
104 and geomorphologic controls on subsurface flow dynamics (Bachmair & Weiler,
105 2012; Savenije, 2010; Freer et al., 1997; Lanni et al., 2013). They indeed model
106 subsurface flow at the soil/bedrock interface (Freer et al., 2002; Tromp-van
107 Meerveld & McDonnell, 2006), which is the most likely to generate saturation
108 excess overland flow in humid and steep terrains with conductive soils (Weiler
109 et al., 2006). They have also been extended to model the coupling between
110 shallow and deep groundwater flow (Broda et al., 2012).

111 First, we describe the partially saturated and fully saturated flow regimes

112 that can be encountered on hillslopes following the two cases of Fan & Bras
113 (1998) (section 2.1) and combine them in a partial differential system deriving
114 from partition considerations (section 2.2). We regularize this system for nu-
115 merical integration, discretize it spatially with a mixed finite element scheme
116 and obtain a system of differential equations (section 2.3). Second, we validate
117 the method and analyze its convergence properties (sections 3.1 and 3.2). Fi-
118 nally we discuss its efficiency and robustness on realistic hillslope cases (section
119 3.3).

120 2. Methods

121 We first recall the hillslope storage Boussinesq equations (Troch et al., 2003;
122 Hilberts et al., 2004; Paniconi et al., 2003) and add to them the overflow con-
123 dition when saturation reaches the surface. We show how the two regimes of
124 partial and total saturation are generally formalized and further propose an al-
125 ternative integrated partition formulation. The issued differential discontinuous
126 equations are then regularized for numerical integration and spatially discretized
127 with a mixed finite element scheme.

128 2.1. Hillslope storage Boussinesq equations

129 2.1.1. Case 1: Subsurface flow only

130 Below land surface, when the soil is not fully saturated, only subsurface flow
131 occurs. Hillslope storage Boussinesq equations physically describe the relation

132 between subsurface flow and saturation (Troch et al. (2003), case 1 of Figure
 133 1a). Following Dupuit-Forchheimer assumption, discharge is proportional to
 134 the saturated thickness. Discharge is further integrated transversally to the
 135 main slope direction to yield a 1D continuity problem where discharge is also
 136 proportional to the width w [m] of the transect profile (Figure 1b). The other
 137 key physical characteristic of the hillslope is its slope θ [rad] describing the
 138 mean evolution of the soil layer height. The integrated flux over the transect
 139 $Q(x, t)$ [$\text{m}^3 \text{s}^{-1}$] is linked to the subsurface water storage $S(x, t)$ [m^2] through
 140 the following set of equations:

$$\left\{ \begin{array}{l} \frac{\partial S}{\partial t}(x, t) = -\frac{\partial Q}{\partial x}(x, t) + N(t)w(x) \\ Q(x, t) = -\frac{kS(x, t)}{f} \left(\cos \theta \frac{\partial}{\partial x} \left(\frac{S}{fw} \right) (x, t) + \sin \theta \right) \\ 0 \leq S(x, t) \leq S_c(x) \\ S(x, t) < S_c(x) \text{ or } -\frac{\partial Q}{\partial x}(x, t) + N(t)w(x) < 0 \text{ ,} \end{array} \right. \quad (1)$$

141 where x [m] represents the distance to the channel varying between 0 at the
 142 river and L at the water divide, N [m s^{-1}] is the infiltration, k [m s^{-1}] is the
 143 hydraulic conductivity, f [-] is the drainable porosity. The subsurface water
 144 storage $S(x, t)$ is defined in Troch et al. (2003) by $S(x, t) = f w(x) h(x, t)$ where
 145 h is the groundwater elevation height [m] (Figure 1). $S(x, t)$ cannot exceed
 146 the maximum subsurface water storage S_c [m^2] defined by $S_c(x) = f w(x) d$

147 where d [m] is the soil depth. For illustration purposes and with no loss of
 148 generality, we assume in what follows that k , f , θ and d are constant. The first
 149 equation of system (1) is the mass balance equation stating that the temporal
 150 variation of storage results from the local variation of subsurface flows and
 151 from the infiltration. The second line is Darcy's equation, integrated vertically
 152 and laterally, written as a function of the subsurface water storage S . The
 153 third equation just reminds that S is positive and cannot exceed the maximum
 154 subsurface water storage S_c . This system applies as long as the condition of the
 155 fourth line is fulfilled, i.e. for partially saturated soil columns ($S(x, t) < S_c(x)$)
 156 or for fully saturated soil columns desaturating ($-\frac{\partial Q}{\partial x}(x, t) + N(t)w(x) < 0$).

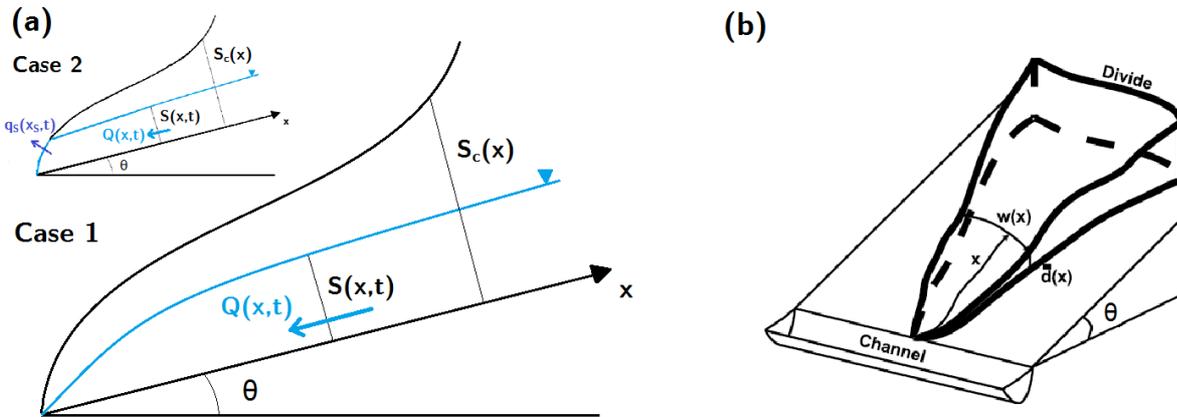


Figure 1: (a) Cross-section view of the hillslope with a constant slope θ . The water table location $S(x, t)$ is indicated in blue. The subsurface flux $Q(x, t)$ is marked by the blue arrow. The saturation excess overland flow $q_s(x_s, t)$ (case 2) is materialized by the purple arrow and the surface by $S_c(x)$. Cases 1 and 2 represent partially saturated and locally fully saturated hillslopes respectively. (b) 3D view of the hillslope with an illustration of the width function $w(x)$ (Adapted from Troch et al. (2003)).

