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Key Points:
- A Dupuit solution is compared to a full-depth solution of saturated flow
- A criterion is established for the validity of Dupuit solutions
- Topographic and hydrogeologic controls are investigated

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Is the Dupuit assumption suitable for predicting the groundwater seepage area in hillslopes?

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Abstract Many physically based hydrological/hydrogeological models used for predicting groundwater seepage areas, including topography-based index models such as TOPMODEL, rely on the Dupuit assumption. To ensure the sound use of these simplified models, knowledge of the conditions under which they provide a reasonable approximation is critical. In this study, a Dupuit solution for the seepage length in hillslope cross sections is tested against a full-depth solution of saturated groundwater flow. In homogeneous hillslopes with horizontal impervious base and constant-slope topography, the comparison reveals that the validity of the Dupuit solution depends not only on the ratio of depth to hillslope length d/L (as might be expected), but also on the ratio of hydraulic conductivity to recharge K/R and on the topographic slope s. The validity of the Dupuit solution is shown to be in fact a unique function of another ratio, the ratio of depth to seepage length d/L_S. For $d/L_S < 0.2$, the relative difference between the two solutions is quite small ($<$14% for the wide range of parameter values tested), whereas for $d/L_S > 0.2$, it increases dramatically. In practice, this criterion can be used to test the validity of Dupuit solutions. When d/L_S increases beyond that cutoff, the ratio of seepage length to hillslope length L_S/L given by the full-depth solution tends toward a nonzero asymptotic value. This asymptotic value is shown to be controlled by (and in many cases equal to) the parameter $R/(sK)$. Generalization of the findings to cases featuring heterogeneity, nonhorizontal impervious base and variable-slope topography is discussed.

1. Introduction

Groundwater seepage areas (also called saturated areas), formally defined as the intersection between the water table and the land surface, play a critical role in many hydrological processes such as groundwater flow, groundwater-surface water interaction, and surface runoff, which all have important environmental implications [*Winter et al*., 1998; *Sophocleous*, 2002]. Different groundwater flow paths converge into these areas, mixing waters of different properties (chemistry, temperature) that impact both near-stream and in-stream conditions [*Winter*, 1999; *Dahl et al*., 2007]. Being saturated, seepage areas create the conditions for wetlands to develop [*Mitsch and Gosselink*, 2007], and control the functioning of riparian areas [*Tabacchi et al*., 1998; *Jencso et al*., 2010]. Soil saturation also prevents any infiltration, so that saturation-excess overland flow occurs after rainfall, a major process contributing to flood peak in adjacent streams [*Kirkby*, 1988]. Seepage areas further constitute surfaces of maximum evaporation and transpiration from the water table [*Salvucci and Entekhabi*, 1995; *Winter*, 1999]; therefore, their spatial extent influences the water balance and vegetation development. Eventually, erosion processes implied by the seepage of groundwater to the land surface control channel initiation [*Montgomery and Dietrich*, 1989]. To address the pressing management issues related to all these processes, predicting groundwater seepage areas is essential.

The extent of groundwater seepage areas depends on a number of topographic and hydrogeologic factors. The underlying free-surface groundwater flow process is strongly nonlinear in terms of seepage area variation [*Bear*, 1972; *Crank*, 1984; *Forster and Smith*, 1988]. Three-dimensional, variably saturated models [e.g., *Freeze*, 1971] can theoretically offer a solution for this problem, but in practice their complexity often prevents their use. Such models require large computing resources even for small-scale problems, and their parameterization is difficult due to important uncertainties generally associated with the necessary detailed subsurface properties.

As an alternative, simplified models based on the Dupuit assumption have been developed. In idealized hillslopes, the Dupuit assumption even allows the derivation of analytical expressions relating the extent of seepage areas to topographic and hydrogeologic factors [*O'Loughlin*, 1981; *Salvucci and Entekhabi*, 1995; *Ogden and Watts*, 2000; *Batelaan and De Smedt*, 2004]. Such expressions are highly valuable since they provide direct insight into the way topographic and hydrogeologic factors control the extent of seepage areas. In more realistic cases, numerical models based on the Dupuit assumption can be used. Such models have largely been adopted by the community. They encompass: topography-based index models such as TOPMODEL [*Beven and Kirkby*, 1979; *Beven et al*., 1995], THALES [*Moore and Grayson*, 1991; *Grayson et al*., 1992], and TOPKAPI [*Todini and Ciarapica*, 2001]; PLASM [*Potter and Gburek*, 1987]; the hillslope-storage Boussinesq model of *Troch et al*. [2003]; HYDRAT2D [*Cohen et al*., 2006]; and MODFLOW [*McDonald and Harbaugh*, 1988] when adopting a single unconfined layer and activating the DRAIN package or the SEEPAGE package [*Batelaan and De Smedt*, 2004].

In order to ensure the sound use of such simplified models, knowledge of the conditions under which they provide a reasonable approximation is critical. The Dupuit assumption consists of a dimensional reduction of saturated flow. Therefore, a fundamental question is: under which conditions does a Dupuit solution provide a reasonable approximation of a full-dimensional solution of saturated flow? This paper deals with this question, which is referred to as the question of validity.

Although the validity of the Dupuit assumption has been extensively studied in regard to flow or water table prediction in a number of configurations [*Bear*, 1972; *Haitjema*, 1987; *Chenaf and Chapuis*, 2007; *Rushton and Youngs*, 2010], it has been poorly studied in regard to seepage area prediction using models such as stated above. *Potter and Gburek* [1986] addressed this issue explicitly, comparing a Dupuit solution to the solution given by a variably saturated model in hillslope cross sections. However, although they examined a number of configurations, they restricted the analysis to a constant-head profile at the downhill boundary, representing a stream. This boundary condition implies that the impervious base of the modeled hillslope lies at the level of the streambed, neglecting deeper groundwater flow. This assumption was also adopted in the comparison of a hillslope-storage Boussinesq model to a variably saturated model [*Paniconi et al*., 2003]. Yet, in many cases, the effectively impervious base lies at depth below the streambed [e.g., *Forster and Smith*, 1988]. Also, due to catchment symmetry, the downhill boundary below the streambed is a groundwater divide (Figures 1a and 1b). This study focuses on such cases.

In this study, a Dupuit solution of the seepage length in hillslope cross sections is tested against a full-depth solution of saturated groundwater flow. The study is conducted for wide ranges of values of topographic and hydrogeologic parameters. Namely, the effect of the depth to impervious base is carefully investigated. The limits of the validity of the Dupuit solution are established, and are shown to be controlled by a simple geometric criterion. Beyond these limits, topographic and hydrogeologic controls are examined and empirical expressions for the seepage length are given.

