Reducing the Backus effect given some knowledge of the dip-equator

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Reducing the Backus effect given some knowledge of the dip-equator

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Abstract.
Geomagnetic field models computed from intensity measurements on or near the Earth's surface are plagued with strong errors known as the "Backus effect". These errors are related to the mathematical non-uniqueness of the solution which can be theoretically removed by the knowledge of the dip-equator location. Here, we present a new method for computing models using intensity data and additional information about the location of the dip-equator. We first give a numerical validation of this approach in an ideal case in which the dip-equator can be assumed to be perfectly located. We then show that in practice, updating a good past model of the field with a model of secular variation can constrain the location of the dip-equator sufficiently for our method to be useful. Implications for the planning of future satellite missions are briefly discussed.

Introduction

Models of the Earth's internal geomagnetic field computed from intensity measurements made at the Earth's surface, or on satellites, are plagued with strong errors known as the "Backus effect". This effect arises because of the non-uniqueness of the solution to the associated theoretical problem (e.g., Backus; 1970), and is related to the tendency of the error on main field models to lie perpendicular to the measured field when a finite dataset is being used (Lowes, 1975). Several different methods have been proposed in order to reduce this effect: the method most commonly used consists of adding some vector ground-based data from observatories or aeromagnetic surveys, especially near the equator where the error is maximum (e.g., Hurwitz and Knapp, 1974; Stern and Bredekamp, 1975; Thomson et al., 1997; Ultré-Guérard and Achache, 1997). However, these methods usually lead to a limited reduction of the Backus effect and are sometimes difficult to implement in practice. In the special case when poor satellite attitude data are available, the method of Holme and Bloxham (1995) can also be used.

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In a recent paper, Khoklov et al. (1997) have shown that if the intensity of the field and the location of the dip-equator are known at the Earth's surface then the field is known everywhere outside the sources to within a global sign. In the present paper, we describe a method of inversion based on this theorem and use it to provide a practical validation of the theorem. We then evaluate the best way of locating the dip-equator and discuss the improvement brought in reducing the Backus effect.

Defining the Backus effect

When modelling the internal part of the near-Earth geomagnetic field it is usual to use a representation of the form of a potential function expressed as a series of spherical harmonics:

$$V(r,\theta,\phi) = a\sum_{n=1}^{\infty}\sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1}(g_n^m \cos m\phi + h_n^m \sin m\phi)P_n^m(\cos\theta) \quad (1)$$

where a is the mean radius of Earth, (r, θ, ϕ) are the spherical coordinates, $P_n^m(\cos\theta)$ is the Schmidt-normalized associated Legendre function of degree n and order m (see e.g., Langel, 1987) and g_n^m and h_n^m are the Gauss coefficients. The magnetic field of internal origin is then given by $\vec{B} = -\vec{\nabla}V$, and the field is modelled by parameter estimation of a truncated set of the Gauss coefficients from some measurements of the Earth's magnetic field.

In this study, a data set of about N_{obs}=5000 vector measurements acquired by the Magsat satellite, launched in November 1979, has been used. An associated scalar data set has been built by computing the intensity $F = (X^2 + Y^2 + Z^2)^{1/2}$ from each vector measurement (X, Y, Z). We computed a reference vector (Ref) model, based on the N_{obs} vector data, up to degree and order 13, by minimizing:

$$\chi^2_{vec} = \sum_{i=1}^{Nobs} W_i[(X_i^{obs} - X_i^{mod})^2 + (Y_i^{obs} - Y_i^{mod})^2 + (Z_i - Z_i^{mod})^2] \quad (2)$$

where the indices correspond to the measured and modelled components at the i-th position, and the weight $W_i = sin\theta_i$ compensates for the increasing density of measurements with the latitude. A (slightly overestimated) value of $\Sigma_{Ref} = 11\ nT$ for the error on this Ref model has been calculated by computing the RMS difference between two vector models computed with two data sets (each of $N_{obs}/2$ observations) built by selecting every other data in the primary data set.

The Backus effect associated with a given inversion procedure is then defined as the difference between the resulting model computed up to degree and order 13 and the Ref model. The RMS value of this Backus effect over the Earth's surface will be denoted RMS BE. The "raw" Backus effect, associated with the intensity-only modelling procedure which consists in minimizing:

$$\chi_F^2 = \sum_{i=1}^{Nobs} W_i \left(F_i - F_i^{mod}\right)^2 \tag{3}$$

where the notation is as in equation (2) but for intensity data, is shown on Figure 1 a (see also Stern et al., 1980). The corresponding RMS BE is $BE_{max} = 144nT$.

