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Stability of relativistic electron trapping by strong whistler or electromagnetic ion cyclotron waves

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In the present paper, we investigate the trapping of relativistic electrons by intense whistler-mode waves or electromagnetic ion cyclotron waves in the Earth’s radiation belts. We consider the non-resonant impact of additional, lower amplitude magnetic field fluctuations on the stability of electron trapping. We show that such additional non-resonant fluctuations can break the adiabatic invariant corresponding to trapped electron oscillations in the effective wave potential. This destruction results in a diffusive escape of electrons from the trapped regime of motion and thus can lead to a significant reduction of the efficiency of electron acceleration. We demonstrate that when energetic electrons are trapped by intense parallel or very oblique whistler-mode waves, non-resonant magnetic field fluctuations in the whistler-mode frequency range with moderate amplitudes around 3 – 15 pT (much less intense than the primary waves) can totally disrupt the trapped motion. However, the trapping of relativistic electrons by electromagnetic ion cyclotron waves is noticeably more stable. We also discuss how the proposed approach can be used to estimate the effects of wave amplitude modulations on the motion of trapped particles. © 2015 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4927774]

I. INTRODUCTION

Relativistic electron trapping (and/or phase bunching) by strong electromagnetic whistler or ion cyclotron (EMIC) waves is believed to play an important role in the formation of small populations of high-energy particles in the Earth’s radiation belts and as well as in bursty precipitations of electrons into the atmosphere. Moreover, nonlinear wave-particle interactions are responsible for rapid wave growth and amplitude modulation of the waves.

Basic theoretical approaches allowing the description of particle trapping by intense waves in the inhomogeneous plasma of the magnetosphere were first laid down in Refs. 28, 29, 50, and 51 (see also reviews of Refs. 6, 57, and 62). Next, modern spacecraft observations provided comprehensive statistical informations about the occurrence rate and other parameters of intense whistler and EMIC waves in the radiation belts, stimulating further investigations of nonlinear wave-particle interactions.

However, the important problem of the stability of the particles’ trapped motion has often been left aside in previous works—except for a few studies devoted to the effects of wave amplitude modulations or the simultaneous interactions of particles with several waves. The latter studies consider the presence of overlapping resonances from two (or more) waves in the system (see Refs. 65 and 87). One most remarkable case of this kind is the situation where sideband waves are present very close to the main wave. Then, one deals with simultaneous resonant interaction with several waves, where particle trapping into one wave is destroyed by electromagnetic field perturbations induced by the other waves.

Besides this resonant destruction of trapping due to sidebands, however, one can also consider a mechanism of non-resonant destruction. When trapped particles are transported by intense waves over quite long time intervals, they can be affected by various small-amplitude non-resonant fluctuations of the background electromagnetic field. Such additional fluctuations, while being generally too weak and too far from resonance to significantly perturb particle trajectories, may be used to control the fine regime of wave-particle resonant interaction and, thus, can eventually result in particle detrapping.

The cumulative effect of such electromagnetic field fluctuations can be estimated following an approach developed in Refs. 10 and 11. Below, we apply this kind of approach to a rather general system of trapped particle motion. The obtained results are used to estimate the stability of particle trapping by intense whistler-mode waves and EMIC waves in the Earth’s radiation belts.

Of course, the considered non-resonant (diffusive) scattering of trapped particles is not as effective as a resonant perturbation: If trapped particles are influenced by a perturbation at the trapping frequency (in case of sideband
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Equation (6) shows that $g \Omega_r$ is the frequency of particle oscillations in the $(\phi, P)$ plane. The factor $g = 1$ for nonrelativistic systems (for which the frequency $\Omega_r$ was initially introduced\cite{28,30,50,51}), while for relativistic systems the factor $g$ is responsible for a modification of the trapping frequency. Although $g \Omega_r$ represents the frequency of trapped particle oscillations only in the vicinity of the bottom of the potential well, this term can be used to estimate the frequency of oscillations of trapped particles over almost the entire region filled by closed trajectories (except near its boundary), because the actual frequency of trapped particle oscillations depends very weakly on $l$—i.e., on the position of a particle within this region (see, e.g., Refs. 4 and 80). It allows us to use hereafter $2\pi/(g \Omega_r)$ as an estimate of the period of trapped particle oscillations. This period is small as compared with the typical timescale of $s$ variations, because $\Omega_r^2 \sim kc \dot{s}$ and $kc \gg \dot{s}$.

