Fast transport of resonant electrons in phase space due to nonlinear trapping by whistler waves

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- Nonlinear wave-particle interaction as a fast transport in phase space
- Comparison of efficiency of quasilinear and nonlinear electron accelerations
- Wave spectrum slope determines the importance of nonlinear acceleration

Supporting Information:
- Readme
- Text S1

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Fast transport of resonant electrons in phase space due to nonlinear trapping by whistler waves

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Abstract We present an analytical, simplified formulation accounting for the fast transport of relativistic electrons in phase space due to wave-particle resonant interactions in the inhomogeneous magnetic field of Earth's radiation belts. We show that the usual description of the evolution of the particle velocity distribution based on the Fokker-Planck equation can be modified to incorporate nonlinear processes of wave-particle interaction, including particle trapping. Such a modification consists in one additional operator describing fast particle jumps in phase space. The proposed, general approach is used to describe the acceleration of relativistic electrons by oblique whistler waves in the radiation belts. We demonstrate that for a wave power distribution with a hard enough power law tail $P(B_w^2) \propto B_w^{-\eta}$ such that $\eta < 5/2$, the efficiency of nonlinear acceleration could be more effective than the conventional quasi-linear acceleration for 100 keV electrons.

1. Introduction

The acceleration of relativistic electrons in the radiation belts is one of the most attractive problems of magnetospheric plasma physics [*Millan and Baker*, 2012]. The main role in this acceleration is played by resonant wave-particle interaction; thus, the acceleration process is often spatially and temporally localized [*Thorne et al.*, 2013; *Reeves et al.*, 2013]. The conventional description of wave-particle resonant interaction is based on a Fokker-Planck equation for the particle velocity distribution, with quasilinear diffusion coefficients derived for a given wave spectrum [*Drummond and Pines*, 1962; *Vedenov et al.*, 1962]. For a system with an inhomogeneous magnetic field, this equation is averaged over the periodic bounce motion of charged particles [see *Albert*, 2005; *Glauert and Horne*, 2005; *Shprits et al.*, 2008, 2009; *Ni et al.*, 2011a, 2011b]. Nevertheless, this approach excludes the consideration of any nonlinear effect such as particle trapping, while the latter can be very important [*Shklyar and Matsumoto*, 2009]. Thus, the quasilinear approach is applicable only under the assumption of small wave amplitudes [*Shapiro and Sagdeev*, 1997]. However, recent spacecraft observations in the Earth's radiation belts strongly suggest the presence of some amount of waves with amplitudes much larger than the threshold for nonlinear interaction [*Cattell et al.*, 2008; *Cully et al.*, 2008; *Li et al.*, 2011; *Wilson et al.*, 2011]. As a result, the description of wave-particle resonant interactions should be crucially modified. One of the possible approaches consists in careful numerical simulations [*Furuya et al.*, 2008; *Tao et al.*, 2012] of nonlinear wave-particle resonant interactions. However, results obtained this way cannot be directly incorporated into the Fokker-Planck equation for the electron distribution function which is solved in radiation belt codes aiming at forecasting space weather [e.g., see *Shprits et al.*, 2008; *Thorne et al.*, 2013]. In this paper, we present an alternative, efficient approach allowing to combine nonlinear effects with the Fokker-Planck equation.

2. Electron Trapping and Energy Jumps

Owing to many recent spacecraft missions, we have access to uniquely detailed information about wave distributions in the Earth radiation belts [*Cattell et al.*, 2008; *Cully et al.*, 2008; *Li et al.*, 2011; *Agapitov et al.*, 2013]. In particular, high-amplitude strongly oblique whistler waves are often observed in the outer radiation belt ($\sim$ 4–6 R_E away from the Earth). These waves propagate in a quasi-electrostatic mode and effectively trap electrons in the Landau resonance [*Bell*, 1986; *Artemyev et al.*, 2012]. The Hamiltonian of the relativistic charged particle (of mass m_e and charge e) interaction with a quasi-electrostatic wave $\Phi \sin\phi$ has 1 degree of freedom (motion along field lines) and a time dependence [*Artemyev et al.*, 2013]:

$$\begin{cases} H = \gamma - \varepsilon u_0(s)\sin\phi \\ \gamma = \sqrt{1 + p_\parallel^2 + 2Ib(s)} \\ \phi = \phi_0 + \int^s k_\parallel(s')\mathrm{d}s' - \omega t \end{cases} \tag{1}$$