157 *2.1.2. Case 2: Subsurface flow and dynamic saturation excess overland flow*

158 If, at the location x_S , the soil is fully saturated and there is a positive flux
 159 balance, the excess in the flux balance is identified to the saturation excess
 160 overland flow q_S [$\text{m}^2 \text{s}^{-1}$] as it cannot be assigned to storage temporal variations
 161 ($\frac{\partial S}{\partial t}(x_S, t) = 0$, case 2 of Figure 1a). The system of equations becomes:

$$\left\{ \begin{array}{l} q_S(x_S, t) = -\frac{\partial Q}{\partial x}(x_S, t) + N(t) w(x_S) \\ \frac{\partial S}{\partial t}(x_S, t) = 0 \\ Q(x_S, t) = -\frac{kS_c(x_S)}{f} \left(\cos \theta \frac{\partial}{\partial x} \left(\frac{S_c}{fw} \right) (x_S, t) + \sin \theta \right) \\ S(x_S, t) = S_c(x_S) \text{ and } -\frac{\partial Q}{\partial x}(x_S, t) + N(t) w(x_S) \geq 0 . \end{array} \right. \quad (2)$$

162 The fourth line ensures that the soil is fully saturated and that there is a positive
 163 flux balance ($-\frac{\partial Q}{\partial x}(x_S, t) + N(t) w(x_S) \geq 0$). As shown by the first equation,
 164 saturation excess overland flow is made up of two terms representing the seepage
 165 flow ($-\frac{\partial Q}{\partial x}$) and the direct precipitations onto saturated areas (Nw) (Dunne
 166 & Black, 1970).

167 Solving the dynamic transition between these two regimes (essentially due
 168 to interactions between the soil water table and the land surface) requires to
 169 change of system of equations (system (1) or (2)) depending on the state of
 170 saturation and subsurface flow (S and Q). It is the last equation in systems (1)

171 and (2) that controls the transition.

172 2.2. Partition Problem

173 To avoid switching between regimes and equation sets, we propose an alter-
 174 native partition formulation. This formulation reconciles systems (1) and (2) in
 175 a single system by partitioning the incoming flux $-\frac{\partial Q}{\partial x} + N w$ in the variation
 176 of the subsurface water storage $\frac{\partial S}{\partial t}$ and in the saturation excess overland flow
 177 q_S :

$$\left\{ \begin{array}{l} \frac{\partial S}{\partial t}(x, t) + q_S(x, t) = -\frac{\partial Q}{\partial x}(x, t) + N(t)w(x) \\ q_S(x, t) = \mathcal{G}\left(\frac{S(x, t)}{S_c(x)}\right) \mathcal{R}\left(-\frac{\partial Q}{\partial x}(x, t) + N(t)w(x)\right) \\ Q(x, t) = \frac{kS(x, t)}{f} \left(\cos \theta \frac{\partial}{\partial x} \left(\frac{S}{fw} \right) (x, t) + \sin \theta \right) \\ 0 \leq S(x, t) \leq S_c(x) . \end{array} \right. \quad (3)$$

178 \mathcal{G} (Figure 2a) is the function defined in $[0, 1]$ by $\mathcal{G}(u) = \mathcal{H}(u - 1)$, \mathcal{R} is the
 179 ramp function (Figure 2b) defined in \mathbb{R} by $\mathcal{R}(u) = u \mathcal{H}(u)$, where \mathcal{H} is the
 180 Heaviside step function (Figure 2c):

$$\begin{aligned} \mathcal{H} : \quad \mathbb{R} &\longrightarrow \{0, 1\} \\ u &\longmapsto \begin{cases} 0 & \text{if } u < 0 \\ 1 & \text{if } u \geq 0 . \end{cases} \end{aligned} \quad (4)$$

181 In fact, system (3) can be readily expressed as a partition formulation by
 182 rewriting its first two equations:

$$\begin{cases} \frac{\partial S}{\partial t}(x, t) = \left(1 - \mathcal{G}\left(\frac{S(x, t)}{S_c(x)}\right) \mathcal{H}\left(-\frac{\partial Q}{\partial x}(x, t) + N(t)w(x)\right)\right) \left(-\frac{\partial Q}{\partial x}(x, t) + N(t)w(x)\right) \\ q_S(x, t) = \left(\mathcal{G}\left(\frac{S(x, t)}{S_c(x)}\right) \mathcal{H}\left(-\frac{\partial Q}{\partial x}(x, t) + N(t)w(x)\right)\right) \left(-\frac{\partial Q}{\partial x}(x, t) + N(t)w(x)\right) \end{cases} \quad (5)$$

183 It is the quantity $\mathcal{G}(S/S_c)\mathcal{H}(-\partial Q/\partial x + Nw)$ that triggers the dynamic
 184 transitions between the two regimes. When it is null, system (3) reduces to (1).
 185 When it is equal to 1, system (3) reduces to (2).

186 The fourth line of system (3) ensures that the subsurface water storage S
 187 remains positive and lower than S_c .

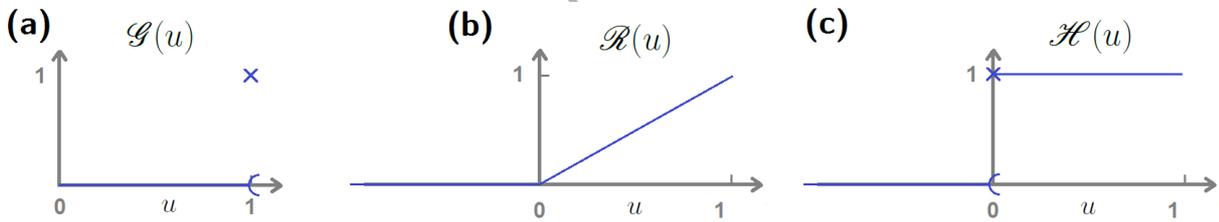


Figure 2: Illustration of (a) the discontinuous function \mathcal{G} (b) the ramp function \mathcal{R} (c) the Heaviside function \mathcal{H} defined by equation (4).

188 2.3. Numerical methods

189 System (3) is a set of partial differential equations with discontinuous right-
 190 hand sides which can be formally defined in the weak sense of distributions. To
 191 solve the partial differential system (3) numerically, \mathcal{G} is challenging because
 192 of its discontinuity at $u = 1$. Various numerical techniques exist to handle

193 the discontinuous functions present in partial derivatives systems (Agosti et al.,
 194 2015; Bouillard et al., 2007; Kumar et al., 2014; Hoffmann et al., 2016). System
 195 (3) can be rewritten, still in a weak sense, as a non linear complementarity
 196 problem, solved with a semi-smooth Newton method (Pang & Gabriel, 1993;
 197 De Luca et al., 1996). Here, to solve system (3), we choose to regularize the dis-
 198 continuous function \mathcal{G} , discretize in space using a mixed finite element method
 199 and use classic methods for temporal integration (variable-step, variable-order
 200 solver).