In a system with constant $s$, the particle trajectory in the $(\phi, P)$ plane does not evolve and, thus, the area surrounded by this particle trajectory is exactly conserved. In the more realistic case of a slow enough variation of $s$ (when the time scale of the $s$ variations is much larger than the period $\sim 2\pi/\Omega_r$), the action $I$ becomes an adiabatic invariant of the system,\cite{8,35} i.e., the area $2\Omega I$ is still conserved to a high degree of accuracy even if the effective potential $U$ varies with $s$ (with time). Thus, $I$ can be used to characterize the trapped particle motion.

Generally, in the saturated stage of wave instability away from the equator, the function $a(s) = \Omega_r^2 A(s)$ does not depend anymore on wave amplitude, being mostly determined by the gradients of the system. For each particular system, $a(s)$ can be found using the expansion of the initial equations of motion around the resonance $\phi = 0$. In the particular case of the gyroresonant (i.e., first cyclotron resonance) interaction of relativistic electrons with whistler-mode waves, the expression for $a$ contains three terms:\cite{20,21,54,56} $\sim \partial a/\partial t$, $\sim (k^2/\Omega_r) \partial \Omega_r/\partial s$, and $\sim v_R \partial \Omega_r/\partial s$. In case of a significant gradient in $k_i(s)$, one would get an additional (fourth) term $\sim v_R \partial k_i/\partial s$ (see, e.g., Refs. 9 and 62). For EMIC waves, the function $a$ can be found in Refs. 5 and 59. It is also worth noting that the function $a \sim \Omega_r^2$ during strong wave-particle interaction such as trapping and nonlinear scattering\cite{9,56} corresponding to a normalized wave amplitude of the same order of magnitude as the gradients of the system parameters: $B_w/B \sim v_R/(\Omega_r R_0)$.

Since we aim at investigating the potential disruption of trapped particle motion by additional magnetic fluctuations in the most general case, we consider below a very generic system corresponding to electron trapping by an intense wave. Such a standard wave-particle system can easily be used for various applications. Although the considered system does not correspond to any particular wave mode, we do use model profiles $g(s)$, $a(s)$, and $w^2(s)$ similar to realistic profiles discussed in Refs. 3, 9, and 62. Namely, we take $g(s) = 1/\sqrt{1 + s^2}$ (for nonrelativistic systems there is a multiplication factor $(v_0/c)^2$ before $s^2$), $a(s) = s^2(1 - s^2)^{-1}$, $w^2 = s^2/(s_0^2 + s^2)$, where $s_0 = 0.15$ is a typical scale length of increase of wave intensity in the Earth’s radiation belts around $L \sim 5$ (see Refs. 1, 9, and 44). The fact that $a = 0$ at the equatorial plane $s = 0$ means that we do not take into account the term $\sim \partial a/\partial t$ (only this term has a finite value at $s = 0$ because the magnetic field gradient $\partial \Omega_r/\partial s$ and wave number gradient $\partial k_i/\partial s$ both vanish at $s = 0$ due to the symmetry of the geomagnetic field model and wave propagation, see details in Refs. 20, 21, 54, and 56). The profiles of coefficients $a(s)$, $w^2(s)$, and $g(s)$ are presented in Fig. 1(c).
Let us first examine the trapped motion of particles in the presence of an intense wave and demonstrate the conservation of $I$. We introduce the dimensionless wave amplitude $\varepsilon = kR_0 B_w/B_0$ and renormalize time as $t \to \omega_0 t$ with $\omega_0 = \sqrt{\Omega_{\text{eff}}}/R_0$. In this case, the Hamiltonian (5) takes the form

$$H = \frac{1}{2} g P^2 + \varepsilon w^2 \cos \phi - a \phi.$$  

(7)

Trapped trajectories exist in the region where $\varepsilon w^2(s) > a(s)$. Consider the solution of Hamiltonians equations $\phi = \partial H/\partial P$, $\dot{P} = -\partial H/\partial \phi$ for a particle trajectory initially trapped at $s = s_0$. We slowly vary $s > s_0$ and study the particle motion in the $(\phi, P)$ plane. Figure 2 shows the particle trajectory and the action $I$:

$$I = \frac{1}{2\pi} \oint Pd\phi = \frac{1}{\pi} \sqrt{\frac{2}{g}} \left[ \sqrt{H - \varepsilon w^2 \cos \phi + a \phi} dl \right].$$  

(8)

where $\phi_{1,2}$ are shown in Fig. 1(b). The variations of parameters $g$, $w$, and $a$ result in the observed evolution of the particle trajectory in the $(\phi, P)$ plane, but the action $I$ is conserved. There are only very small amplitude $\sim \chi^{-1/2}$ ($\chi = kR_0 \gg 1$) oscillations of $I$, corresponding to the fact that integral (8) is calculated for a frozen $s$, while there are actually small variations of $s$ within one period of trapped particle oscillations in the $(\phi, P)$ plane.