where $(s, p_\parallel)$ are the parallel coordinate and momentum of the particle normalized to field line curvature radius R_0 and $m_e c$, respectively, $I = \text{const}$ is the normalized magnetic moment, $b(s) > 1$ is the magnetic field magnitude normalized to its minimum (equatorial) value, wave-frequency ω and time t are normalized by R_0/c, while the wave number $k_\parallel(s)$ is normalized by R_0 (see details in *Artemyev et al.* [2013]). The dimensionless wave amplitude $\varepsilon = e\Phi_0/m_e c^2 \ll 1$ denotes the ratio of wave energy to particle rest energy (here Φ_0 is the amplitude of the wave scalar potential). For simplicity in this letter, we do not take into account the wave packet structure of whistler waves in the radiation belts. However, *Artemyev et al.* [2012] showed that effects of such a structure can substantially influence electron acceleration in Landau resonance only at very high latitudes where the coherence of waves within one packet significantly decreases. Thus, we simply describe the variation of amplitude of the wave during its propagation along a field line by a function $u_0(s) \in [0, 1]$ derived from Cluster statistics (see details in *Artemyev et al.* [2012]).

In the vicinity of the Landau resonance $\dot\phi = 0$ (i.e., $\dot s = v_R = \omega/k_\parallel$), system (1) can be expanded as

$$\begin{cases} \ddot\phi = -A + B\cos\phi \\ A = \frac{Ik_\parallel - v_R^2\gamma^2 k_\parallel'}{1+2Ib(s)}, \; B = \varepsilon k_\parallel^2 u_0/\gamma \end{cases} \tag{2}$$

where the resonant gamma factor is $\gamma = \sqrt{1+2Ib(s)}/\sqrt{1-v_R^2}$. The phase portrait of system (2) is shown in Figure 1a for $A < B$. It is worth emphasizing that this phase portrait is actually representative of any many physical systems where particles interact with waves propagating along inhomogeneous magnetic fields [*Shklyar and Matsumoto*, 2009]. The separatrix demarcates trajectories of trapped particles (closed trajectories oscillating around $\dot\phi = 0$) and transient particles. If the area S surrounded by the separatrix increases with time, then new particles can be trapped by the wave [*Shklyar*, 1981; *Arnold et al.*, 2006]. An example of such a trapping process obtained by numerically solving system (1) is presented in Figure 1c. First, the particle oscillates between mirror points in the geomagnetic field (see schematic view of the system in Figure 1b). Then, the resonant interaction of the particle with the wave results in particle trapping and motion along with the wave. The particle escapes from resonance at higher latitude (when A becomes larger than B, see details in *Shklyar and Matsumoto* [2009]). This single wave particle resonant interaction corresponds to a substantial decrease of the particle pitch angle as well as a significant increase of its energy (see Figure 1d). Here a dipolar magnetic field $b(s)$ has been assumed and the variation of $k_\parallel$ along the field line corresponds to whistler mode wave propagation at the Gendrin angle (see details in *Artemyev et al.* [2013]). The wave amplitude is taken as $\Phi_0 k_\parallel \sim 50$ mV/m [*Cattell et al.*, 2008; *Agapitov et al.*, 2014]. The unit of normalized time (R_0/c) corresponds roughly to a quarter of the particle bounce period. Thus, the acceleration of trapped particles takes less than one bounce period (see Figure 1d). Such a fast acceleration can be described as a particle jump in velocity space (or in the energy/pitch angle space) for systems averaged over bounce motion. Such jumps are essentially nondiffusive and cannot in principle be accounted for by the diffusion equation. Consequently, they have to be modeled as an additional term in the Fokker-Planck equation.

To incorporate effects of nonlinear particle acceleration with the quasilinear effects described by the Fokker-Planck equation for distribution function $f(E, \alpha_0)$, one should first determine the probability of trapping, Π. For given initial pitch angle and energy, this probability is the ratio of particles trapped during the next resonant interaction to the whole number of particles passed through the resonance. This probability can be found as the ratio of trapped to resonant phase fluxes [see *Shklyar*, 1981; *Arnold et al.*, 2006]. For system (2), an analytical expression for Π was derived in *Artemyev et al.* [2013] under the assumption of wave propagation at the Gendrin angle:

$$\Pi = \frac{\sqrt{\varepsilon}}{4\pi\omega_{pe}A}\frac{\partial S}{\partial s} \tag{3}$$