2. Methods

2.1. Hillslope Model and Processes

A generic hillslope cross section is considered in which the base as well as both uphill and downhill vertical boundaries are impervious (Figure 1b): water enters and leaves the groundwater system only across the water table. This system models half an ideally symmetrical and isolated first-order catchment (Figure 1a); potential leakage of deep groundwater toward downhill catchments is not considered. A hypothetical stream, whose geometrical dimensions are neglected, is assumed to drain both direct runoff and groundwater discharge at the valley bottom, maintaining the hydraulic head at the land surface level. The study focuses primarily on homogeneous hillslopes with horizontal impervious base and constant-slope topography (Figure 1b). More realistic cases featuring heterogeneity, nonhorizontal impervious base, and variable-slope topography are discussed in section 4.3.

The study deals exclusively with fully saturated groundwater flow; the reference model to which the Dupuit solution is compared does not include the unsaturated zone. Whereas various phenomena related to the infiltration process are thus disregarded, this allows for a strict test of the Dupuit assumption (dimensional reduction of saturated flow). For simplicity, a uniform recharge rate is specified at the water table level (where the water table is below the land surface). However, the Dupuit assumption has been shown to be invalid around small recharge areas [*Haitjema*, 1987]; therefore, cases with highly localized recharge areas could imply more restrictions than in this study regarding the validity of Dupuit solutions. Finally, the

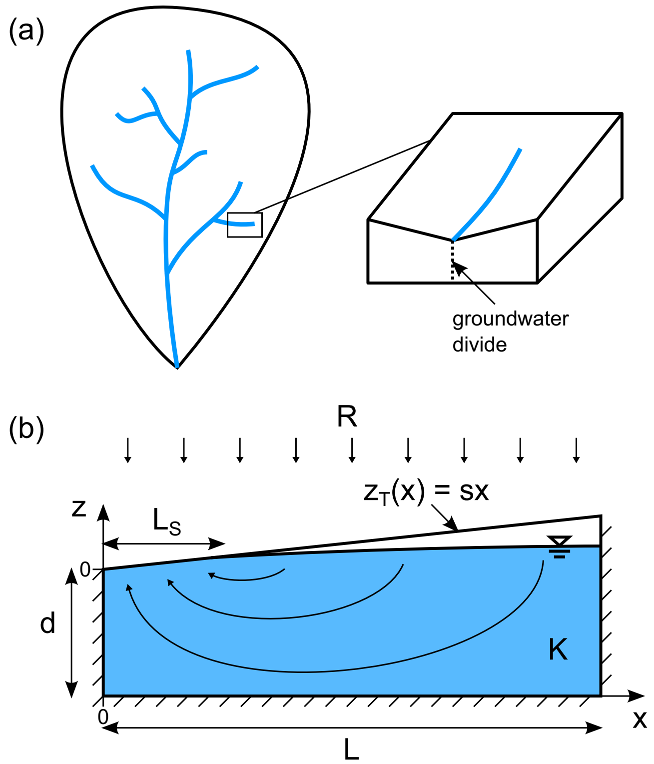

Figure 1. (a) Schematic top view of a watershed and 3-D view of an ideally symmetrical and isolated first-order catchment. (b) Generic 2-D hillslope, modeling the cross section of one half of the ideal first-order catchment shown in Figure 1a. The length of the seepage area, L_S, is the main feature analyzed.

system is studied at steady state. Such a system does not capture transient behaviors, but is sufficient to test the core of the Dupuit assumption. The associated findings are expected to form a solid and necessary base for the analysis of more complete formulations.

The system parameters are: depth to impervious base below the streambed d [L], hillslope length L [L], hydraulic conductivity K [LT^{-1}], available recharge rate R [LT^{-1}], and topographic slope s [-]. The study focuses on the seepage length L_S. The system is made nondimensional by using hillslope length L and recharge rate R as reference quantities for lengths and velocities, respectively. Three dimensionless parameters remain: the ratio of depth to hillslope length d/L [-], the ratio of hydraulic conductivity to recharge K/R [-] and the topographic slope s [-]. Consistent with this normalization, the ratio of seepage length to hillslope length L_S/L [-] is studied. In order to study a wide range of possible environments, the analysis is carried out for ranges of parameter values covering many orders of magnitude: $10^{-5} \leq d/L \leq 10^{2}$, $10^{0} \leq K/R \leq 10^{5}$, and $10^{-3} \leq s \leq 10^{-1}$.

2.2. Dupuit Solution

Let us assume [after *Dupuit*, 1863] that the flow lines crossing the vertical section located at $x=L_S$ (upper point of the seepage area) are horizontal. At this point, the water table becomes confounded with the land surface topography, so that the hydraulic gradient can be assumed equal to the topographic slope. The flow across the vertical section at $x=L_S$ is then given by

$$Q_{L_S}=\int_{-d(L_S)}^{z_T(L_S)} K(L_S,z)\frac{\partial z_T}{\partial x}(L_S)dz \qquad (1)$$

where $z_T(x)$ [L] and $d(x)$ [L] are the land surface topography and the depth to impervious base at position x, respectively, and $K(x,z)$ is the hydraulic conductivity at position (x,z) (origin of both axes being taken at the valley bottom as in Figure 1b). Uphill, the recharge flow is given by

$$Q_R=R(L-L_S) \qquad (2)$$

For a global flow conservation, the seepage length must adapt such that Q_{L_S} becomes equal to Q_R, thus leading to an equation for L_S. For the case of homogeneous hillslopes with horizontal impervious base and constant-slope topography, the solution for L_S/L is

$$\frac{L_S}{L}=\frac{1-\frac{sKd}{R\,L}}{1+\frac{s^2K}{R}} \qquad (3)$$

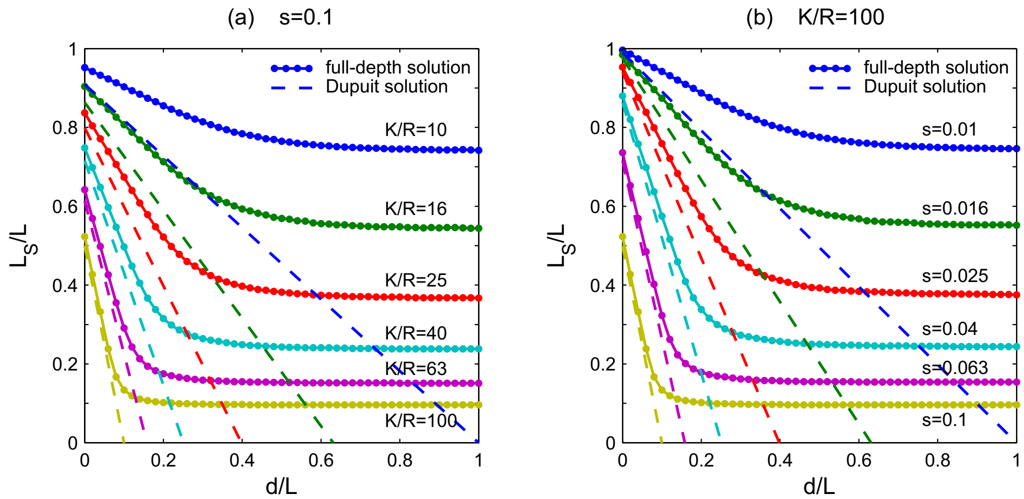

Figure 2. L_S/L as a function of d/L: full-depth and Dupuit solutions. (a) Results for $s=0.1$ and different values of K/R. (b) Results for $K/R=100$ and different values of s.