201 2.3.1. Regularization

202 To smoothen the transition between partially and fully saturated regimes,
 203 we use the convex function \mathcal{G}_r (Haddou & Migot, 2015) defined by :

$$\begin{aligned} \mathcal{G}_r : [0, 1] &\longrightarrow [0, 1] \\ u &\longmapsto \exp\left(-\frac{1-u}{r}\right) \end{aligned} \quad (6)$$

204 where $r > 0$. The regularized function \mathcal{G}_r converges to \mathcal{G} when r tends to 0. The
 205 function \mathcal{G}_r is continuous in $[0, 1]$, convex and differentiable in $[0, 1[$. The left
 206 derivative in 1 ($\lim_{\substack{x \rightarrow 1 \\ x < 1}} \mathcal{G}'_r(x) = \mathcal{G}'_r(1^-) = \frac{1}{r}$) ensures a sharp transition between
 207 the partially and fully saturated regimes. The regularization coefficient $r > 0$
 208 controls the stiffness of the transition between the two states (Figure 3).

209 Replacing the discontinuous function \mathcal{G} by its continuous counterpart \mathcal{G}_r
 210 leads to the system:

$$\left\{ \begin{array}{l} \frac{\partial S}{\partial t}(x, t) + q_S(x, t) = -\frac{\partial Q}{\partial x}(x, t) + N(t)w(x) \\ q_S(x, t) = \mathcal{G}_r\left(\frac{S(x, t)}{S_c(x)}\right) \mathcal{R}\left(-\frac{\partial Q}{\partial x}(x, t) + N(t)w(x)\right) \\ Q(x, t) = -\frac{kS(x, t)}{f} \left(\cos \theta \frac{\partial}{\partial x} \left(\frac{S}{fw} \right) (x, t) + \sin \theta \right) \\ 0 \leq S(x, t) \leq S_c(x) \end{array} \right. \quad (7)$$

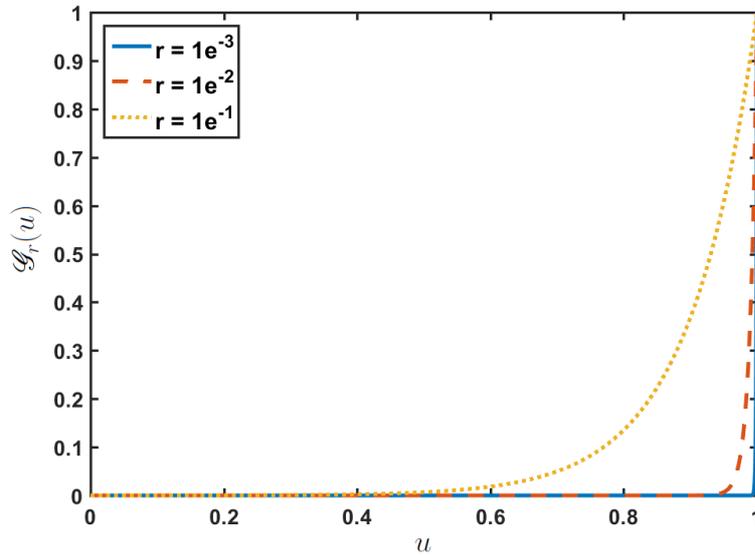


Figure 3: Function \mathcal{G}_r regularized from function \mathcal{G} (figure 2). Function \mathcal{G}_r is defined by equation (6) and represented for different values of the regularization parameter r .

211 2.3.2. Space and time discretization

212 We discretize the 1D hillslope in N_x elements and use a mixed finite ele-
 213 ment method to derive from the system (7) estimates of the soil storage S , the
 214 subsurface flux Q and the excess overland flux q_S (Douglas & Roberts, 1985).
 215 S and q_S are discretized at the cell centers and Q at their edges following the

216 classic mixed finite element methodology. As for regular grids, finite differences
 217 lead to the same expression as mixed finite element (Chavent & Roberts, 1991),
 218 we express spatial derivatives by their finite difference approximation. In prac-
 219 tice, the continuity equation (third equation in system 7) is discretized at the
 220 cell centers and the integrated Darcy equation (second equation of system 7)
 221 is discretized at the edges. Q and $\frac{\partial}{\partial x} \left(\frac{S}{fw} \right)$ are defined at the cell edges by
 222 imposing their continuity between two adjacent cells. The multiplicative factor
 223 $\frac{kS}{f}$ at the cell edge is taken as the arithmetic average between the neighboring
 224 cell centres. The discretization in space of the system (7) eventually leads to
 225 a semi-discrete system of ordinary differential equations. We use the variable
 226 time step and variable order ode15s MATLAB[®] solver based on the backward
 227 differentiation formulas (BDF) of orders 1 to 5 (Shampine et al., 1999).

228 2.3.3. Metrics for convergence analysis

229 We define the metrics for the convergence analysis of the discretization
 230 scheme using a discrete L^2 norm. Let $\{t_i, i = 1, \dots, N_t\}$ a set of given times
 231 in $[0, T]$ with $t_0 = 0$ and $\{x_k, k = 1, \dots, N_{ref}\}$ a set of given points in $[0, L]$
 232 with $x_0 = 0$. We define the discrete norm of a function $f(x, t)$ with given values
 233 $f(x_k, t_i)$ by:

$$\|f\|^2 = \sum_{i=1}^{N_t} \sum_{k=1}^{N_{ref}} |f(x_k, t_i)|^2 (t_i - t_{i-1}) (x_k - x_{k-1}) \quad (8)$$

234 In order to compare a function f with a reference function g , we introduce the
 235 relative error metric ϵ_f defined by:

$$\epsilon_f = \frac{\|f - g\|}{\|g\|} \quad (9)$$

236 with adapted computations when f and g display different spatial discretization
 237 schemes. In practice, the times t_i are provided by user-defined external time-
 238 steps and the points x_k are cell centers or cell edges of a reference mesh.

239 3. Results

240 We assess the partition formulation (7) discretized with a mixed finite ele-
 241 ment scheme on several numerical experiments. We use two numerical exper-
 242 iments of Troch et al. (2003) without any overland flow generated for the first
 243 one, and with locally saturated hillslope for the second one. First, we use one
 244 of the numerical experiments of Troch et al. (2003) where no overland flow is
 245 generated. Then we use a numerical experiment of Troch et al. (2003) where
 246 the hillslope saturates locally. We also design experiments with transitions from
 247 purely subsurface flow dynamics to simultaneous subsurface and saturation ex-
 248 cess overland flows. On this latter experiment, we exhibit that the choice of
 249 the discretization scheme preserves the mass balance, we determine the con-
 250 vergence of the spatial discretization scheme, analyze the internal time steps
 251 influence and assess the sensitivity of the partition formulation to the regular-

252 ization parameter r . Third we carry extensive numerical testing to assess the
 253 robustness of the regularized partition formulation on realistic experiments.