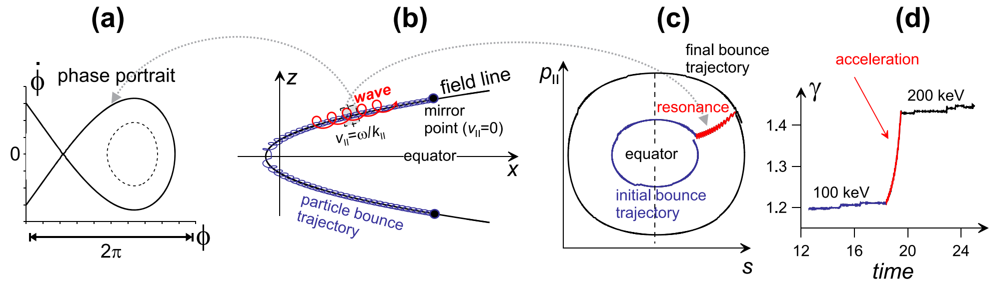

Figure 1. (a) Phase portrait of system (2) for $A < B$. (b) Schematic view of the system. (c) Particle trajectory in $(s, p_{\|})$ space. Black and blue colors are used for final and initial bounce oscillations, while red color shows trapped particle motion. (d) Particle gamma factor as a function of time.

where ω_{pe} is the plasma frequency to electron gyrofrequency ratio at the equator. Probability Π in equation (3) should be evaluated at the resonance point where $\dot{s} = \omega/k_{\|}$. The area S is determined by $A(s)$ and $B(s)$ functions [see *Arnold et al.*, 2006; *Artemyev et al.*, 2013]. Equation (3) is valid for large enough wave amplitudes $B > \sqrt{\epsilon}A$ where $B \sim \epsilon$ according to equation (2). It should remain a good approximation for wave propagation near and above the Gendrin angle but not too close to the resonance cone [*Artemyev et al.*, 2013; *Agapitov et al.*, 2014].

Equation (3) can be checked by comparisons with test particle simulations. In Figure 2, the analytical and numerical probabilities Π have been plotted for four values of the equatorial pitch angle and various energies. The numerical probability is obtained as the ratio of the number of particles trapped during their first resonant interaction with the wave to the total number of resonant particles. Each trajectory is modeled during a quarter of the bounce period. The analytical probability of trapping Π describes quite well the numerical results.

3. Nondiffusive Transport of Particles in Phase Space

For given values of energy and equatorial pitch angle (E^{**}, α_0^{**}), one can define the number of particles $N = f\delta E\delta\alpha_0$ contained in the domain $E^{**} \pm \delta E$, $\alpha_0^{**} \pm \delta\alpha_0$. The scales of this domain $(\delta E, \delta\alpha)$ correspond to the resonance width in the space (E, α_0). Considering this restricted phase space domain, particles are either coming into it or leaving it as a consequence of their interaction with the waves. The latter case corresponds to an outward flux of $\Pi_- N_-$ particles transported during each time step into the domain centered at $(E^{**} + \Delta E, \alpha_0^{**} + \Delta\alpha_0)$. $\Delta E(E^{**}, \alpha_0^{**})$ and $\Delta\alpha(E^{**}, \alpha_0^{**})$ denote the changes of the particle energy and pitch angle during one trapping event (analytical expressions for ΔE, $\Delta\alpha$ have been derived in *Artemyev et al.* [2013]), and the time step is hereafter taken as half of the bounce period. Both Π_- and N_- are evaluated at (E^{**}, α_0^{**}). To this outward flux, one must add an inward flux corresponding to $\Pi_+ N_+$ particles transported to the domain centered at (E^{**}, α_0^{**}). Π_+ and N_+ are evaluated at (E^*, α_0^*) given by equations $E^* + \Delta E(E^*, \alpha_0^*) = E^{**}$, $\alpha_0^* + \Delta\alpha_0(E^*, \alpha_0^*) = \alpha_0^{**}$. Consequently, there is a competition between two opposite phase flows (see scheme in Figure 3).