2.3. Full-Depth Solution

Let us denote Ω the saturated flow region, with boundaries consisting of prescribed zero-flux boundaries Γ_N, a free surface Γ_F (the water table below the land surface), and a seepage area Γ_S. The full governing equations of this problem are [*Neuman and Witherspoon*, 1970]

$$\nabla \cdot (K\vec{\nabla}h)=0 \ \ in \ \ \Omega \tag{4}$$

$$K\vec{\nabla}h \cdot \vec{n}=0 \ \ on \ \ \Gamma_N \tag{5}$$

$$K\vec{\nabla}h \cdot \vec{n}=Rn_z \ \ on \ \ \Gamma_F \tag{6}$$

$$h=z \ \ on \ \ \Gamma_F \tag{7}$$

$$h=z \ \ on \ \ \Gamma_S \tag{8}$$

where h [L] is the hydraulic head and $\vec{n}$ [-] is the unit vector normal to the surface and oriented outward (n_z being its vertical component). The position of Γ_F and the extension of Γ_S are a priori unknown: these are part of the solution, both adapting to the topographic and hydrogeologic parameters. Equations (6–8) form a free surface condition constrained by a land surface. This boundary condition constitutes the source of the nonlinearity of the system. First, the "pure" free surface condition (combination of equations (6) and (7)) can be shown to give a nonlinear equation in terms of hydraulic head at the free surface [*Bear*, 1972]. Second, the land surface constraint (expressed in equation (8)) sets an upper threshold for the free surface.

A large number of numerical methods and software can solve the above set of equations. Here we use a locally adaptive finite volume method with a nonlinear solver for determining the free surface position [*Bresciani et al.*, 2012], implemented in the H2OLAB platform [*Erhel et al.*, 2009]. This method is chosen for its accuracy and efficiency, which allows a large range of configurations to be explored in a reasonable amount of time. The mesh consists of a grid of rectangular cells. An adaptive meshing strategy is required to deal with the geometric parameters (d/L and s) and the result of interest (L_S), which vary over many orders of magnitude require. The adopted strategy consists of refining the grid both horizontally and vertically toward the valley bottom ($x=0$, $z=0$), and of adapting the grid steps as well as the convergence criterion of the free surface iterations as a function of the parameters. The adaptation is made such that the relative accuracy of the seepage length is similar for all the tested parameter values, while the total number of cells remains independent of all parameters but d/L. In the vertical direction, the part above $z=0$ is discretized with 100 cells. Below $z=0$, the vertical grid step increases with depth following a geometric progression with common ratio 1.1. In the horizontal direction, the number of cells is kept constant and is equal to