Let us consider a whole cycle of charged particle motion, including particle trapping and escape from resonance. To this aim, we start the numerical integration at $s = 0$, choose a large enough $\varepsilon$, and select a trajectory which is trapped (see Fig. 3). The conservation of $I$ for trapped particles means that we can use it to determine the moments of time when a particle enters and escapes from the resonance. At the start of trapping, the value of $2\pi l$ is equal to the area surrounded by the boundary of the region filled by closed trajectories in the $(\phi, P)$ plane. This boundary is called the separatrix, and the corresponding area $S$ is defined as (see, e.g., Refs. 8, 12, 27, and 48)

$$S = 2 \sqrt{\frac{2a}{g}} \int_{\phi_{-}}^{\phi_{+}} \sqrt{\frac{\varepsilon w^2}{a}(\cos \phi_{-} - \cos \phi) - \phi_{-} + \phi} d\phi,$$  

(9)

where $\phi_{-} = \phi_{-}(s)$ is a solution of the equation $\varepsilon w^2(s) \sin \phi = -a(s)$ different from $\phi_{1}$ (both $\phi_{1}$ and $\phi_{-}$ are shown in Figs. 1(a) and 1(b)). The area $S(s)$ varies with $s$, while $I$ is conserved. Thus, the trapped particles escape from the resonance when $S$ becomes equal to $2\pi l$ while $dS/ds < 0$ (i.e., when the area surrounded by the separatrix becomes smaller than the area surrounded by the trapped particle trajectory so that the particle escapes into the region with open trajectories). An example of particle escape from the resonance is shown in Fig. 3: it occurs exactly when $S$ becomes equal to $2\pi l$ (or to the value of $S$ at the start of trapping).

Figures 2 and 3 demonstrate that the trapped motion is the result of the conservation of the adiabatic invariant $I$ in the main wave-particle system. However, if an additional (external) force inducing variations of $I$ were to be present too, then $2\pi l$ could become equal to $S$, leading to a corresponding escape of the particle from resonance. In Section III, we consider the effects of such an external force and demonstrate that it may eventually lead to the destruction of the adiabatic invariant $I$.

III. MAGNETIC FIELD FLUCTUATIONS

In this section, we consider the effect of additional magnetic field fluctuations on trapped particle motion. For the sake of generality, these fluctuations are assumed to be mostly non-resonant, i.e., their frequency, although being of the same order as the main wave frequency, is assumed to differ sufficiently from it to allow neglecting resonant interactions between particles trapped by the main wave and these fluctuations (in contrast to cases considered in Refs. 52 and 65). Moreover, we consider here fluctuations parallel to the geomagnetic field line (transverse fluctuations will be briefly addressed in Section V; see also Ref. 17). It may
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$P_i^2$ calculated within consecutive time steps $\tau$ (i.e., between successive changes of $\partial b$)
\[
\text{Var}(P) = \frac{1}{M} \sum_{i=0}^{M} P_i^2 = \frac{\sum_i P_i^2 \tau}{\sum_i \tau} \approx \frac{g \Omega_x}{2\pi} \int P^2 dt = \frac{\Omega_x}{2\pi} \int P d\phi = \Omega_x I, \quad (14)
\]
where we took into account the smallness of the time step $\tau$. Substituting Eq. (14) into Eq. (13), we obtain
\[
\text{Var} (\Delta I) = \frac{\omega^4 \alpha^2}{\Omega_x B_0^2} I \sigma^2. \quad (15)
\]

The evolution of $I$ can be viewed as a random walk and described by the diffusion equation for the distribution $\Psi(I)$ of trapped particles as a function of $I$
\[
\frac{\partial \Psi}{\partial t} = \frac{1}{D_{II}} \frac{\partial^2 \Psi}{\partial I^2}, \quad (16)
\]
In Eq. (16), we introduced the diffusion coefficient $D_{II} = \text{Var}(\Delta I)/\tau$. Performing a change of coordinate from $t$ to $s$ (via $s = t R_0/R$) in Eq. (16) and introducing $u_R = v_R/v_R0$, with $v_R0$ the resonant velocity at the equator, we get
\[
\frac{\partial \Psi}{\partial h} = \frac{1}{h} \frac{\partial^2 \Psi}{\partial I^2}, \quad (17)
\]
where
\[
h = \frac{\omega^2 R_0 \sigma \tau}{\Omega_x v_R0 B_0^2} \int w(s') a_I^2(s') ds'. \quad (18)
\]
The boundary of the region filled by trapped trajectories is determined by the value of the area $S$ surrounded by the separatrices (see Eq. (9)). Again omitting all numerical factors of order unity, we get $S \approx \Omega_x$ (we assume that $\omega v^2 > a$ and, thus, in dimensional form $S \sim \omega \sqrt{\omega v \sim \Omega_x}$). Renormalizing $i = 2\pi I/S$, we can rewrite Eq. (17) in dimensionless form
\[
\frac{\partial \Psi}{\partial K} = \frac{1}{h} \frac{\partial^2 \Psi}{\partial I^2}, \quad (19)
\]
where
\[
K = \frac{2\pi \omega^4 R_0^4 \sigma \tau}{\omega^2 \rho v_R0 B_0^2} \int w(s') \mu_R(s') \left( \frac{c}{v_R0} \right)^2 k R_0 F_1(s) = K_0 F_1(s). \quad (20)
\]