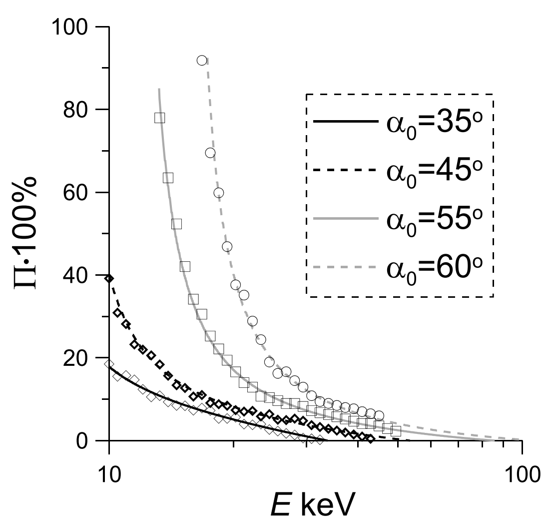

Figure 2. Comparison of the analytical probability (curves) given by equation (3) and numerical probabilities (symbols). To obtain each point of the numerical probability we run 10^6 particles with the same initial energy E and equatorial pitch angle α_0—only initial wave phase ϕ_0 is changed. System parameters are typical for the Earth radiation belts: $\omega_{pe} = 4.5$, the wave amplitude is ~50 mV/m, and the ratio of wave frequency to electron equatorial gyrofrequency is 0.35.

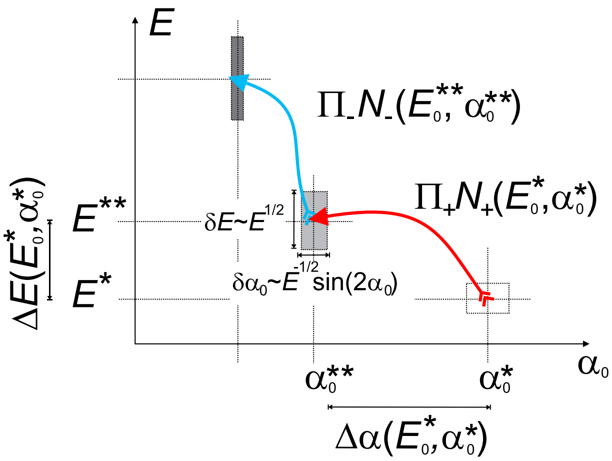

Figure 3. Schematic view of particle jumps in the phase space (E, α_0).

To determine the width of resonances in the pitch angle/energy space, we introduce the resonance width $\delta v_\| \approx \sqrt{(1-v_R^2)\varepsilon u_0/\gamma}$ in the parallel velocity space (see description in *Shklyar and Matsumoto* [2009] and *Arnold et al.* [2006]):

$$\delta E = m_e c^2 \gamma v_R \delta v_\| = m_e c^2 v_R \sqrt{(1-v_R^2)\varepsilon u_0 \gamma}$$

$$\delta\alpha = \frac{\delta v_\|}{2v_R}\sin 2\alpha = \frac{\sqrt{(1-v_R^2)\varepsilon u_0/\gamma}}{2v_R}\sin 2\alpha$$

These expressions should be evaluated at the resonance point where $\dot{\phi} = 0$. The local particle pitch angle α can be obtained from the adiabatic invariance $\sin^2\alpha = \sin^2\alpha_0 b(s)$.

Finally, to describe jumps of particles in phase space (E, α_0), we introduce two operators $\hat{L}_\pm$ for the distribution function $f(E, \alpha_0)$:

$$\hat{L}_\pm = \tau_b^{-1}\int_0^\infty\int_0^{\pi/2}\frac{\Pi(\alpha_0', E')\mathrm{X}_\pm(E', \alpha_0')}{\delta E(\alpha_0', E')\delta\alpha_0(\alpha_0', E')} f(E', \alpha_0')\,\mathrm{d}E'\mathrm{d}\alpha'$$

$$\mathrm{X}_+(E', \alpha_0') = \bar{\Theta}(E - E' - \Delta E(\alpha_0', E')) \cdot \bar{\Theta}(\alpha_0 - \alpha_0' - \Delta\alpha_0(\alpha_0', E'))$$

$$\mathrm{X}_-(E', \alpha_0') = \bar{\Theta}(E - E')\bar{\Theta}(\alpha_0 - \alpha_0')$$