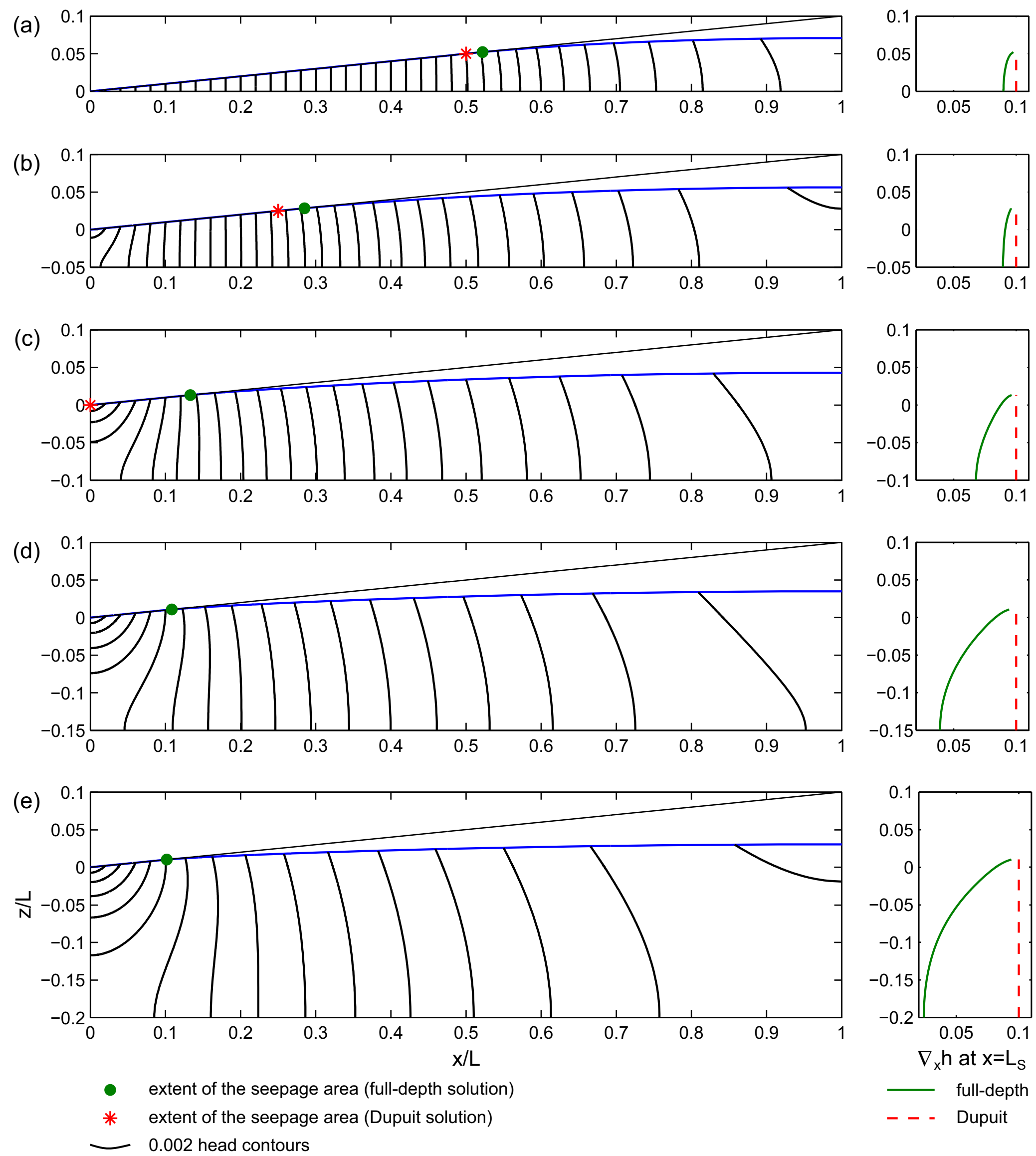

Figure 3. (left) Head contours and (right) horizontal hydraulic gradient profile at $x = L_S$, for (a) $d/L = 0$, (b) $d/L = 0.05$, (c) $d/L = 0.1$, (d) $d/L = 0.15$, and (e) $d/L = 0.2$. Other parameters are kept constant: $K/R = 100$ and $s = 0.1$. In Figures 3d and 3e, the Dupuit solution gives a negative seepage length, which is not physically acceptable (not reported).

1000, allowing the achievement of highly accurate results. Representative cases were checked to ensure that further mesh refinement or stricter convergence criteria had no effect on the results.

3. Results

The Dupuit solution predicts a linear decrease of L_S/L with d/L (equation (3)). A similar behavior can be observed in the full-depth solution for relatively small values of d/L, as illustrated in Figures 2a and 2b. In this part of the curve, although the Dupuit solution systematically underestimates the seepage length, the two solutions show reasonable agreement. As d/L becomes larger, the Dupuit solution goes to zero. More precisely, $L_S/L = 0$ for $d/L = R/(sK)$; beyond that, the solution is not physically acceptable ($L_S/L < 0$, equation (3)). On the other hand, the full-depth solution rapidly becomes independent of d/L, and tends toward a nonzero asymptotic value.

The results are further illustrated in Figures 3a–3e, which shows head contours in the full-depth solution for five increasing values of d/L in the case $K/R = 100$ and $s = 0.1$. The extent of the seepage area is also

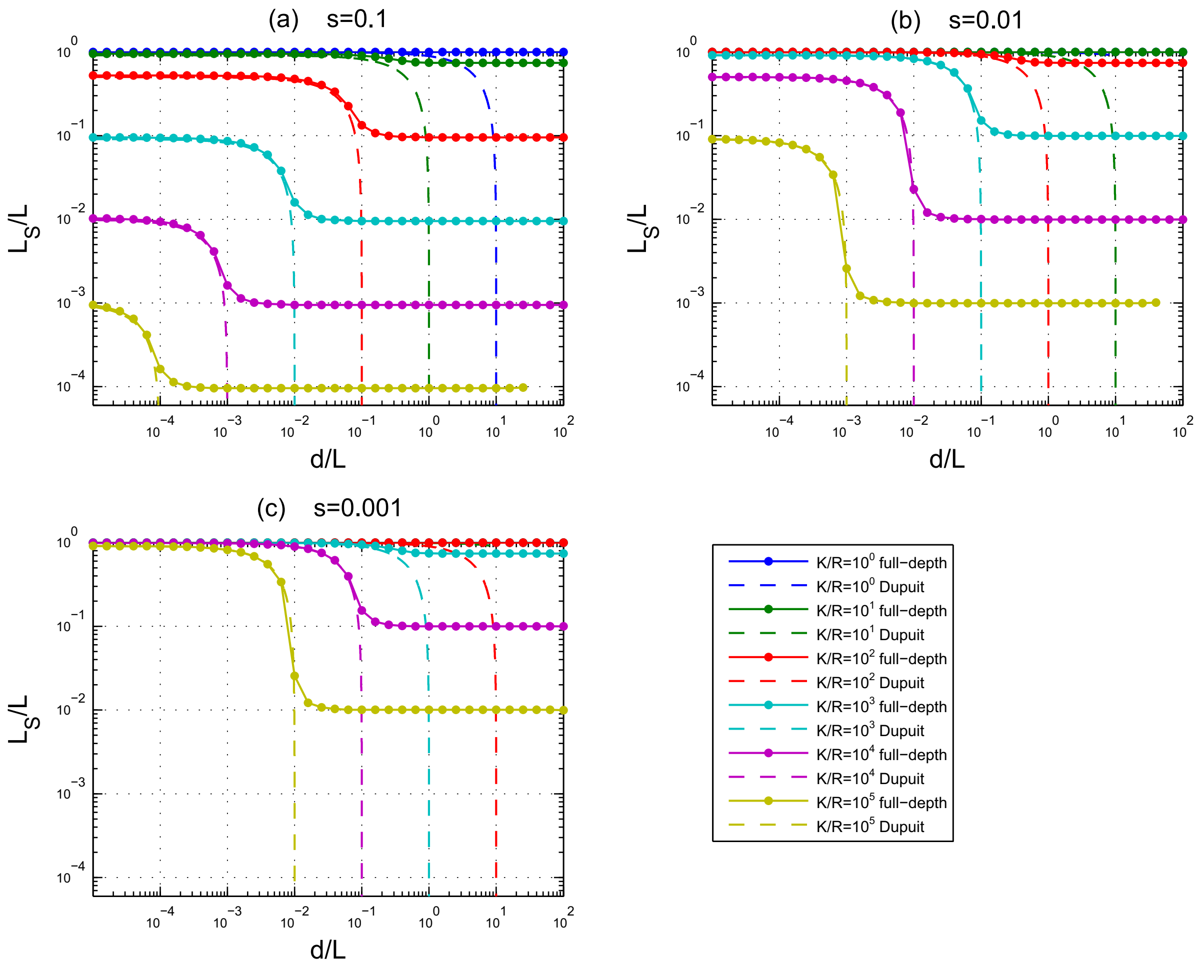

Figure 4. L_S/L as a function of d/L (log-log plots): full-depth and Dupuit solutions. Results for different values of K/R and (a) $s = 0.1$, (b) $s = 0.01$, and (c) $s = 0.001$. In Figures 4b and 4c, several small values of K/R constantly give $L_S/L \approx 1$ and overlap.

indicated by a red star and a green dot for the Dupuit and full-depth solutions, respectively, as well as the vertical profile of the horizontal hydraulic gradient at $x = L_S$. For relatively small values of d/L (Figures 3a and 3b), the full-depth solution reveals quite vertical head contours around $x = L_S$ and a horizontal hydraulic gradient profile almost constant with depth. In this case, the Dupuit solution logically gives a reasonable result. Figure 3c shows the particular case where $d/L = R/(sK)$, for which the Dupuit solution gives $L_S/L = 0$. In this case, the horizontal hydraulic gradient profile decreases significantly with depth, explaining the important difference between the two solutions. For larger d/L, high curvatures of the head contours can be observed around $x = L_S$ (Figures 3d and 3e), in which case it is not surprising that the Dupuit solution fails.