Equation (19) describes the temporal evolution of the distribution $\Psi(i)$ of trapped particles. Thus, for a given initial condition, one can easily calculate the number of particles remaining trapped for a given value of $K$. Solutions of Eq. (19) are shown in Fig. 4 for three types of initial distributions: (a) A uniformly filled region of trapped particles, (b) trapped particles present only at the bottom of the potential well, and (c) only recently trapped particles with $i \sim 1$ are present. The right panel of Fig. 4 shows that in all three cases, the number of trapped particles diminishes as $K$ increases, ultimately decreasing to $50\% - 20\%$ already for $K = 0.2$. Consequently, to estimate the amount of detrapped (escaped) particles in realistic systems, one simply needs to evaluate $K = K_0 F_1(s)$, where $F_1(s)$ is of order unity. As trapped motion corresponds to $s \in [0, 1]$, the dependence of $K$ on $s$ can be further neglected, considering it merely as a multiplicative factor $\sim 1$. In the end, the occurrence of a significant detrapping of electrons is simply equivalent to the condition $K_0 = 0.2$, where $K_0$ is a function of the wave amplitude, the intensity of magnetic field fluctuations, and other wave parameters. Solving this equation $K_0 = 0.2$ for different realistic systems, we can deduce the intensity of magnetic field fluctuations required to detrapp a significant population of particles.

**A. Turning acceleration by whistler waves**

In this subsection, we consider particle trapping in the first cyclotron resonance with parallel propagating whistler-mode waves (one of the most classical examples of electron acceleration in the radiation belts, see, e.g., Refs. 15, 28, 29, 50, and 51 and reviews Refs. 62 and 76). In this regime of particle acceleration, the resonant velocity $v_R$ is
\[
v_R = (\omega \gamma - \Omega_x)/k \gamma. \quad (21)
\]

**FIG. 4.** Three examples of evolution of distributions of trapped particles $\Psi(i)$. All particles with $i > 1$ are assumed to escape from the system within a very short time interval. The right panel shows the evolution of the number of trapped particles for these three examples.
For particles with energy \( \gamma < \Omega_0/\omega \) (i.e., for energies lower than \( \sim 1 \) MeV), \( \nu_p \) is negative and trapped particles propagate in the direction opposite to the wave. However, if in the course of trapped particle acceleration, the energy increases enough to get \( \gamma > \Omega_0/\omega \), then the trapped particle’s direction of propagation is reversed (the so-called turning acceleration, see Refs. 25, 54, and 69). The total duration of trapped motion is rather long, leaving some room for the occurrence of a diffusive destruction of the trapping process.

As a first step, one can simply take \( \gamma \sim 1 \) and thus \( |v_{\|0}| = \Omega_0/(1-\omega/\Omega_0)/k \) to estimate the disturbance to the trapping process of 50 – 150 keV electrons induced by additional magnetic field fluctuations. Equation (20) can then be rewritten as

\[
K_0 = \frac{2\pi \sigma N^4 \Omega_0 R_0}{\omega B_v B_0} \left( \frac{\omega}{\Omega_0} \right)^4 \left( 1 - \frac{\omega}{\Omega_0} \right)^2, \tag{22}
\]

where \( N = k/c \) is the wave refractive index

\[
N^2 = \frac{k^2 c^2}{\omega^2} = \frac{\omega^2 p_e}{\Omega_0^2} \left( 1 - \frac{\omega}{\Omega_0} \right), \tag{23}
\]

where \( \omega p_e \) is the plasma frequency. Evaluating \( N^2 \) at the equatorial plane, we rewrite Eq. (22) as

\[
K_0 = \frac{2\pi \sigma \omega p_e R_0 B_0}{\omega B_v B_0} \left( \frac{\omega}{\Omega_0} \right)^{5/2} \left( 1 - \frac{\omega}{\Omega_0} \right)^{7/2}. \tag{24}
\]