254 3.1. Comparison with previous numerical experiments

255 3.1.1. Case without overland flow

256 We check the partition formulation (7) on the numerical experiments of Troch
 257 et al. (2003). We choose the straight hillslope experiments with a slope of 5% as
 258 it is the less likely to fully saturate the soil column and to generate saturation
 259 excess overland flow. For this case, boundary conditions are $S(x = 0, t) = 0$ at
 260 the channel outlet and $Q(x = L, t) = 0$ at the water divide. The hillslope is 100
 261 m long, spatially discretized with $N_x = 100$ elements. We apply a regularization
 262 parameter $r = 1 \times 10^{-3}$ and consider two infiltration cases. The first case is a
 263 recharge experiment with a constant infiltration ($N = 10 \text{ mm d}^{-1}$) on an initially
 264 dry hillslope ($S(x, t = 0) = 0$). The second case is a free drainage experiment
 265 ($N = 0 \text{ mm d}^{-1}$) on a hillslope initially partially saturated (uniformly saturated
 266 at 20% of its maximum capacity i.e. $S(x, t = 0) = 0.2 \times S_c(x)$).

267 Figure 4 shows a close agreement of the two methods in both recharge and
 268 drainage experiments with the marked storage accumulation near the river (left
 269 part of the graphs). The sharp saturation gradient next to the river comes
 270 from the imposed Dirichlet condition of zero storage at the river. This test
 271 demonstrates the consistency of our numerical approximation of system (7)
 272 with the method of Troch et al. (2003) which is based on the discretization of

273 system (1).

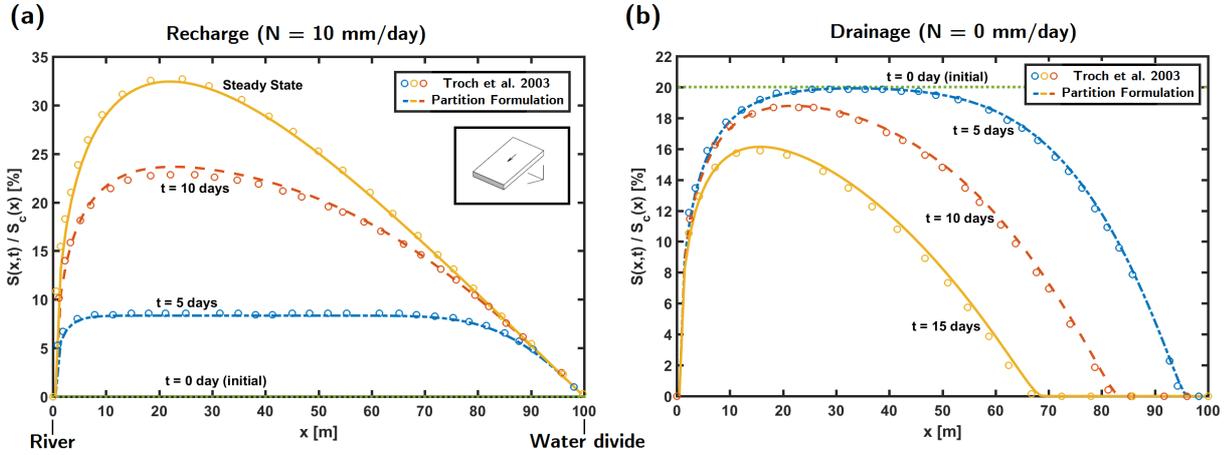


Figure 4: Relative subsurface water storage $S(x, t)/S_c(x)$ (expressed in %) along the hillslope for the (a) recharge and (b) drainage experiments. Insert shows a sketch of the hillslope of slope 5% (Troch et al., 2003).

274 3.1.2. Case with overland flow

275 We further assess the partition formulation on a steep convergent hillslope
 276 (insert of Figure 5, slope of 30%, Troch et al. (2003)). The hillslope saturates
 277 close to its outlet. It is initially partially saturated, $S(x, t = 0) = 0.2 S_c(x)$,
 278 and progressively drains to the river where the saturation remains imposed
 279 $S(x = 0, t) = 0$. Hillslope is limited on its upper side by a water divide condi-
 280 tion $Q(x = L, t) = 0$.

281 The saturation profiles and the subsurface flow to the river remain close to
 282 those of Troch et al. (2003) (Figure 5). The main benefit of the partition for-
 283 mulation (7) is to provide the saturation excess overland flow generated both
 284 spatially and temporally, which amounts to 16% of the total outflow to the river
 285 (Figure 5b). The river outflow is not bounded by the limited interface of the

286 subsurface to the river in this convergent configuration. It may be significantly
 287 enhanced by the generation of saturation excess overland flow when the water
 288 table intercepts the land surface (Figure 5b).

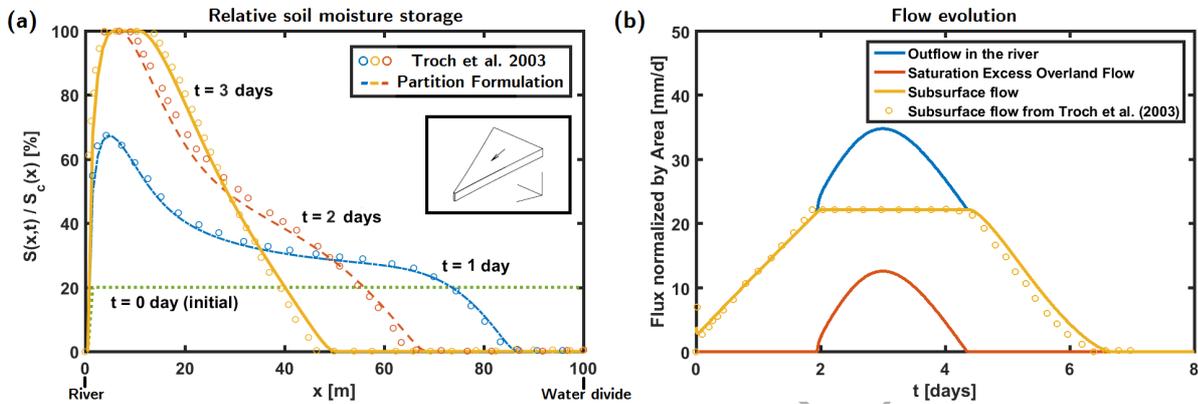


Figure 5: (a) Relative subsurface water storage $S(x,t)/S_c(x)$ (expressed in %) along the 30% sloping convergent hillslope shown in the insert (Troch et al., 2003) during the drainage of partially saturated initial conditions. (b) Outflow evolution in the river for the same drainage experiment.

289 3.2. Saturation excess overland flow on a convergent hillslope

290 To check that the saturation excess overland flow is well modeled, we consider
 291 a **steep** periodic hydrologic forcing $N(t)$ on a hillslope with **another** convergent
 292 shape (shown in insert of Figure 6, Troch et al. (2003)). Both the convergence
 293 of the hillslope and the high value of infiltration tend to generate saturation ex-
 294 cess overland flow **with dynamically generated fully saturated areas**. Boundary
 295 conditions are modified at the river to fully saturated soil ($S(x = 0, t) = S_c(x)$)
 296 but remain no flow at the water divide ($Q(x = L, t) = 0$). Initially, the hillslope
 297 is dry ($S(x, t = 0) = 0$). $N(t)$ is a square wave with a period of 10 days and val-
 298 ues alternating between 0 and 30 mm d⁻¹. We spatially discretize the hillslope

299 with $N_x = 100$ elements and apply a regularization parameter $r = 1 \times 10^{-3}$.