The same behavior is observed when varying K/R (Figure 2a) or s (Figure 2b). K/R and s nevertheless modulate the results significantly, both in a similar way. For larger K/R or larger s, the following observation can be made. In the first part of the curve (at relatively small values of d/L), L_S/L is smaller and decreases more rapidly with d/L. At higher values of d/L, the characteristic value of d/L beyond which the full-depth solution departs from the Dupuit solution is smaller. Thus, the validity of the Dupuit solution is not only controlled by d/L, but also by K/R and s. Eventually, the asymptotic value reached in the full-depth solution is smaller.

Results for the entire range of parameter values are given in Figures 4a–4c, where the log-log scale was chosen to display the full range of orders of magnitude investigated. The behaviors described above apply for all the tested parameter values. Moreover, the characteristic value of d/L beyond which the full-depth solution departs from the Dupuit solution seems to be of the same order as the value of d/L that nullifies the Dupuit solution, which is known to be $R/(sK)$. This result is important, since it gives a first-order scaling

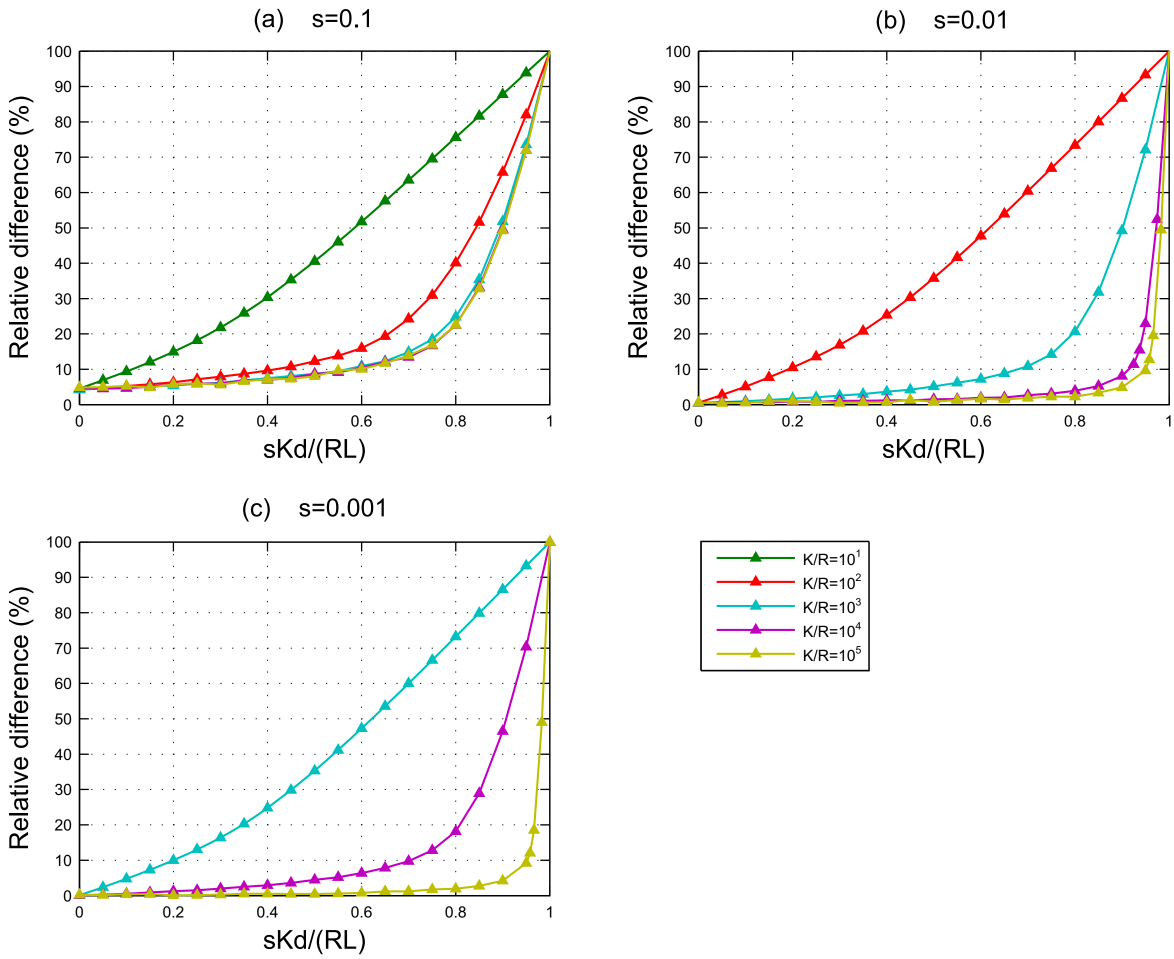

Figure 5. Relative difference between the full-depth and the Dupuit solutions, defined by $(\{L_S\}_{full-depth}-\{L_S\}_{Dupuit})/\{L_S\}_{full-depth}\times 100$, as a function of the ratio *sKd*/(*RL*). Results for different values of *K*/*R* and (a) $s=0.1$, (b) $s=0.01$, and (c) $s=0.001$.

criterion for the validity of the Dupuit solution; the Dupuit solution is valid only if $d/L << R/(sK)$. In order to refine this criterion, the relative difference between the full-depth solution and the Dupuit solution was plotted as a function of the ratio $sKd/(RL)$ (Figures 5a–5c). (Note: in Figure 5, as in the following Figure 6, values of *K*/*R* that always give $L_S/L \approx 1$ (see Figure 4) are not reported since the limit of accuracy related to the horizontal grid step was reached.) As expected, the difference is small for small values of $sKd/(RL)$. However, the figure reveals that the difference is not uniquely determined by $sKd/(RL)$ (for this to be true, all the curves should collapse to a single curve). For $sKd/(RL) < 0.2$, the difference remains smaller than 15% for the range of parameter values tested; therefore, $sKd/(RL) < 0.2$ could be suggested as a criterion of validity of the Dupuit solution. However, this criterion is not optimal and cannot be applied outside of the range of parameters investigated.

4. Discussion

4.1. Geometric Rule

The Dupuit solution assumes a constant head with depth, which makes possible a depth-integrated equation. Intuitively, this collapse of the vertical dimension should be valid for systems that have much larger horizontal extent than vertical extent. For systems with a constant-head profile at the downhill boundary, a Dupuit solution indeed performs better for a small ratio of depth to hillslope length [*Potter and Gburek*, 1986]. For the system studied here, which features a no-flow downhill boundary, this condition is necessary but not sufficient; Figures 4a–4c show that the Dupuit solution fails even for small absolute values of *d*/*L*.