We use the empirical plasmatrough density model from Ref. 61 to estimate the ratio \( \omega p_e/\Omega_0 \approx L \) as a function of \( L \)-shell. In the Earth’s magnetosphere, the ratio \( R_0 \Omega_0/c \) is about \( \sim 1.1 \times 10^3 L^{-2} \) for \( B_0 \approx 3.06 \times 10^5 \)nT/L^3. Thus, Eq. (24) finally takes the form

\[
K_0 \approx 2.3 \times 10^{-3} \frac{2\pi}{\omega t} \left( \frac{\sigma p T^2}{B_v n^2} \right)^4 \left( \frac{\omega}{\Omega_0} \right)^{5/2} \left( 1 - \frac{\omega}{\Omega_0} \right)^{7/2}. \tag{25}
\]

We consider quasi-stationary magnetic field fluctuations with an effective frequency \( 2\pi/\tau \) comparable to the frequency \( \omega \) of intense whistler-mode chorus waves responsible for particle trapping. Thus, one has \( 2\pi/\omega t \sim 1 \). This generic model of fluctuations can be regarded as a very rough representation of a peaked noise spectrum. We plot in Fig. 5 (left panel) the power of magnetic fluctuations \( \sigma^2 \) required to reach two values of \( K_0 \) for \( L = 5 \) and various values of \( \omega \Omega_0/\nu \) and wave amplitude \( B_w \). For a smaller wave frequency, stronger magnetic field fluctuations are necessary to significantly detrap the population of particles (i.e., to get \( K_0 = 0.2 \)). A realistic average level of fluctuations \( \sigma^2/\omega t \sim 6 - 20 \) pT can markedly disrupt the trapping process even for rather high amplitudes \( B_w \sim 0.3 - 1 \) nT of the main wave. For higher frequency waves (e.g., upper-band chorus waves with \( \omega \Omega_0 > 0.5 \)), relatively weak fluctuations with \( \sigma^2/\omega t \leq 10 \) pT are sufficient to totally destroy the trapped motion (since \( K_0 \) reaches one). Finally, for higher energy (\( \sim 0.3 - 1 \) MeV) electrons, the \( \gamma \) factors must be retained, leading to an overall increase of \( K_0 \) by a factor \( \sim \gamma (\omega \Omega_0/\omega) / (\Omega_0 - \gamma \omega) > 1 \). In the alternative case of really coherent whistler-mode fluctuations with \( \ell_f \approx k \), \( |v_{\|0}| \) should be replaced by \( |v_{\|f}| = |v_{\|0} - \omega f/\nu| \). It corresponds to an extra multiplicative factor \( \sim 1 - \gamma \omega/\Omega_0 \), yielding ultimately similar \( K_0 \) values as in Fig. 5 (left panel).

To demonstrate the effect of \( I \) invariant destruction, we have numerically integrated the Hamiltonian equations for Hamiltonian (10). With time normalized by \( \omega_s \), this Hamiltonian takes the form

\[
H = \frac{1}{2} \dot{p}^2 + \frac{e\nu}{2} \cos \phi - a_0 \dot{\phi} - a_1 \delta b \dot{\phi}, \tag{26}
\]

where \( \delta b \) is given by the equation

\[
\delta b \equiv \frac{1}{60} \frac{2\pi}{\omega t} \left( 1 - \frac{\omega}{\Omega_0} \right)^{3/2} \left( \frac{L}{5} \right)^2 \sqrt{\frac{\sigma^2}{\omega^2}} f(t), \tag{27}
\]

where \( f(t) \) is a random function with a time-step of variation equal to \( \tau \) and an amplitude equal to one. We assume that \( f
has a uniform distribution within the interval \([-1,1]\). For \(L \sim 5, \omega/\Omega_0 \sim 0.5, \) and \(\tau \approx 2\pi/\omega\) we have \(\delta b \approx \sqrt{\sigma/p^T_2}/(i)60). Figure 6 shows several examples of particle trajectories for \(a(0) = 1/(1 - s^2)\) and different values of \(\sigma\). Initially, all particles are located close to the bottom of the potential well \((I_0,\) the initial value of \(I,\) is significantly smaller than \(S/2\pi)\). In the absence of additional fluctuations \((\sigma = 0),\) the adiabatic invariant \(I\) is conserved and the particle escapes from the resonance when \(S = 2\pi I_0\). In a system with magnetic field fluctuations \((\sigma \neq 0),\) however, the conservation of \(I\) is broken and the invariant may increase or decrease with time (the profile \(I(t)\) depends on realizations of the random function \(f\) along the trace). However, decreasing \(I\) does not change the type of motion (a particle remains trapped), but increasing \(I\) may cause detrapping. For that purpose, in Fig. 6, we chose trajectories with increasing \(I.\) A stronger fluctuation level corresponds to a larger rate of \(I\) variation and, as a result, to earlier particle escape from resonance when \(I = S/2\pi\). The bottom panel of Fig. 6 shows the effective increase of the area surrounded by the particle trajectory in the \((\phi, P)\) plane.