300 Starting from dry conditions, hillslope progressively fills with quickly rising
301 saturation next to the river because of the convergence conditions (Figure 6a or
302 Video 1). The saturation profile at $t = 12$ days does not show any seepage while
303 at $t = 26$ days and at $t = 35$ days, an extended seepage front has developed in
304 the 20 m next to the river. Even though saturation profiles are close at $t = 26$
305 days and $t = 35$ days, seepage is almost twice as large at $t = 26$ days because of
306 the presence of direct precipitations onto saturated areas (Figure 6b or Video 1).
307 At $t = 35$ days, only seepage flow occurs. Breakdown of the mass balance at the
308 hillslope scale shows the partition of the incoming infiltration in global storage
309 variations, saturation excess overland flow generated and outflow in the river
310 (Figure 7 or Video 1). Despite the intermittent infiltration, the discharge in the
311 river is steadily increasing because of the progressive filling of the hillslope after
312 the dry initial conditions. Discharge in the river remains always larger than the
313 saturation excess overland flow.

314 *3.2.1. Mass balance error analysis*

315 Interest of the mixed finite element scheme is to preserve mass balance at
316 the scale of the discretized spatial elements. A detailed analysis of the local
317 mass balance error with the numerical experiment described previously shows
318 that statistics (mean and 99th percentile) carried on the spatial and temporal
319 distributions of the mass balance errors are of the order of $10^{-13} \text{ m}^3 \text{ s}^{-1}$ and



Video 1: Video representing (a) the temporal evolution of the terms of the hillslope mass balance (infiltration, storage, river discharge, saturation excess overland flow) and the total mass balance error (b) the subsurface water storage profile measured on the left axis (blue line) and the saturation excess overland flow measured on the right axis (red line). River is on the left at $x=0$ m, hillslope divide is on the right at $x=100$ m.

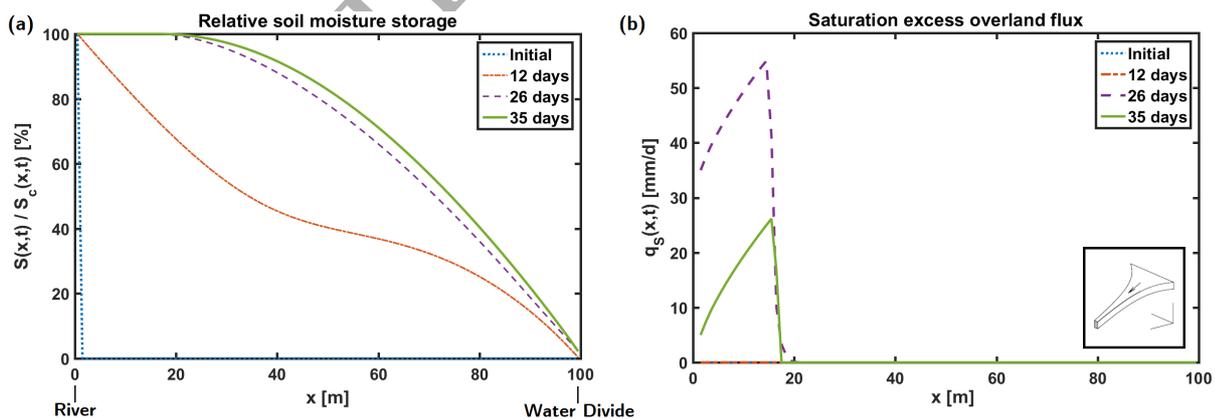


Figure 6: (a) Subsurface water storage and (b) Saturation excess overland flow profile along a convergent hillslope. Sketch of the convergent hillslope is shown in the insert (Troch et al., 2003).

320 remain smaller than the tolerance of the ode15s solver (absolute tolerance fixed
 321 at $10^{-10} \text{ m}^3 \text{ s}^{-1}$). When **normalized** by the forcing terms (second line of table
 322 1), mass balance errors are not found significative (in average equal to $1.6 \times$
 323 10^{-7}). Thus the applied spatial discretization scheme guarantees local mass
 324 conservation. At the scale of the entire hillslope, Figure 7 illustrates this mass
 325 balance preservation globally.

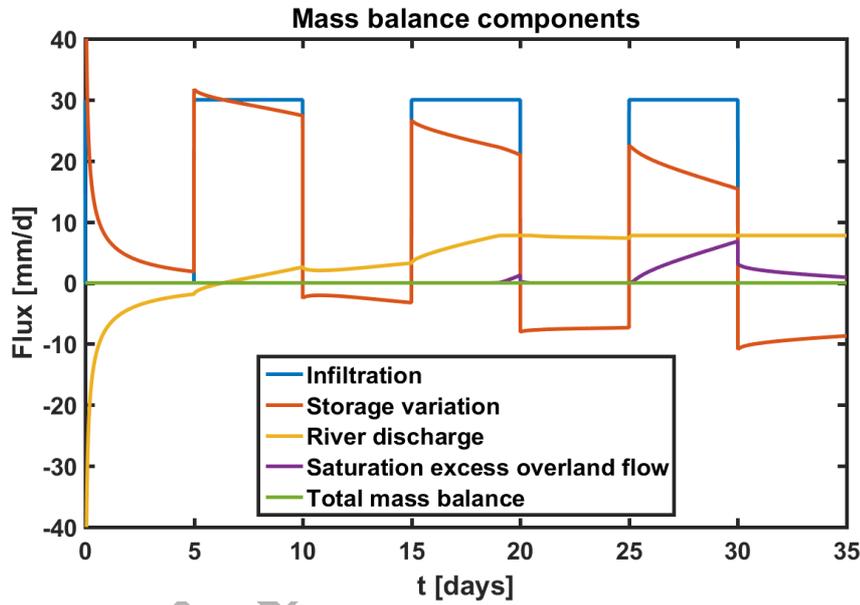


Figure 7: Temporal evolution of the terms of the hillslope mass balance (first equation in system 7) in terms of storage variation, river discharge and saturation excess overland flow. The sum of the terms (the mass balance error) remains equal to 0.

	mean	std	1 st %ile	99 th %ile
Mass balance [$\text{m}^3 \cdot \text{s}^{-1}$]	3.0×10^{-13}	6.0×10^{-12}	0	3.8×10^{-13}
Normalized Mass balance [-]	1.6×10^{-7}	3.2×10^{-6}	0	2.0×10^{-7}

Table 1: First line: statistics on the mass balance error [$\text{m}^3 \text{ s}^{-1}$] for the 100 discretized elements of the hillslope. Second line: normalized mass balance statistics [-] adimensioned by the rainfall (30 mm d^{-1}). Absolute tolerance of the ode15s solver was set to 10^{-10} for this numerical experiment.