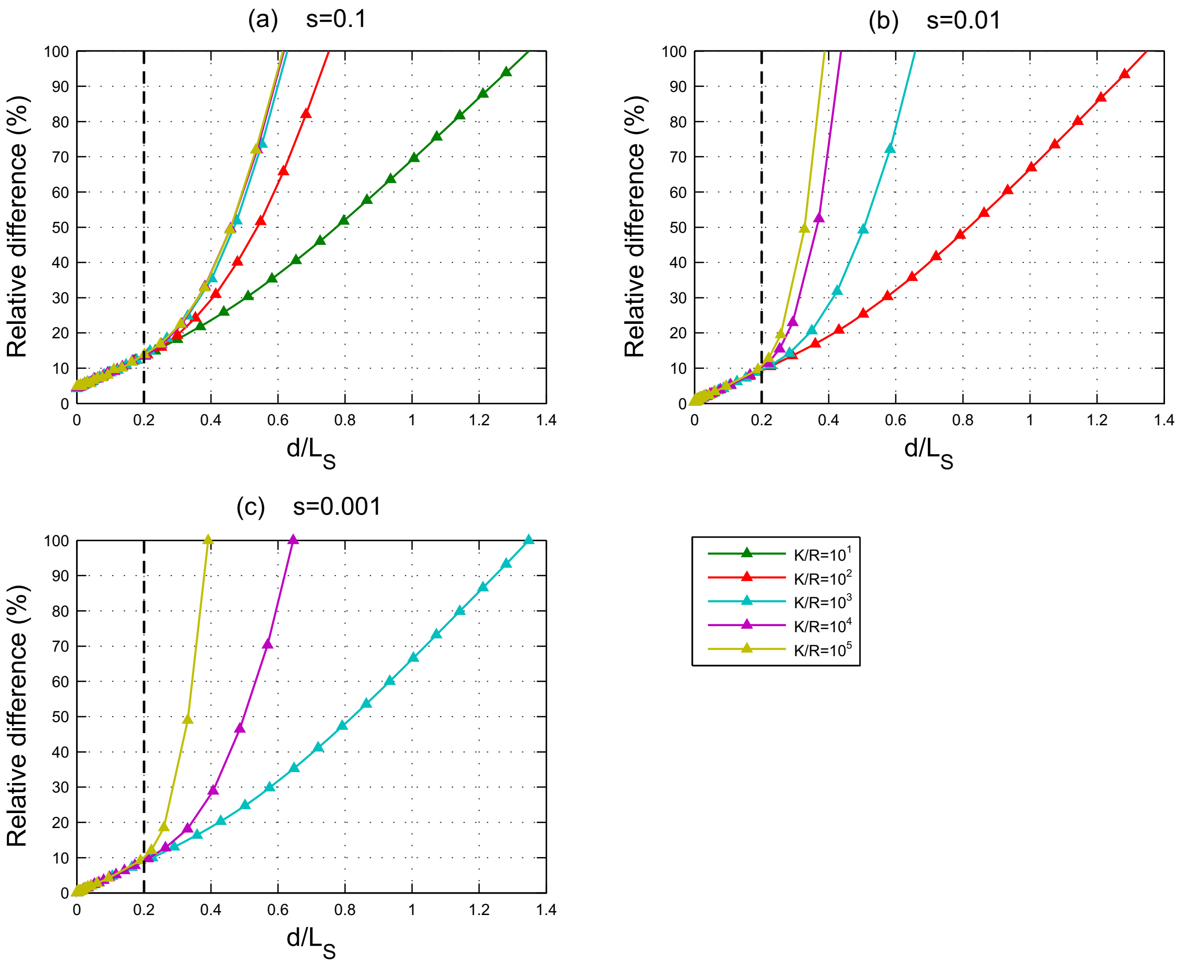

Figure 6. Relative difference between the full-depth and the Dupuit solutions, defined by $(\{L_S\}_{full-depth}-\{L_S\}_{Dupuit})/\{L_S\}_{full-depth}\times 100$, as a function of the ratio d/L_S. Results for different values of K/R and (a) $s=0.1$, (b) $s=0.01$, and (c) $s=0.001$. The vertical dashed line highlights a cutoff value of the ratio d/L_S beyond which the relative difference increases dramatically.

Nevertheless, another geometric rule can be considered: in general, the Dupuit assumption cannot be applied to regions where the vertical flow component is significant relative to the horizontal flow component. As a rule of thumb, some authors suggest that the Dupuit assumption is invalid at a distance $x<2b$ from a vertical flow feature, where b [L] is the saturated thickness [*Bear*, 1972; *Haitjema*, 2006]. The Dupuit solution studied here relies on the validity of the Dupuit assumption at $x=L_S$. Although the entire seepage area potentially constitutes a vertical flow feature (since the seepage boundary condition allows discharge to take place), vertical flow appears only significant close to the downhill boundary (Figure 3b). Thus, the hypothesis can be made that the validity of the Dupuit assumption is determined by the distance between the downhill boundary and $x=L_S$ relative to the saturated thickness. The saturated thickness is best measured below the stream for this purpose, since it is the no-flow downhill boundary that is responsible for the vertical flow feature (note that for $d=0$, there is no vertical flow close to the downhill boundary (Figure 3a)). To test this hypothesis, the relative difference between the full-depth solution and the Dupuit solution was plotted as a function of the ratio d/L_S (Figures 6a–6c). The figure reveals that for $d/L_S<0.2$, the relative difference is small, increases linearly with d/L_S, and most importantly, is independent of K/R (all the curves collapse to a single curve for each value of s) and almost independent of s (the values are identical for $s=0.001$ and $s=0.01$, but slightly larger for $s=0.1$). For $d/L_S>0.2$, the relative difference becomes a function of K/R and s and increases dramatically. It seems natural to take this cutoff value ($d/L_S=0.2$) as a limit of the validity of the Dupuit solution. At the cutoff, the relative difference is equal to 10% for $s=0.001$ and $s=0.01$, and to 14% for $s=0.1$. A slightly larger difference for larger topographical slopes is likely due to a larger ratio of average saturated thickness to hillslope length.