Numerical validation of the uniqueness theorem

A method based on the uniqueness theorem can be tested if we assume that the dip-equator is known to pass through a finite number of points ($N_{eq} = 500$). The corresponding least squares inversion procedure consists in minimizing:

$$\chi_{eq}^2 = \chi_F^2 + W_{eq} \sum_{j=1}^{Neq} \left(Z_j^{mod}\right)^2 \tag{4}$$

where Z_j^{mod} is the Z-component predicted by the model at one of the N_{eq} points defining the equator and W_{eq} is the weight applied to the dip-equator constraint. Different values of this weight have been explored and the optimum one (leading to the lowest RMS BE) has been retained. To test the method in the most ideal case, we first used N_{eq} points located on the equator defined by the Ref model. The location of these points have been found (to within 10^{-6} degree of the assumed equator) by using the root finding method of Ridders (Press et al., 1992). The resulting Backus effect is presented on Figure 1 b. The RMS BE value is reduced from $BE_{max} = 144\ nT$ to $BE_{id} = 21\ nT$, which is reasonnably comparable to $\Sigma_{Ref} = 11nT$ given that BE_{id} is computed from a model derived with only $N_{eq} + N_{obs} = 5500$ data, while Σ_{Ref} is computed from models each with $3N_{obs}/2 = 7500$ data. This result is a practical validation of the uniqueness theorem. Repeating this test by using the Ref model only up to degree 12 or 8 for locating the equator leads to RMS BE of 21 nT and 37 nT. This shows that degrees 9 to 12 are important in properly locating the equator and reducing the Backus effect but not degree 13. The same is true for degrees above 13, the influence of which has been tested by using a degree 20 reference model while only using degrees up to 13 to compute the equator (leading to an RMS BE of 23nT).

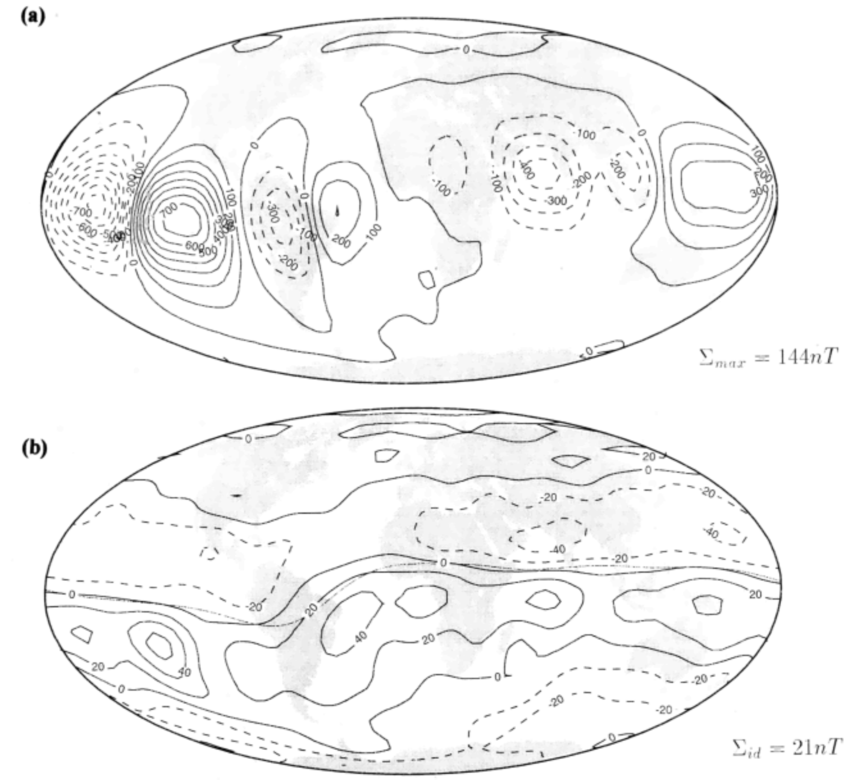

Figure 1. a: Vertical component of the raw Backus Effect at the Earth's surface (nT). **b:** Same as **a** but for the Backus effect remaining when imposing the location of the equator in the ideal case.