326 *3.2.2. Convergence with the spatial discretization*

327 Following the convergence metrics defined in equation (9), we compare differ-
 328 ent simulations of the same numerical experiments with N_x varying between 300
 329 and 700 to the reference simulation composed of 1100 cells. The regularization
 330 parameter r is fixed to 1×10^{-3} . Thus we determine the convergence rate $\epsilon_{Q_{N_x}}$
 331 as a function of N_x . We show the convergence results only for Q (figure 8) as
 332 they are similar for S and q_S . It shows a fast increase in precision compared to
 333 the reference solution. The magnitude of the slope in a log/log scale is equal to
 334 -0.95 . Convergence rate is close to the theoretical rate equal to -1 demonstrated
 335 for linear and smooth problems, $\epsilon_{Q_{N_x}} = O\left(\frac{1}{N_x}\right)$ (Chavent & Roberts, 1991).

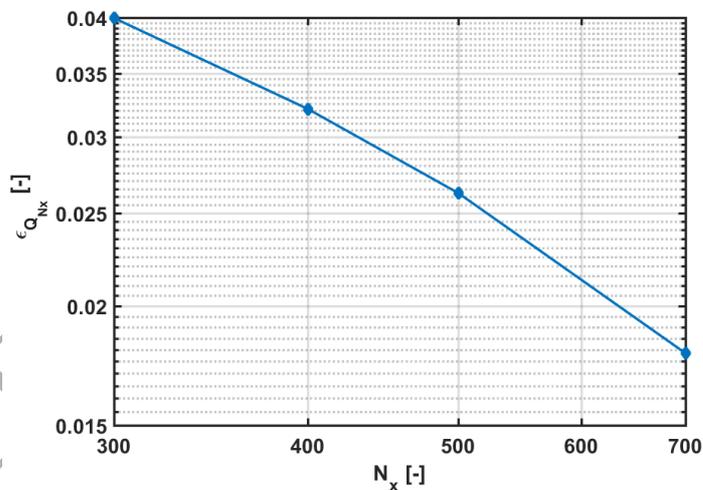


Figure 8: Spatial convergence analysis of Q on the convergent hillslope. Simulations with N_x varying from 100 to 1000 are compared to the reference simulation with $N_x = 1100$. The error metric $\epsilon_{Q_{N_x}}$ is defined by equation (9).

336 *3.2.3. Time steps analysis*

337 Accuracy is also obtained by adapting the time step and the order of the
338 scheme in ode15s to solve system (7). Figure 9a shows that the total number
339 of internal timesteps n_t increases approximately linearly with N_x ($n_t \simeq 40N_x$).
340 The adaptive method used by ode15s is efficient to provide optimal balance
341 between accuracy and efficiency. We cannot separate the effects of the spatial
342 discretization from the number of computed internal timesteps on the gain in
343 accuracy as we use the integrated solver ode15s which automatically adapts
344 its timesteps. Figure 9b highlights at which time ode15s refines its internal
345 timesteps during the simulation. It shows the cumulated number of internal
346 timesteps used by ode15s as a function of the time simulated for different spatial
347 discretization (N_x between 100 and 700). The number of internal timesteps
348 increases rapidly at the beginning of the simulation (for $t < 5$ days) and just
349 after the infiltration events ($t \geq 10, 20$ or 30 days). These refinements are
350 due to strong gradients appearing in the soil matrix at the beginning of the
351 simulation when the hillslope discretized elements equilibrates with the fixed
352 saturated channel bank and when the infiltration stops. The more the hillslope
353 is discretized, the more pronounced is the refinement. This may be to satisfy the
354 relative error tolerance which is more difficult to attain for finer discretizations.

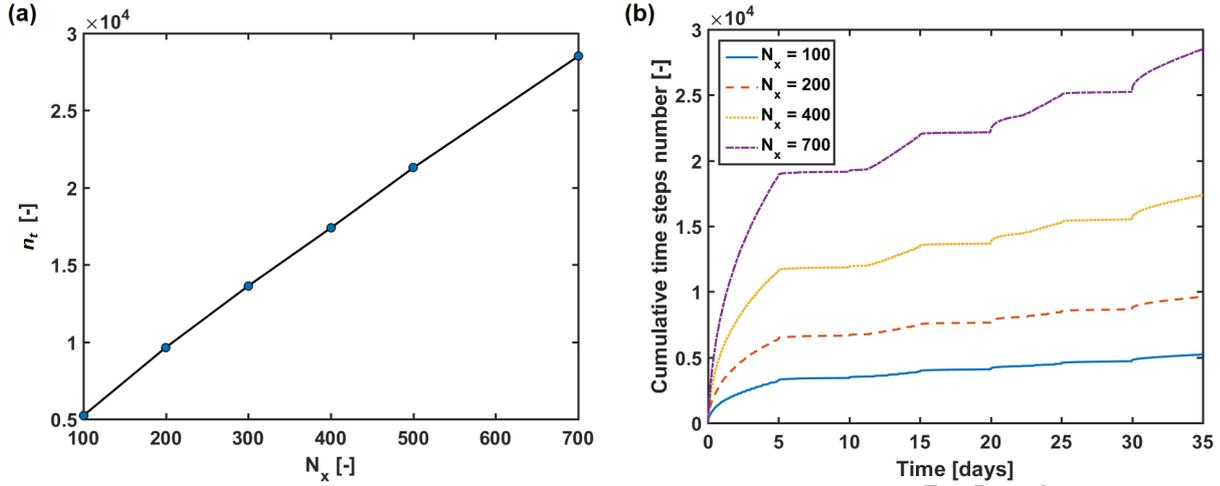


Figure 9: (a) Total number of internal timesteps n_t used by ode15s as a function of the number of the elements N_x used to discretize the hillslope. (b) Evolution of the cumulative number of internal timesteps used by ode15s during a simulation for different numbers of cells N_x .

3.2.4. Convergence with the regularization parameter r

We also determine the convergence rate of ϵ_{S_r} and $\epsilon_{q_{S_r}}$ as a function of the regularization parameter r (equation (9)) for a fixed spatial discretization with $N_x = 100$. The number r varies between 0.2 and 10^{-6} and is compared to the reference solution with $r_{ref} = 2 \times 10^{-7}$ ($N_{ref} = N_x = 100$). We only consider ϵ_{S_r} and $\epsilon_{q_{S_r}}$ since the regularization parameter controls the presence or absence of saturation excess overland flux depending on soil saturation. ϵ_{S_r} and $\epsilon_{q_{S_r}}$ scale with $r^{1.4}$ and $r^{0.94}$ demonstrating the fast convergence with the regularization parameter r . The built-in ode15s solver performs well even with stiff partition functions ($r = 2 \times 10^{-7}$).