### B. Landau resonance with oblique whistler waves

Recent spacecraft observations have revealed the existence of a substantial population of very intense oblique whistler-mode waves in the radiation belts.\(^2\),\(^\!*\),\(^9\),\(^\!*\)\(^4\) These waves propagate with a large angle \(\theta > \theta_g\) between their \(k\)-vector and the background magnetic field (where \(\cos \theta_g = 2\omega/\Omega_0\) and \(\theta_g\) is the so-called Gendrin angle\(^26\). Propagating in an almost electrostatic mode and having a large parallel electric field,\(^5\) such oblique waves are able to trap energetic \((\sim 10 - 100\) keV\) electrons via the Landau resonance.\(^9\),\(^3\)\(^5\) The corresponding frequency of trapped motion \(\Omega_r\) is determined by the electric field amplitude \(E_w: \Omega_r \approx \sqrt{k e E_w/mw}(s).\) The corresponding resonant velocity is \(v_{r0} = \omega/k_{\|} = \omega/(k \cos \theta)\). Accordingly, we consider \(\theta = \theta_g\) and rewrite Eq. (20) for \(K_0\)

\[
K_0 \approx \frac{2\pi}{\omega T E_w} / 2.7\text{ mV/m} = \frac{(S)}{5} \frac{\omega}{\Omega_0}. \tag{29}
\]

Typical amplitudes of intense oblique chorus wave are within the range \(E_w \in [10, 200]\) mV/m (see, e.g., Refs. 2, 18, and 74). For \(\tau \sim 2\pi/\omega\) and given values of \(K_0,\) we plot in Fig. 5 (central panel) the corresponding fluctuation amplitude \(\sigma^{1/2}\) as a function of main wave amplitude \(E_w.\) At \(L \sim 5,\) where a substantial population of oblique chorus waves has been observed,\(^4\) a moderate level of additional magnetic field fluctuations \(\sqrt{\sigma} \in [1, 10]\) \(pT\) appears to be sufficient to significantly destroy the trapped particle motion (at larger \(L\)-shells, the required level of \(\sigma\) is even smaller). If one considers coherent magnetic fluctuations corresponding to parallel whistler-mode waves with \(\omega_f \sim \omega\) and \(k_f \sim k,\) \(|v_{r0}|\) must be replaced by \(|v_{r0}| = |v_{r0} - \omega/k|\) in \(K_0.\) It yields an additional multiplicative factor \(\sim 1/(1 - \cos \theta_g) > 1\) in \(K_0\) and therefore a slightly stronger effect.

### C. Cyclotron resonance with EMIC waves

Strong EMIC waves are observed by spacecrafts mainly during geomagnetically disturbed conditions and inside regions of enhanced plasma density on the duskside.\(^4\),\(^3\)\(^8\)\(^1\) These waves play an important role in the scattering of relativistic and especially ultra-relativistic electrons and can ultimately precipitate them into the atmosphere.\(^3\),\(^3\)\(^9\),\(^6\)\(^8\)\(^7\) Moreover, the very high intensity of EMIC waves leads to nonlinear wave-particle interactions, including the trapping of relativistic electrons.\(^5\),\(^3\)\(^8\),\(^5\)\(^9\) Let us consider here the hydrogen band of EMIC waves.\(^4\),\(^3\)\(^8\) The dispersion relation of such parallel EMIC waves is
where the hydrogen plasma frequency is $\omega_{ph} = n^{1/2} \omega_{pe}$, and the proton gyrofrequency is $\Omega_p = n \Omega_0$, with $n = m_p/m_H$ the electron to proton mass ratio. Since one has $\omega \ll \Omega_0$, the corresponding electron velocity at cyclotron resonance is simply $v_{\text{esc}} \approx \Omega_0/\gamma (\gamma \Omega_0)$ (e.g., see Ref. 68), while $\gamma$ can reach $10 \sim 15$ and must therefore be retained. Using Eq. (30), we rewrite Eq. (20) as

$$N = \frac{ck}{\omega} \left( \frac{\omega^2_{ph}}{\Omega_H (\Omega_H - \omega)} \right).$$

(30)