The time scale of such a change of f is the bounce period $\tau_b \approx (4/\sqrt{1-\gamma^{-2}})(1.3802 - 0.6397\sin^{3/4}(\alpha_0))$ [*Shprits et al.*, 2008]. The function $\bar{\Theta}(x - x')$ is equal to one for $|x - x'| < \delta x$ and is zero otherwise (δx is $\delta\alpha_0$ or δE). Thus, the nonlinear evolution of the distribution function due to particle trapping is described as

$$\partial f(E, \alpha_0)/\partial t = (\hat{L}_+ - \hat{L}_-)P_w \tag{4}$$

where P_w is the dimensionless probability of observation of high-amplitude waves. This probability P_w does not depend on particle energy or pitch angle and can be included into the renormalization of time $t \to tP_w$. For given (E, α_0), the operator $\hat{L}_+$ determines the amount of particles transported into the domain $(E \pm \delta E, \alpha_0 \pm \delta\alpha_0)$ while $\hat{L}_-$ determines the amount of particles evacuated from it. For a general description of wave-particle interactions, operators (4) can be directly included into the Fokker-Planck equation (see the supporting information).

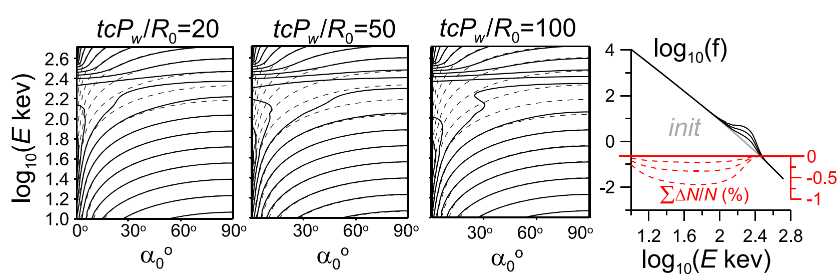

Figure 4. (first to third panels) Two-dimensional maps of level lines of electron distribution $f(E, \alpha_0)$ at three moments of time are shown by solid curves. The initial electron distribution is shown by dotted curves. (fourth panel) The integrated distribution $\int f(E, \alpha_0)\sin\alpha_0 \mathrm{d}\alpha_0$ at these three moments of time, with the initial distribution $f_0 \sim (\gamma^2 - 1)^{-6}\sin^2\alpha_0$ added in grey. The energy-integrated variation of the number of particles is shown in red with separate axes of coordinates (also in red). We consider a single wave propagating at the Gendrin angle with an amplitude of 50 mV/m at the equator. Magnetic field model is taken from *Bell* [1986]. Dependences of parameters ΔE, $\Delta\alpha_0$ on α_0, γ are calculated by methods described by *Artemyev et al.* [2013] (see also the supporting information).

An example of solution of equation (4) is shown in Figure 4. We use a typical initial distribution of particles $f_0 \sim (\gamma^2 - 1)^{-6} \sin^2 \alpha_0$ and consider a single wave with an amplitude ∼50 mV/m. Two-dimensional maps in Figure 4 show the evolution of $f(E, \alpha_0)$ with time. There is an increase of the phase space density (PSD) in the small pitch angle range and at $E \sim 100$–200 keV due to the transport of trapped particles into this region. After a certain time, the PSD forms a plateau around $\alpha_0 < 30°$ and $E \in [50, 250]$ keV. Figure 4 (fourth panel) shows the evolution of the integrated distribution function $\int f(E, \alpha_0) \sin \alpha_0 d\alpha_0$: the resonant nonlinear acceleration results in the formation of a local beam-like maximum of PSD around $E \sim 150$ keV. It is worth noting that operators $\hat{L}_\pm$ conserve the total number of particles in the system, since we only deal with particle transport in phase space. Figure 4 (fourth panel) shows the integrated variation of the number of particles $\Delta N(E_{\max}) = \int^{E_{\max}} (f_t - f_{t=0}) dE$. It can be seen that $\Delta N \to 0$ as $E_{\max} \to \infty$.

4. Discussion and Conclusions

The probability P_w determines how often particles can interact with high-amplitude waves. We can now estimate the value of P_w required to make the mechanism of nonlinear acceleration relatively important not only for individual particles but also for the energization of large amounts of electrons. To this aim, nonlinear acceleration can be compared with analytical estimates of quasi-linear energy diffusion coefficients [*Mourenas et al.*, 2012a] and lifetimes [*Mourenas et al.*, 2012b] of electrons interacting with waves of average amplitudes ∼10 pT [*Agapitov et al.*, 2013] at a distance ∼4–6 R_E from the Earth:

$$\left(D_{EE}/E^2\right) [\mathrm{s}^{-1}] \approx 2 \cdot 10^{-7} \cdot (\gamma - 1)^{-3/2}$$
$$\tau_L [\mathrm{s}] \approx 8 \cdot 10^4 \cdot \gamma(\gamma^2 - 1)^{1/2}$$