We have assessed the numerical methods proposed in section 2.1. The mixed finite element method gives an accurate estimation of fluxes and subsurface

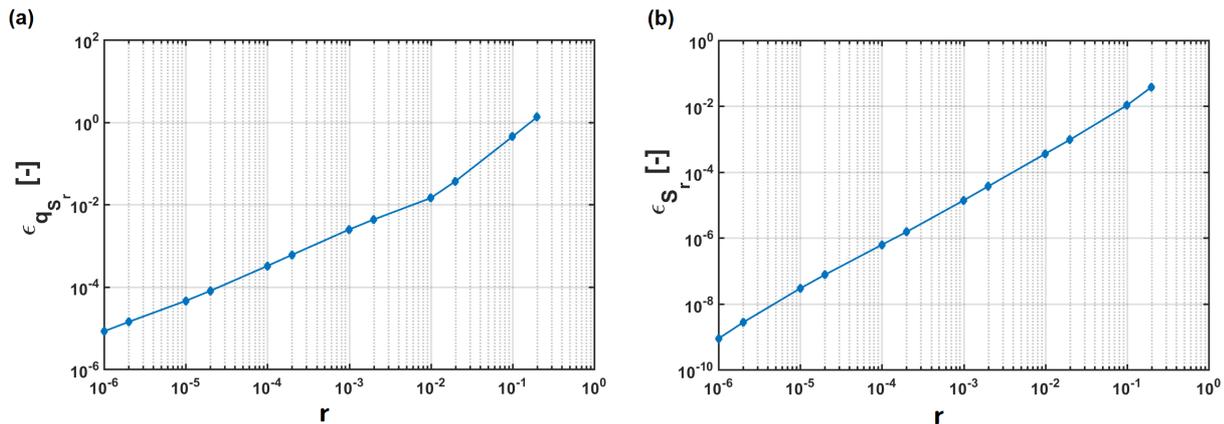


Figure 10: Convergence of (a) q_S and (b) S with the regularization parameter r . Errors are compared to the reference solution taken at $r = 2 \times 10^{-7}$.

367 water storage, as shown by the comparison with the previous results of Troch
 368 et al. (2003). It ensures accurate local and global mass balance preservation.
 369 The partition formulation with discontinuous functions and the proposed reg-
 370 ularization are relevant methods to account for seepage in efficient and simple
 371 ways. They are well suited to follow dynamic transitions between partially and
 372 fully saturated regimes. They show good convergence rates with the regulariza-
 373 tion parameter and lead to problems that can be integrated with classic ordinary
 374 differential equation solvers.

375 3.3. Extensive testing for robustness evaluation

376 To evaluate the robustness of the method, we run realistic simulations on
 377 1109 real hillslope shapes, with varied geologic parametrizations and with five
 378 different infiltration time series. This represents 8320 simulations. Hillslope
 379 structures are extracted from a 5m LiDAR Digital Elevation Model of Brittany.
 380 Hillslope shapes can be classified in the two categories of the head basin hill-

381 slopes (Figure 11a) and the riverside hillslopes (i.e. the hillslopes located along
382 river channels, Figure 11b). Head basins hillslopes have a typical divergent
383 and then convergent shape favoring the apparition of saturated source areas
384 near the streams. Hillslopes located along river channels are mainly divergent.
385 Both hillslope types display some complexities in their width functions. Several
386 geologic parametrizations are tested with different values of hydraulic conduc-
387 tivities, soil depths and drainable porosities. These parameters are drawn from
388 truncated lognormal distributions for the hydraulic conductivity and the soil
389 depth and from a truncated normal distribution for drainable porosity (Table
390 2). Infiltration time series are taken as simple synthetic cases or deduced from
391 real precipitations. The two synthetic cases are a steady infiltration time series
392 of 0.75 mm d^{-1} and a square periodic (period of 10 days) infiltration time series
393 of 35 days varying between 0 and 1.5 mm d^{-1} . Three realistic infiltration cases
394 are derived from three different precipitation chronicles of 15, 61 and 365 days.
395 For the steady infiltration time series, we start the simulation from an initially
396 dry soil profile. For the other infiltration time series, we take the steady sub-
397 surface water storage profile under the average infiltration of the time series as
398 initial conditions.

399 Figure 12 illustrates the simulation methodology for a 1 km^2 hillslope of
400 the Pleine Fougères watershed (Kolbe et al., 2016) located in North Brittany
401 (Figure 12a) with the realistic 61 days infiltration time series (Figure 12b). Hy-

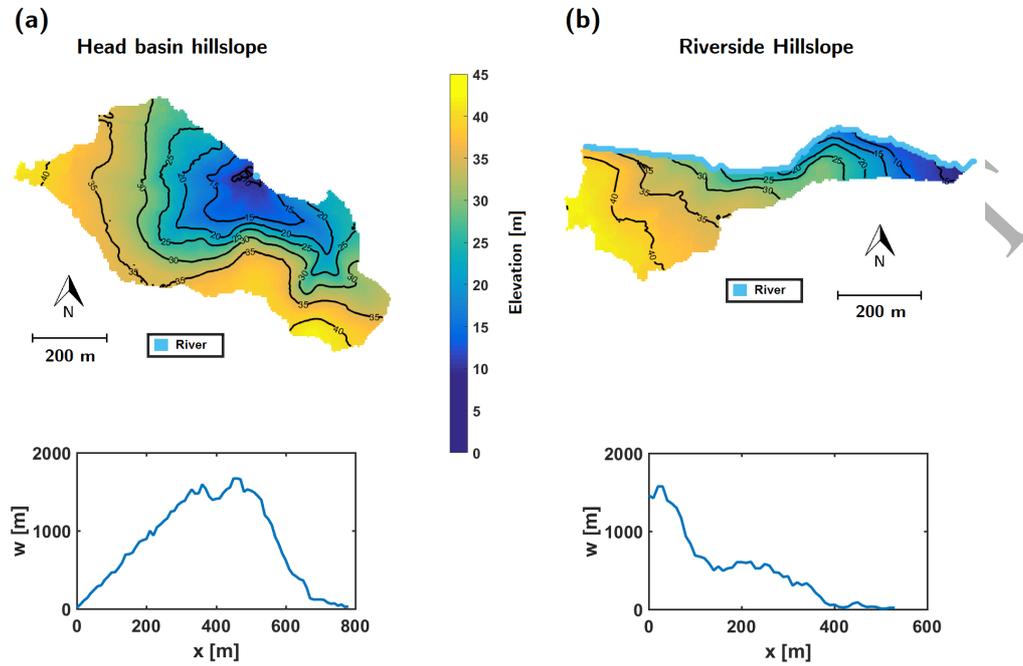


Figure 11: Two major classes of hillslopes: (a) Head basin hillslopes located at the spring of river. (b) Riverside hillslopes located along the river channel. Notice the different behaviour of their width functions.

Parameters	Distribution type	mean	std	Interval
f [%]	Normal	30	10	[5,50]
k [m h ⁻¹]	Log Normal	1	10	[0.05,15]
d [m]	Log Normal	1	10	[0.2,11]

Table 2: Parameters of the truncated distributions used for the parametrization of the geological properties f (drainable properties), k (hydraulic conductivity) and d (soil depth). Intervals indicate the truncation range.