Practical reduction of the Backus Effect

In practice no Ref model is available for the epoch under study. But a vector model updated to the appropriate epoch with a secular variation (SV) model can possibly be used as a proxy. Since any future vector model of the field would be of a quality comparable to our Magsat Ref model, we tested the inversion with the 1980 scalar data set and an equator defined by the Ref model after introducing some errors comparable to the errors that present quality SV models would introduce as a result of updating a 5 years old model. Part of this error would arise from the uncertainties in the SV models themselves. This is taken into account by adding some noise proportional to five times the uncertainties involved in the 1995 SV model of Macmillan et al. (1997) up to degree 8 and proportional to five times the SV itself (estimated by linearly extrapolating the spectrum of this SV model at the Earth's surface) for degrees 9 to 13. A systematic error will also arise as a result of the field not evolving as a linear function of time, as predicted by the SV model. To assess this second type of error, a study using the ufm model computed by Bloxham and Jackson (1992) has been performed. Starting from a given epoch t_0, we compared the model predicted at $t_0 + 5$ using 5 years of constant SV and the true model at $t_0 + 5$. This error is then estimated every 5 years for t_0 between 1900 and 1990 and a global RMS value is finally computed for each Gauss coefficient. Additional noise proportional to these RMS values is then assumed as a result of relying on a linear SV prediction. The power spectra of all these assumed sources of errors have been plotted as a function of the degree on Figure 2. Ten different artificial sets of errors on the Gauss coefficients have then been computed by adding gaussian noise for each type of error (the errors being independent). The resulting SV error has been added to the exact vector model. Using the dip-equator derived from the resulting models together with the original 1980 scalar data set leads to RMS BE ranging from 22nT to 26nT, close to the ideal value of $BE_{id} = 21nT$.

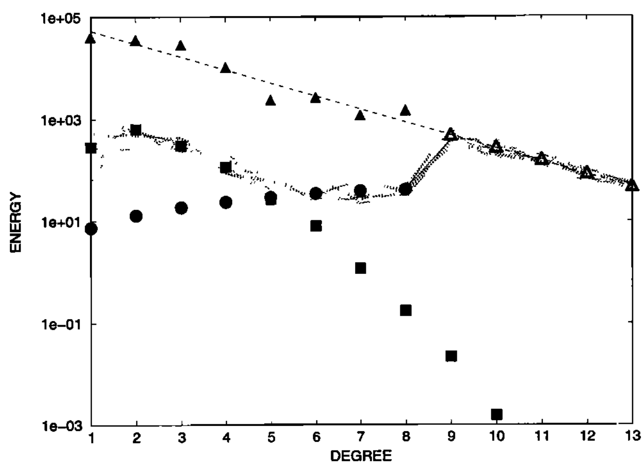

Figure 2. Spectra (defined by the classical $W_n = (n+1)\sum_{m=1}^{n}(g_n^{m2} + h_n^{m2})$ formula, Langel 1987) corresponding to the 1995-SV model of Macmillan et al. (1997) integrated over 5 years (full triangle) and of its associated uncertainties (full circles). Empty triangles correspond to an extrapolation up to degree 13 of this 5 years SV spectra. Squares represent the spectra of the error resulting from 5 years of linear extrapolation of the SV. Also shown the spectra of the noises we actually used in our computation. Units are $(nT)^2$.

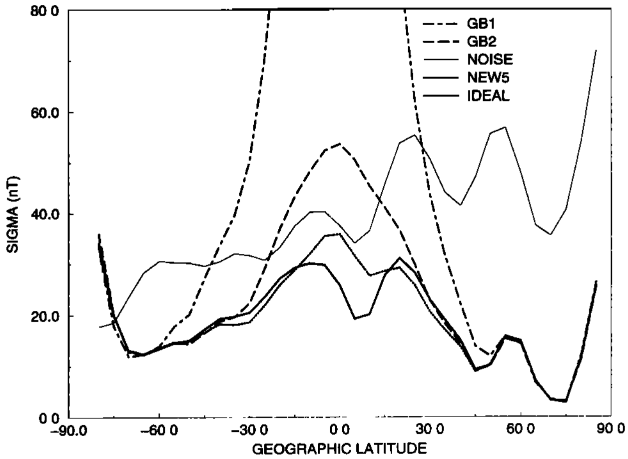

Figure 3. RMS of the error (in nT) as a function of the latitude for the different cases (see Text).

