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Quantized Vector Potential and the Photon Wave-function

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Abstract. The vector potential function $\vec{\alpha}_{k\lambda}(\vec{r},t)$ for a k-mode and λ-polarization photon, with the quantized amplitude $\alpha_{0k}(\omega_k) = \xi\omega_k$, satisfies the classical wave propagation equation as well as the Schrodinger's equation with the relativistic massless Hamiltonian $\tilde{H} = -i\hbar c\vec{\nabla}$ and finally an equivalent quantum equation for the vector potential amplitude operator $\tilde{\alpha}_0 = -i\xi c\vec{\nabla}$. Thus, $\vec{\alpha}_{k\lambda}(\vec{r},t)$ behaves as a wave function for the photon within a non-local representation that can be suitably normalized. It is deduced that the probability for detecting a k-mode photon around a point on the propagation axis depends on the square of the angular frequency. Taking into account the left and right circularly polarized states and weighting $\vec{\alpha}_{k\lambda}(\vec{r},t)$ by $\omega_k\sqrt{2\varepsilon_0}$ we define a six components function as a general function for a k-mode photon. The square of the modulus of the defined general function gives the energy density at a given coordinate which depends on the fourth power of the angular frequency. The amplitudes of the electric and magnetic fields of a single k-mode photon free of cavity are also calculated and it is shown that they are proportional to the square of the angular frequency. In this way, the influence of the photon electric and/or magnetic fields on the energy levels of atoms and molecules might be used for a non-destructive photon detection.

Introduction

The challenging difficulties encountered for establishing a wave function for the photon are well developed in the literature [1-5]. Indeed, the photon as a relativistic massless particle constitutes a particular case and a wave function within the quantum mechanical concept can hardly be defined. However, in recent years, the experimental evidence of the entangled states with single photons sources has demonstrated the permanent violation of Bell's inequality entailing that the existence of hidden variables within a local quantum mechanical representation is excluded [6, and references therein]. At the same time it revealed the necessity of developing a non-local representation for the photon, through a real wave function, in which new variables are not explicitly excluded. Hence, we explore here the possibility of establishing a photon wave function through the vector potential with quantized amplitude. We also analyze the resulting expressions of the intrinsic electric and magnetic fields of a free photon opening perspectives for new experiments that might be used for a non-destructive photon detection method.

Vector potential amplitude quantization

A detailed unit analysis of the general solution of Maxwell's equations for the vector potential has shown that it is proportional to the frequency. Consequently, for a *k*-mode photon with angular frequency ω_k the vector potential amplitude α_{0k} can be written [7-10]

$$\alpha_{0k} = \xi \omega_k \tag{1}$$

where ξ is a constant.

In a first approximation the value of the constant ξ has been evaluated [9,11] by normalizing the mean energy of a plane electromagnetic wave over a period to Planck's expression for the energy of a single photon $\hbar\omega_k$, where $\hbar = h/2\pi$ is Planck's reduced constant, getting

$$|\xi| \propto \left| \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{\hbar}{8\alpha_{FS}\varepsilon_0 c^3}} \right| = \left| \frac{\hbar}{4\pi e c} \right| = 1.747\ 10^{-25}\ V\ m^{-1} s^2 \tag{2}$$

with α_{FS} =1/137 the Fine Structure constant, ε_0 the vacuum electric permittivity, *c* the speed of light in vacuum and *e* the electron charge.

Hence, in the plane wave representation the vector potential for a *k*-mode and λ-polarization photon can be written

$$\vec{\alpha}_{k,\lambda}(\vec{r},t) = \omega_k \left(\xi \hat{\varepsilon}_\lambda e^{i\left(\vec{k}\cdot\vec{r} - \omega_k t + \theta\right)} + cc \right) = \omega_k \vec{\Xi}_{k\lambda}(\omega_k, \vec{r}, t) \tag{3}$$

where *cc* is the complex conjugate and λ takes two values corresponding to the left (*L*) and right (*R*) circular polarization expressed by the unit vector ε_λ.