Particle energy grows due to stochastic diffusion like $E \approx \sqrt{D_{EE} t} \approx 0.02 \cdot (t[\mathrm{s}])^{2/3}$ keV. To gain 50 keV, 100 keV particles therefore need $t \sim 10^5$s. On the other hand, the mechanism of nonlinear acceleration can transport a substantial particle population with initial energies ∼100 keV up to the energy range ∼150 keV in $t_{\mathrm{NL}} \sim 20\, R_0/P_w c \sim 1\mathrm{s}\, /P_w$ (see Figure 4). Thus, the effectiveness of nonlinear acceleration will be comparable to, or larger than, the effectiveness of quasi-linear acceleration for $P_w > 10^{-5}$; i.e., there should be at least one wave with amplitude ∼50 mV/m (or ∼1 nT for propagation at the Gendrin angle) for 10^5 waves of 10 pT average amplitude. This can occur if wave power B_w^2 follows a power law distribution such that $P_w(B_w^2) \propto B_w^{-\eta}$ with $\eta < 2.5$. Recent spacecraft observations suggest that such kinds of power law distributions could be rather common in the Earth radiation belts [e.g., *Cully et al.*, 2008].

A comparison between the time scale of nonlinear acceleration $t_{\mathrm{NL}} \sim 20\, R_0/P_w c$ and the lifetime of electrons τ_L suggests a possible scenario of acceleration. For 100 keV electrons and $P_w \sim 10^{-4}$, we get $t_{\mathrm{NL}}/\tau_L \sim 1/10 \ll 1$, and there is enough time for cyclotron diffusion (or trapping) to increase the pitch angle of these particles, avoiding their direct scattering in the loss cone. Thus, a portion of accelerated particles can survive and take part in the formation of a new energy distribution. For a smaller value of P_w (such that $t_{\mathrm{NL}}/\tau_L \sim 1$), conversely, the fast transport of electrons toward the loss cone may result in intense bursty precipitations. Spacecraft observations of strong correlations between such precipitations and intense whistler waves [e.g., *Kersten et al.*, 2011] support this scenario. Future works should check the actual accuracy of the new term describing nonlinear acceleration by detailed comparisons with particle simulations, which is beyond the scope of the present letter. Moreover, we have shown that the parameter P_w (or the shape of the P_w distribution of amplitudes) determines the importance of nonlinear effects as compared with quasi-linear ones. Thus, accurate measurements by the Van Allen Probes of the wave power distribution function over a wide range of amplitudes will be crucial in the future to assess statistically the relative importance of our new term $(\hat{L}_+ - \hat{L}_-)P_w$.

To derive the main equations used in this paper, several approximations have been made. First of all, we consider whistler waves propagating at the Gendrin angle. The final conclusions should remain valid for most highly oblique waves, provided that their propagation angle remains at least 1° smaller than the resonance cone angle [*Agapitov et al.*, 2014]. For such very oblique high-amplitude waves, Landau resonance can be more effective than cyclotron resonances [*Artemyev et al.*, 2013], justifying our approximation that the magnetic moment of electrons is conserved. For high-amplitude parallel waves, however, the effect of trapping into cyclotron resonances should be taken into account [e.g., *Shklyar and Matsumoto*, 2009]. Nonlinear acceleration due to trapping into cyclotron resonance should be more effective for high-energy (> 500 keV) electrons which cannot be trapped into Landau resonance [*Artemyev et al.*, 2013]. Thus, the generalization of the proposed approach for parallel whistler waves is an important task. We also omit several potentially important effects: wave frequency modulation [e.g., *Demekhov et al.*, 2006] and loss of phase correlation at high latitudes [*Tsurutani et al.*, 2011]. The role of all these effects should be estimated in the future.

In this paper, we show that an essentially nonlinear effect (like particle trapping) can actually be combined with the classical quasilinear description of wave-particle resonant interaction. To this aim, one should first determine the main properties of the nonlinear process (probability of trapping, corresponding energy gains, and pitch angle jumps) and then construct the new operator for the particle distribution function. This approach directly shows that for hard enough spectral distributions of wave power, the effects of nonlinear wave-particle interaction may dominate in the formation of the particle energy/pitch angle distribution.

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