Discussion

To test the usefulness of our new method, we finally checked that it leads to better results than the trivial method consisting in directly updating the past model with the SV model. This method would lead to an error directly measured by the noise we added to the Ref model in the previous computation. For a typical noise that led to a RMS BE of 22nT with our new method, we therefore computed $\Sigma_{NOISE}(\lambda)$, the RMS value of this noise over a band of latitude λ and $\Sigma_{New5}(\lambda)$, the RMS error of our model with respect to the Ref model over the same band of latitude (Figure 3). For further comparisons, we also computed $\Sigma_{id}(\lambda)$, the RMS error corresponding to the ideal case when the equator is perfectly located and $\Sigma_{GB1}(\lambda)$ and $\Sigma_{GB2}(\lambda)$, the RMS errors associated with two additional models computed by introducing some vector ground based data and minimizing:

$$\chi^2_{obs} = \chi^2_F + W_{vec}\sum_{i=1}^{Nobs}\left[\left(X_i - X_i^{mod}\right)^2 + \left(Y_i - Y_i^{mod}\right)^2 + \left(Z_i - Z_i^{mod}\right)^2\right] \quad (5)$$

where χ^2_F is defined by (3) and W_{vec} is the weight applied to the vector observatory data assumed to be available at existing observatory locations ($\approx$ 140 points). Observed 1980 ground values have been used in the first model ($\Sigma_{GB1}(\lambda)$) and values predicted by the Ref model in the second model ($\Sigma_{GB2}(\lambda)$). These models are representative of the traditional method of reducing the Backus effect. All these functions have been plotted on Figure 3. We first recover the well-known fact that taking the raw observations at the existing observatories and using the classical computation does not lead to a significant reduction of the Backus effect (see $\Sigma_{GB1}(\lambda)$), the main reason for this being that these observations are biased by local crustal anomalies (Ultré-Guérard and Achache, 1997). Using the observed differences to update the prediction of a reference model after 5 years at each of these observatories could correct for the crustal biases. These observations would be almost exactly compatible with the satellite intensity data, and $\Sigma_{GB2}(\lambda)$ shows that in this ideal case the traditional approach could lead to a significant reduction of the Backus effect. But Figure 3 also shows that this would not lead to the best possible reduction. Indeed, our new method clearly does a better job even though it relies on a predictive SV model including some errors. It also leads to errors that are significantly lower than those we would get by a straightforward updating of the past model. Of course relying on an a posteriori (rather than predictive) SV model such as one that would take into account the observed ground based field differences (which would result in a significant reduction in the error linked to the linear extrapolation), would lead to an even better reduction. In fact Figure 3 shows that a vector model updated to the epoch of interest with an error less than 40nT at the equator can be used to define the dip-equator we need to compute a model following our method. The error in the resulting model is then simply constrained at the equator by our ability to update the location of this equator, and elsewhere by the quality of the intensity data.

Conclusion

We have described a method of inversion of satellite intensity data when some knowledge of the dip-equator location is given. When the dip-equator is exactly known, this method reduces the Backus effect as predicted by the uniqueness theorem of Khoklov et al. (1997). In practice, the dip-equator required to invert the intensity data at some epoch can be inferred with enough accuracy from the updating of a past good model (computed with vector satellite data such as Magsat for the present day or Ørsted, Champ, Sac-C in the future). When the past model is within 5 years of the epoch of interest, the error on the model computed with the updated dip-equator is lower than the error made when using the more traditional method relying on additional vector observatory data. Our results show therefore that satellite intensity data can prove very valuable when flown within five years of a Magsat quality vector satellite. This suggests that at least one vector data satellite should be flown every 10 years, and that easy to handle scalar magnetometers (for which no attitude measurements are required) could be flown on board any available satellites when no vector data satellites are in operation. Of course, observatory data are still critical sources of data. By providing a continuous survey of the time variation of the geomagnetic field, especially near the equator, they make it possible to build the SV models required to update the location of the dip-equator. In this respect, it is important to note that any future observatory would prove especially valuable if located near the equator.

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