The general equation for the vector potential of an electromagnetic wave considered as a superposition of plane waves now writes

$$\vec{A}(\vec{r},t) = \sum_{k,\lambda} \omega_k \left[\xi \hat{\varepsilon}_\lambda e^{i\left(\omega_k t - \vec{k}\cdot\vec{r} + \varphi\right)} + \xi^* \hat{\varepsilon}_\lambda^* e^{-i\left(\omega_k t - \vec{k}\cdot\vec{r} + \varphi\right)} \right] = \sum_{k,\lambda} \omega_k \vec{\Xi}_{k\lambda}(\omega_k, \vec{r}, t) \tag{4}$$

According to the relation (1) the quantized vector potential amplitude is an intrinsic property of a single photon state defining the wave function (3).

The vector potential as a wave function for the photon

It can be easily shown [7,9,12] that the vector potential function (3) satisfies the classical wave propagation equation in vacuum

$$\vec{\nabla}^2 \vec{\alpha}_{k,\lambda}(\vec{r},t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{\alpha}_{\kappa,\lambda}(\vec{r},t) = 0 \tag{5}$$

as well as the *vector potential – energy* (*wave – particle*) equation

$$i\begin{pmatrix}\xi\\ \hbar\end{pmatrix}\frac{\partial}{\partial t}\vec{\alpha}_{k;\lambda}(\vec{r},t)=\begin{pmatrix}\widetilde{\alpha}_0\\ \widetilde{H}\end{pmatrix}\vec{\alpha}_{k,\lambda}(\vec{r},t) \tag{6}$$

where the vector potential amplitude operator $\widetilde{\alpha}_0=-i\xi c\vec{\nabla}$ and the relativistic Hamiltonian for a massless particle $\widetilde{H}=-i\hbar c\vec{\nabla}$ have the eigenvalues $\xi\omega_k$ and $\hbar\omega_k$ respectively [7-9]. Consequently, the vector potential function $\vec{\alpha}_{k\lambda}(\vec{r},t)$ with the quantized amplitude $\xi\omega_k$ behaves as a real wave function for the photon within a non-local representation. Obviously, when considering the propagation of a k-mode photon along e.g. the z axis, unlike the momentum operator $\widetilde{P}_z$ which is well defined in quantum mechanics $\vec{P}_z=\hbar\vec{k}_z=\frac{h}{\lambda_k}\hat{e}_z\to\widetilde{P}_z=-i\hbar\frac{\partial}{\partial z}$, it is not evident how to define a position operator and this difficulty has been widely commented in the literature [1,4,6,9,13]. In fact, Heisenberg's uncertainty for the position and momentum for a photon with a wavelength λ_k propagating along the z-axis can be written

$$\delta z\,\delta P_z\geq h\to\delta z\,\delta\left(\frac{1}{\lambda_k}\right)\geq 1 \tag{7}$$

The physical meaning of Heisenberg's relation is that the photon as an integral entity cannot be localized precisely on the propagation axis z except with a significant uncertainty which is of the order of the wavelength λ_κ. This fact is a natural consequence that lays in the intrinsic wave-particle nature of the photon and has been widely commented in the literature [6, 13, 15-17]. As quoted by many authors, despite the fact that the *point-particle* model used in the QED formalism is a practical mathematical tool, in reality the photon cannot be defined within a length shorter than its wavelength. Thus, what we can essentially say is that when a k-mode photon is emitted at a coordinate z_e at an instant t_e and propagates in vacuum along the z-axis the probability $P_k(z)$ to be localized at time t and at the coordinate $z = z_e + c\,(t-t_e)$ has rather to be considered within a volume having roughly the value λ_k^3 [6, 16, 17] around the point z and according to (3) it is proportional to the square of the angular frequency

$$P_k(\vec{r})\propto\left|\vec{\alpha}_{k\lambda}(\vec{r},t)\right|^2\propto\xi^2\omega_k^2 \tag{8}$$

Considering the normalization factor $1/\xi\omega_{\max}$ for $\vec{\alpha}_{k\lambda}(\vec{r},t)$ and taking $\omega_{\max}\sim 10^{25}$ rad Hz, which corresponds to highly energetic gamma photons with wavelengths much shorter than the electron radius, then the detection probability becomes $P_k(\vec{r})\propto\omega_k^2/\omega_{\max}^2$ and tends to 1 for $\omega_k=\omega_{\max}$. It is also worthy noticing that, disregarding the physical units, equation (8) also tends naturally to one for the same value $\omega_k=\omega_{\max}=10^{25}$ rad Hz and can be practically used as a direct indication. Another normalization possibility consists of considering $\omega_{\max}\sim 10^{43}$ rad Hz corresponding roughly to Planck's energy 10^{19} GeV. However, we do not actually have any experimental evidence of photons with frequencies as high as 10^{43} Hz.