The top row of panels shows one example of oblique whistler-mode waves (strong parallel electric field and weak magnetic field). In this spectrum, the central peak at $\sim 1.5$ kHz is surrounded by fluctuations (for instance at $\sim 1.8$ kHz or $\sim 3$ kHz) with a magnetic field power two-three orders of magnitude smaller than the main wave intensity. Magnetic field data show that the wave amplitude reaches $\sim 30$ pT, while the amplitude of fluctuations is about $\sim 1 \sim 2.5$ pT. Figure 5 (central panel) shows that such fluctuations can significantly influence the dynamics of trapped particles. In the case of the intense weakly oblique whistler wave at $\sim 2.4$ kHz shown in bottom panels of Fig. 7, the intensity of nearby magnetic field fluctuations is also significant. The main wave amplitude is about $\sim 250$ pT, while surrounding fluctuations (at $\sim 1.7$ kHz or $\sim 3.9$ kHz) have amplitudes $\sim 2 \sim 8$ pT. The latter level of fluctuations is high enough to result in particle escape from the resonance (see Fig. 5 (left panel)). Therefore, we can conclude that the effects discussed in our paper can actually occur under realistic conditions in the Earth’s radiation belts.

IV. SPACEMARK OBSERVATIONS

To derive equations describing the effects of magnetic field fluctuations on trapped particle motion, we assume that the timescale of these fluctuations is comparable with the period of the main wave trapping the electrons. To illustrate the presence of such fluctuations in real systems, we show two examples of observations of intense whistler-mode waves (highly oblique and parallel) in the radiation belts by one of the two Van Allen Probes spacecrafts (see Fig. 7). The top row of panels shows one example of oblique whistler-mode waves (strong parallel electric field and weak magnetic field). In this spectrum, the central peak at $\sim 1.5$ kHz is surrounded by fluctuations (for instance at $\sim 1.8$ kHz or $\sim 3$ kHz) with a magnetic field power two-three orders of magnitude smaller than the main wave intensity. Magnetic field data show that the wave amplitude reaches $\sim 30$ pT, while the amplitude of fluctuations is about $\sim 1 \sim 2.5$ pT. Figure 5 (central panel) shows that such fluctuations can significantly influence the dynamics of trapped particles. In the case of the intense weakly oblique whistler wave at $\sim 2.4$ kHz shown in bottom panels of Fig. 7, the intensity of nearby magnetic field fluctuations is also significant. The main wave amplitude is about $\sim 250$ pT, while surrounding fluctuations (at $\sim 1.7$ kHz or $\sim 3.9$ kHz) have amplitudes $\sim 2 \sim 8$ pT. The latter level of fluctuations is high enough to result in particle escape from the resonance (see Fig. 5 (left panel)). Therefore, we can conclude that the effects discussed in our paper can actually occur under realistic conditions in the Earth’s radiation belts.

V. ALTERNATIVE MECHANISMS OF DESTRUCTION OF TRAPPING MOTION

In the present study, the destruction of trapped particle motion is due to external, non-resonant magnetic field fluctuations, included as externally driven variations of the main system parameters. However, there are two other important sources of similar variations.

A. Modulation of wave amplitude

Intense whistler waves are usually propagating in the form of relatively long wave packets often exhibiting a fine subpacket structure corresponding to fast modulations of the wave amplitude.\textsuperscript{2,3,7,33,77,81} For resonant whistler-mode waves with $\theta = 0^\circ$, these modulations concern the wave magnetic field component transverse to the geomagnetic field line, while, in the case of very oblique quasi-electrostatic whistlers, they concern the wave electric field parallel component. Such fast modulations can effectively reduce the timespan of charged particle trapped motion\textsuperscript{2,73} and may be considered as fluctuations of the parameter $\epsilon$. In this case, the second term in the Hamiltonian (7) can be divided into nonperturbed $(\omega^2 \cos \phi)$ and perturbed $(\delta \omega^2 \cos \phi)$ parts. The corresponding change of invariant $I$ over one time step ($\sim \tau \ll 2\pi/\Omega_p$) can be written (in analogy with Eq. (11)) as
The magnetic field, while the dimensionless variable main equations coincide with equations derived for fluctuations. Thus, the term effect similar to the effect of magnetic field fluctuations. result in high-frequency variations of the term a disturbance (relative to the background magnetic field) electric field component. Middle panels present one of the transient components of wave magnetic field. Spectra for the intervals indicated by colors are displayed in the right panels.

\[ \Delta I = \frac{1}{\Omega_0 g} \Delta H = \frac{1}{g} \Omega_0 \frac{\delta l}{c} \sin \phi \Delta \phi. \]  

(33)

For \( \text{Var}(\Delta I) \), we obtain (see Eqs. (12)–(15))

\[ \text{Var}(\Delta I) \sim \Omega_0^2 I \tau^2 \sigma_z/2, \]  

(34)

where \( \sigma_z = \text{Var}(\delta l/c) \) and \( \text{Var}(\sin \phi) = 1/2 \). Thus, the corresponding parameter \( K_0 \) is

\[ K_0 \approx \frac{\pi \Omega_0^2 \tau R_s}{v_{R0}} \sigma_z = \frac{\pi c}{v_{R0}} \Omega_0 \tau \sigma_z. \]  

(35)

Equation (35) shows that the trapped particle motion is more unstable for high wave amplitude and a larger timescale of fluctuations \( \tau \). Thus, for charged particle trapping by very intense waves, the effect of strong modulations of the main wave amplitude can be more dangerous than the essentially diffusive effects of additional non-resonant magnetic field fluctuations studied in Section III.