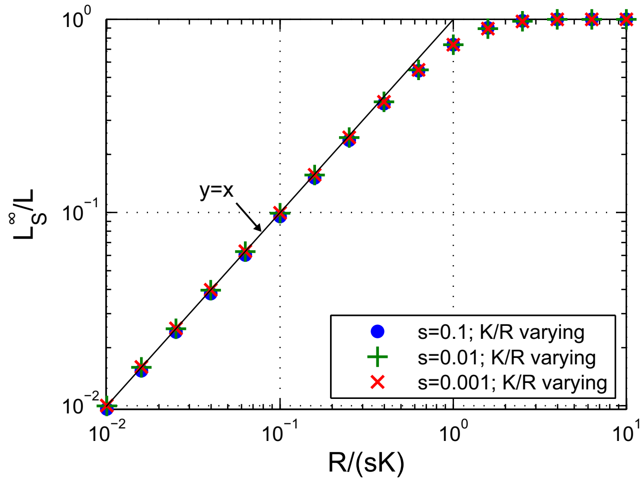

Figure 7. Asymptotic value L_S^∞/L, obtained for $d/L \to \infty$, as a function of $R/(sK)$ for values of K/R and s varying independently (graph restricted to the part where all the studied parameter values overlap). It is seen that all the points collapse to a single curve, proving that L_S^∞/L is controlled by the single, combined parameter $R/(sK)$. For $R/(sK) < 0.4$, the points collapse to the curve $y = x$.

The aforementioned rule of thumb [*Bear*, 1972; *Haitjema*, 2006] suggests that the Dupuit solution should be valid for $d/L_S < 0.5$. Here, a more restrictive condition is found ($d/L_S < 0.2$). Although these two conditions are of the same order, the difference is significant since adherence only to the less restrictive condition would most often yield a dramatically wrong result (Figures 6a–6c).

4.2. Topographic and Hydrogeologic Controls on "Deep" Systems

This section discusses the topographic and hydrogeologic controls on relatively "deep" systems ($d/L_S >> 0.2$), for which L_S/L has reached an asymptotic value (as given by the full-depth solution). First, the fact that the asymptotic value is nonzero reveals the limited influence of the aquifer depth on the capacity of hillslopes to transfer recharge (Q_R would reach its maximum potential value only if $L_S = 0$). Because the capacity to transfer recharge is classically related to transmissivity, defined as the saturated thickness multiplied by the hydraulic conductivity, this result might appear counterintuitive. In fact, the concept of transmissivity implicitly assumes the validity of the Dupuit assumption, which, as shown previously, can be invalid.

The asymptotic value of L_S/L, denoted by L_S^∞/L in the following, is smaller for larger values of K/R or s (Figures 4a–4c). In fact, changes in either K/R or s seem to have identical effects on L_S^∞/L. This suggests that the single, combined parameter sK/R controls L_S^∞/L. Plotting L_S^∞/L as a function of its inverse, $R/(sK)$, for independently varying values of K/R and s, confirms this hypothesis (Figure 7). In addition, the relationship

$$\frac{L_S^\infty}{L} = \frac{R}{sK} \tag{9}$$

holds for $R/(sK) < 0.4$. The simplicity of this expression is quite remarkable. Given the nonlinearity of the equations, and that in "deep" systems the flow field around $x = L_S$ is fundamentally two-dimensional (horizontal and vertical; Figure 3e), an analytical demonstration is not necessarily obvious.

4.3. Generalization to Nonhorizontal Impervious Base, Heterogeneous Hydraulic Conductivity, and Variable-Slope Topography

In this paper, the analysis was carried out for the basic case of homogeneous hillslope with horizontal impervious base and constant-slope topography. However, realistic cases can feature nonhorizontal impervious base, heterogeneity, and variable-slope topography. These features can also be accounted for in a Dupuit solution. In the following, generalization of the above findings to such cases is discussed. A number of examples are examined (cases A–D), for which the detailed features and the Dupuit solution are indicated in Table 1 and Table 2, respectively.

4.3.1. Nonhorizontal Impervious Base

As explained above, the validity of the Dupuit assumption depends on the saturated thickness below the stream. Thus, the criterion of validity must be independent of the topography of the impervious base (as long as it varies smoothly, such that it does not create any additional, significant vertical flow feature). As an example, let us consider the particular case of an impervious base parallel to the land surface (case A). In Figure 8a, the ratio of seepage length to hillslope length is plotted for varying ratio of depth to hillslope

Table 1. Detailed Features of the Hillslope Cases Tested in the Generalization Section (Section 4.3)

	Land Surface Topography		Geology	
	Features	Dimensionless Parameters	Features	Dimensionless Parameters
Case A	Constant slope $z_T(x)=sx$	$s=0.01$	Parallel base Homogeneous	$\frac{K}{R}=1000$
Case B	Constant slope $z_T(x)=sx$	$s=0.01$	Infinite base Exponential decay: $K(\tilde{z})=K_0 exp(-\tilde{z}/d_C)$	$\frac{K_0}{R}=1000$
Case C	Concave $z_T(x)=ax+\frac{1}{2}bx^2$	$a=0.005$ $bL=0.01$	Horizontal base Homogeneous	$\frac{K}{R}=1000$
Case D	Convex $z_T(x)=ax+\frac{1}{2}bx^2$	$a=0.015$ $bL=-0.01$	Horizontal base Homogeneous	$\frac{K}{R}=1000$

length. In this example, the geometric rule developed earlier (validity of the Dupuit solution for $d/L_S < 0.2$) applies very well.

In relatively "deep" systems ($d/L_S >> 0.2$), there is virtually no impervious base. In this case, the asymptotic value L_S^∞/L is controlled by $R/(sK)$, as in the basic case. In the example provided (case A), equation (9) gives a very good estimation of L_S^∞/L (Figure 8a).

4.3.2. Heterogeneous Hydraulic Conductivity

In heterogeneous cases, the challenge for generalizing the criterion of validity of the Dupuit solution is to define an effective depth to impervious base. In the basic (homogeneous) case, heterogeneity is in fact implicit; the concept of a depth to impervious base assumes a sharp interface between a high and a low hydraulic conductivity value (sufficiently low to be neglected). Thus, in heterogeneous cases, the effective depth to impervious base could simply be defined as the depth below which the hydraulic conductivity becomes negligibly low. However, except in ideal cases where the hydraulic conductivity decreases monotonously with depth, heterogeneity is generally randomly distributed. The effective depth should then be the sum of all zones displaying non-negligible hydraulic conductivity. It also seems obvious that different weights must be given to zones of different values of hydraulic conductivity, since zones of low values should contribute less to the effective depth than zones of high values. A natural way to define the effective depth is thus offered by the participation ratio S_2 [-] $\in [0;1]$ [*Sornette et al.*, 1993; *Le Goc et al.*, 2010]. This ratio characterizes the portion of space occupied by the highest values of a field. It compares moments of different orders, and is mathematically defined as

$$S_2=\frac{M_1(\Phi)^2}{M_0(\Phi)M_2(\Phi)} \tag{10}$$

where $M_k(\Phi)$ is the kth moment of the field Φ. In order to obtain an effective depth, S_2 is calculated for vertical profiles of the hydraulic conductivity field. The moments are thus given by

$$M_k(K)=\int_{-d}^{z_T}[K(z)]^k dz \tag{11}$$

The portion of depth that effectively contributes to horizontal flow, noted $d^*[L]$, can then be defined as

$$d^*=S_2 d \tag{12}$$

Table 2. Dupuit Solution for the Hillslope Cases Tested in the Generalization Section (Section 4.3, Details in Table 1)