Now, taking into account the left (*L*) and right (*R*) circularly polarized states ($\lambda = L,R$) and weighting the vector potential by the factor $\omega_k \sqrt{2\varepsilon_0}$ we define the six components general function

$$\Phi_{k,(L,R)}(\vec{r},t) = \omega_k \sqrt{2\varepsilon_0} \begin{pmatrix} \vec{\alpha}_{k,L}(\vec{r},t) \\ \vec{\alpha}_{k,R}(\vec{r},t) \end{pmatrix} \tag{9}$$

Obviously, for both *L* and *R* polarizations $\Phi_{k,(L,R)}(\vec{r},t)$ satisfies also the wave propagation equation (5) and the *vector potential – energy* equation (6). The square of the modulus of the defined general function $\left|\Phi_{k,(L,R)}(\vec{r},t)\right|^2$ gives the energy density of the electromagnetic field composed of a single mode

$$W_k(\vec{r},t) = \left|\Phi_{k,(L,R)}(\vec{r},t)\right|^2 = 2\varepsilon_0 \xi^2 \omega_k^4 \tag{10}$$

It is worth noticing that the last expression depends on the fourth power of the angular frequency as in the case of the energy density of a radiating dipole [13, 14].

We can also deduce the amplitude of the electric field $\left|\vec{\varepsilon}_k\right|$ of a free single *k*-mode photon [18] which can be expressed through the square of the angular frequency

$$\left|\vec{\varepsilon}_k\right| = \left|-\frac{\partial}{\partial t}\vec{\alpha}_{k\lambda}(\vec{r},t)\right| \propto \xi \omega_k^2 \tag{11}$$

Considering a plane wave representation the corresponding magnetic field $\left|\vec{\beta}_k\right|$ of the photon is equally proportional to the square of the angular frequency

$$\left|\vec{\beta}_k\right| \propto \sqrt{\varepsilon_0 \mu_0}\, \xi \omega_k^2 \tag{12}$$

where μ_0 is the vacuum magnetic permeability.

It is of fundamental interest to investigate experimentally the last results, *e.g.* the coupling of left and right circularly polarized photons gives rise to a linearly polarized state whose spatial region around $\lambda_k / 4$, where the electric field is maximum, corresponds (in the visible-UV range) to a considerable distance of many nanometers and can be crossed by a large number of atoms or molecules. Under these conditions, high resolution laser spectroscopy might be employed for the detection of the Stark shifts induced by the photon electric field on the atomic or molecular energy levels. Such an experimental investigation could confirm the dependence of the single photon electric/magnetic fields on the square of the angular frequency. This might also open perspectives on the development of non-destructive photon detection through the measurement of the influence of the photon electric or magnetic fields upon the atomic or molecular energy levels.

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Conclusion

We have seen that the vector potential function $\vec{\alpha}_{k\lambda}(\vec{r},t)$ with the quantized amplitude $\alpha_{0k}(\omega_k) = \xi\omega_k$ satisfies the propagation equation and the coupled *vector potential – energy* equation. It can therefor play the role of a real wave function for the photon in a non-local representation that can be suitably normalized. A *k*-mode photon as an integral physical entity subsists over a period (thus over a wavelength λ_k) through the oscillation of the quantized vector potential and propagates along the wave function. Weighting $\vec{\alpha}_{k\lambda}(\vec{r},t)$ by $\omega_k\sqrt{2\varepsilon_0}$ we may define a six components general function $\Phi_{k,(L,R)}(\vec{r},t)$ for the photon including both circular polarizations. The square of the modulus of $\Phi_{k,(L,R)}(\vec{r},t)$ gives the energy density on a point of the propagation axis. The oscillation of the vector potential gives birth to intrinsic electric and magnetic fields within a single free photon state over a wavelength.

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