402 draulic conductivities, soil depth and drainable porosity are respectively set at
403 1 m/h, 2 m and 10%. The simulation exhibits a coexistence of subsurface flow
404 along with saturation excess overland flow with two different dynamics (Figure
405 12c). Subsurface flow dynamic displays smooth response following infiltration
406 events while saturation excess overland flow, made up of both seepage flow and
407 direct precipitations onto saturated areas, responds instantaneously to infiltra-
408 tions with highly peaked flows. Out of the 8320 cases tested, no errors were
409 reported by the MATLAB[®] ode15s solver. These extensive numerical experi-
410 ments on contrasted hillslopes with sharp infiltration times series demonstrate
411 the robustness of the numerical methods proposed, even in the case of steep
412 rainfall time series (Figure 12).

413 4. Conclusion

414 Hillslope response to precipitations is characterized by sharp transitions be-
415 tween two different regimes defined as a function of soil saturation. For partial
416 soil column saturation, flows are restricted to the subsurface. For fully saturated
417 soil column, flows occur both in the subsurface and on the surface as saturation
418 excess overland flow. The hillslope response is highly impacted by the dynamic
419 transition between these regimes. We propose a partition formulation with a
420 mass balance equation where the storage variation is equal to the sum of the lo-
421 cal variation of the infiltration and the subsurface flow minus the overland flow.
422 We derive a regularized model which can be discretized in space and time by

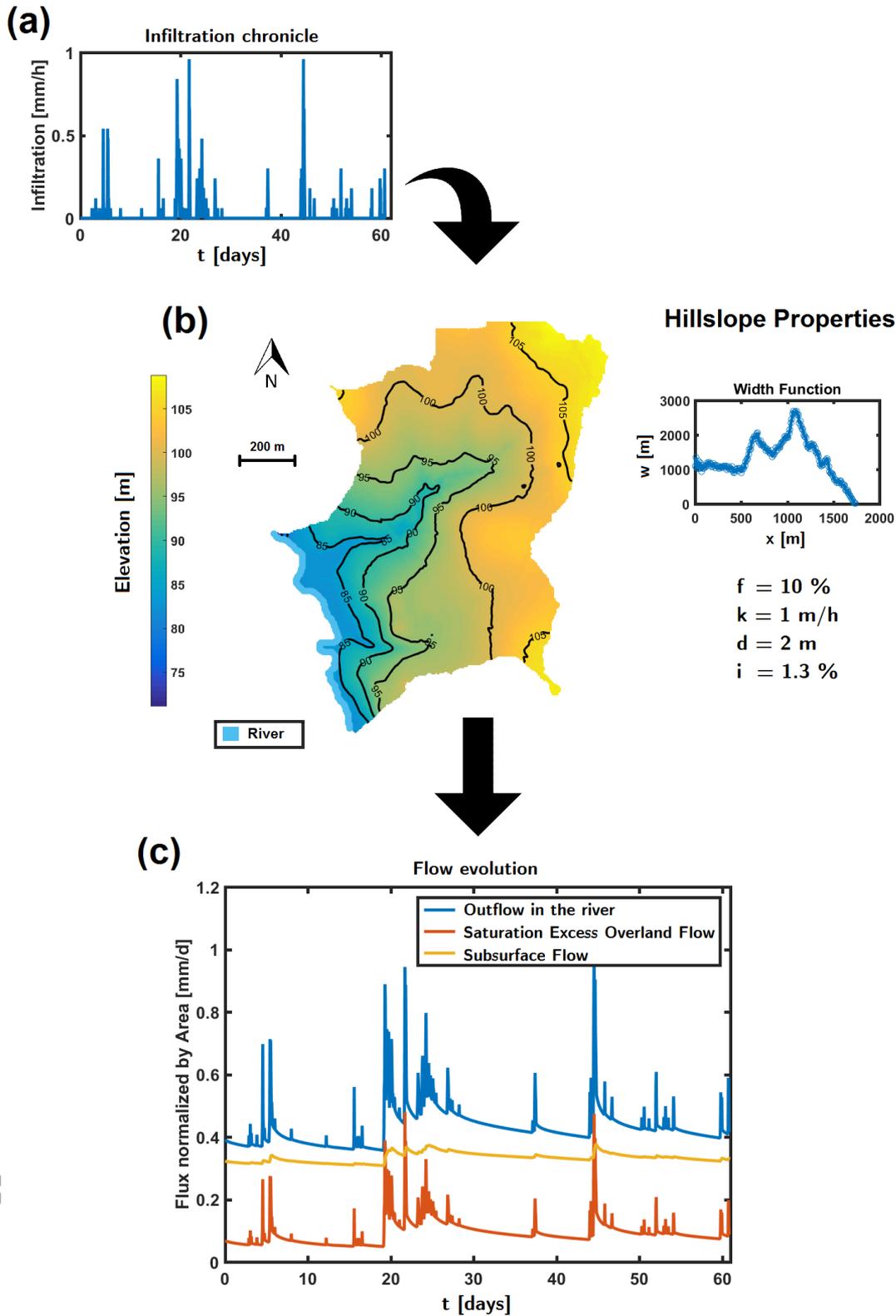


Figure 12: Results of the simulation for a real hillslope from a catchment in Brittany (France) (Kolbe et al., 2016) with (a) the infiltration chronicle, (b) the hillslope shape and properties (width function and pedologic parameters) and (c) the hillslope response as subsurface and saturation excess overland flow and the sum of both (outflow in the river).

423 classic schemes. We choose a mixed finite element method which ensures local
424 and global mass balance preservation along with an implicit temporal scheme
425 to deal with sharp transitions.

426 The regularized discrete scheme has been validated against previously pub-
427 lished results without saturation excess overland flow. With saturation excess
428 overland flow, additional numerical experiments on a convergent hillslope show
429 good convergence properties for both flow and saturation. Extensive tests on
430 8320 cases with different hillslope shapes and infiltration time series demon-
431 strate the overall robustness of the method. The regularized and discretized
432 partition formulation thus appears accurate and robust.

433 This model may be especially relevant for hillslopes with steep, shallow and
434 conductive soils on top of poorly weathered bedrocks that promote the gen-
435 eration of saturation excess overland flow in riparian areas where surface flow
436 routing can be neglected (Weiler et al., 2006). These conditions are typically en-
437 countered in Brittany (Merot et al., 2003; Montreuil & Merot, 2006). Integrating
438 this model at the regional scale requires to subdivide the watershed into rep-
439 resentative sub-units hillslopes. This can be done by automatically extracting
440 contour-based hillslopes (Moretti & Orlandini, 2008) based on the representative
441 elementary watersheds (Reggiani et al., 1998).

442 More generally this formulation provides a well-established mathematical for-
443 mulation for dynamic transitions between partially and fully saturated regimes.

444 It might be extended to 2D shallow aquifers. Adapted partition conditions
445 might also be formulated for shallow water equations on top of 2D Boussinesq
446 subsurface flows with the additional capacity to account for infiltration pro-
447 cesses.

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