B. Externally driven pitch-angle diffusion

The term \( a \) in the Hamiltonian (5) depends on the magnetic moment \( \mu \) of trapped particles. In the case of Landau resonance, \( \mu \) is conserved, while particle trapping into cyclotron resonance results in a variation of \( \mu \) with \( s \) (for details, see Ref. 62). Thus, externally driven fluctuations of \( \mu \) should result in high-frequency variations of the term \( a \), with a final effect similar to the effect of magnetic field fluctuations. Thus, the term \( a \sim \mu \) can be separated into two parts: undisturbed \( a_0 \) and fluctuating \( a_1 = a_0(\delta \mu/\mu) \). In this case, the main equations coincide with equations derived for fluctuations of the magnetic field, while the dimensionless variable \( K \) from Eq. (20) takes the form

\[ K = \frac{2\pi \sigma_0 R_s \tau}{\Omega_0 v_{R0}} \sigma_u \frac{w(s)}{a_0^2(s)} \int_0^\infty a_0^2(s')ds', \]  

(36)

with \( \sigma_u = \text{Var}(\delta \mu/\mu) \) and \( \tau \) the timescale of \( \mu \) variations, giving

\[ K_0 = \frac{2\pi c}{v_{R0}} \Omega_0 \tau \sigma_u. \]  

(37)

Equation (37) provides an estimate of the role of external pitch-angle (or magnetic moment) diffusion. However, the quasi-linear pitch-angle diffusion induced by a wide spectrum of low amplitude whistler-mode waves seems to be insufficiently strong (even for highly oblique waves) to produce a measurable change in the pitch-angle of particles within one bounce period of energetic electrons between their mirror points. Nevertheless, small-scale intense electrostatic structures recently observed in the radiation belts could bring forth a much more efficient pitch-angle scattering. Being generated in the form of wide packets with wide frequency ranges, these structures could interact with <20 keV electrons and the corresponding pitch-angle diffusion could result in a destruction of trapped motion. Moreover, disturbances to the current of thermal electrons caused by such electrostatic structures often generate magnetic field perturbations, which could serve as a source for some of the magnetic field fluctuations considered in Section III.

VI. CONCLUSIONS

In this paper, we have proposed an approach allowing to estimate the effect of additional non-resonant magnetic field fluctuations on the motion of relativistic electrons trapped by intense waves. Such an approach is principally based on the consideration of trapped electron oscillations in the phase

FIG. 7. The data are captured on November 1, 2012 by one Van Allen Probes spacecraft. Electric and magnetic field waveforms were produced by the EFW and EMFISIS detectors. The top panels show a very oblique wave with \( \theta \sim 70^\circ \). The bottom panels present waves with \( \theta \leq 10^\circ \). Left panels show the electric field component. Middle panels present one of the transverse components of wave magnetic field. Spectra for the intervals indicated by colors are displayed in the right panels.
plane where a corresponding adiabatic invariant can be introduced. Magnetic field fluctuations can break the conservation of this invariant and ultimately result in electron detrapping. This general approach has been used here to estimate the stability of trapped motion in three systems describing electron acceleration in the Earth’s radiation belts.

We show that the first and second systems describe electron trapping into the first cyclotron and Landau resonances by intense parallel and oblique whistler-mode waves. For typical wave amplitudes (~0.1 – 1 nT or ~10 – 100 mV/m), additional magnetic field fluctuations with an intensity ~1 – 100 pT² can significantly disrupt the trapped motion. The fluctuation intensity necessary for significant perturbations of the trapped motion varies with L-shell as ~L⁻¹⁵, and it is larger for a smaller frequency \( \Omega / \Omega_c \) of the main wave.

The third system concerns electron trapping into the first cyclotron resonance by intense EMIC waves. For typical wave amplitudes ~1 nT, significant perturbations of the trapped motion can be obtained only for much higher frequency magnetic field fluctuations at a high intensity ~10,000 pT². Thus, trapped motion appears more stable than in the two previous cases. It can be perturbed only for high-frequency EMIC waves with \( \Omega / \Omega_c \geq 0.5 \).

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