	Dupuit Solution
Case A	$\frac{L_S}{L}=1-\frac{sK}{R}\frac{d}{L}$
Case B	$\frac{L_S}{L}=1-\frac{sK}{R}\frac{d_C}{L}$
Cases C and D	$a\frac{d}{L}-\frac{R}{K}+\left(\frac{d}{L}bL+a^2+\frac{R}{K}\right)\frac{L_S}{L}+\frac{3}{2}bLa\left(\frac{L_S}{L}\right)^2+\frac{1}{2}(bL)^2\left(\frac{L_S}{L}\right)^3=0$ [a]

[a]The analytical solution of this equation was taken from http://www.wolframalpha.com.

The criterion of validity of the Dupuit solution is then suggested to be $d^*/L_s < 0.2$. For a homogeneous case, it can be verified that $S_2=1$ and $d^*=d$.

To illustrate the use of the participation ratio, let us consider the case of hillslopes characterized by an exponential decay of the hydraulic

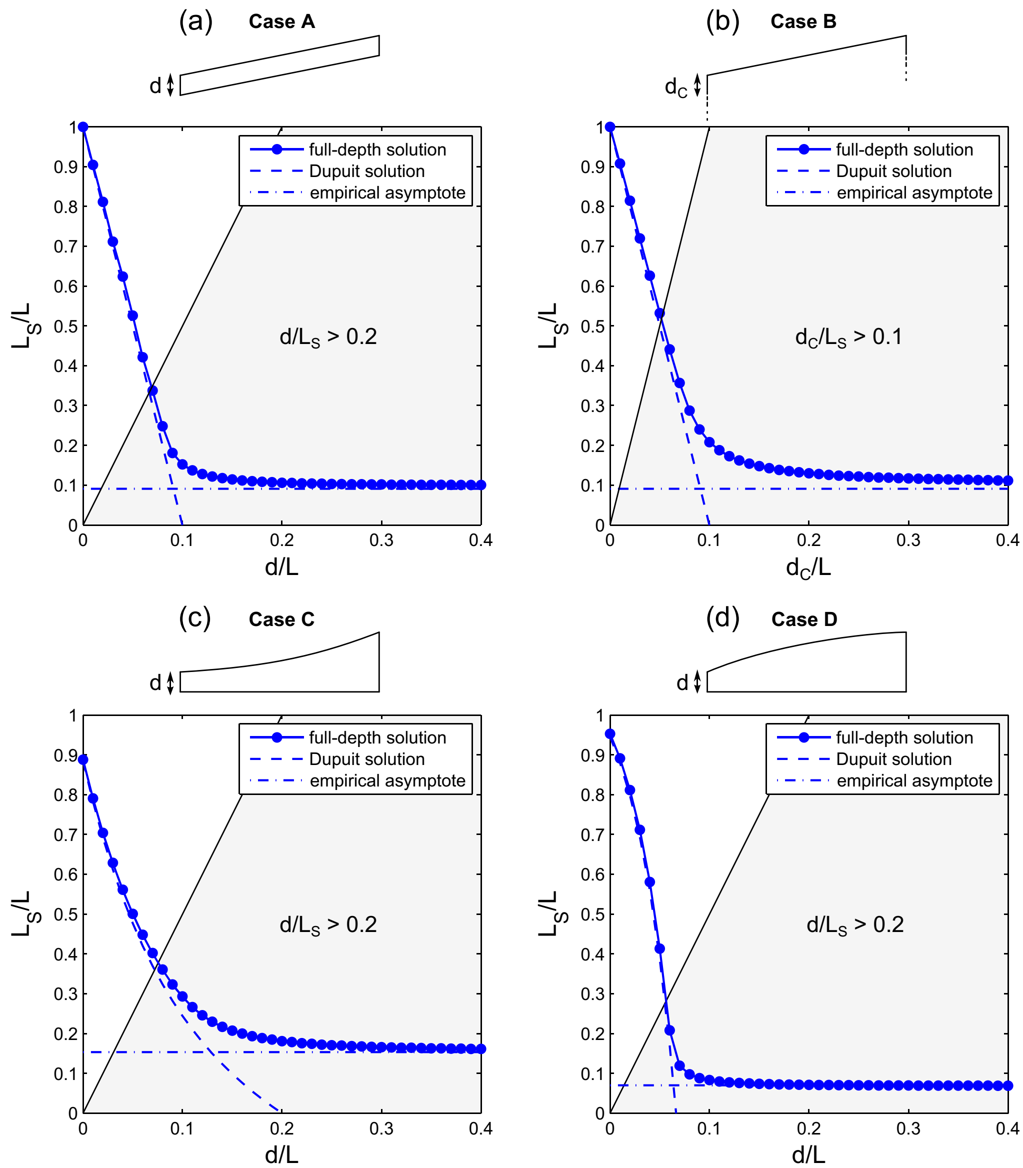

Figure 8. (a–d) L_S/L as a function of d/L (or d_C/L in case B) in cases A–D, which are sketched above the graphs (details given in Table 1). Results reported for the full-depth solution, the Dupuit solution (expected to be invalid in the shaded region defined by $d/L_S > 0.2$), and the empirical asymptote (equation (9)).

conductivity with depth, a commonly adopted hillslope model [*Beven and Kirkby*, 1979]. The hydraulic conductivity is then given by $K(\tilde{z})=K_0 exp(-\tilde{z}/d_C)$, where K_0 is the hydraulic conductivity at the land surface, $\tilde{z}$ is the depth measured from the land surface, and d_C is a characteristics depth. In that case, it can be shown that $d^*=2d_C$. The criterion of validity of the Dupuit solution is then $d_C/L_S < 0.1$. In the example examined (case B), this proves to be a very sensible criterion (Figure 8b).

In order to generalize the results of the relatively "deep" systems to heterogeneous cases, an effective hydraulic conductivity has to be computed. An extensive literature exists on this topic, which is too complex to be treated here. We refer to *Sanchez-Vila et al.* [2006] for an overview of the different approaches to this issue. In the particular case where the hydraulic conductivity decays exponentially with depth, taking $d^* \to \infty$ implies that the hydraulic conductivity field becomes essentially equivalent to a homogeneous field of value K_0. In this case, L_S^∞/L is simply controlled by $R/(sK_0)$, as in the basic case. Figure 8b shows that in case B, L_S/L tends toward $R/(sK_0)$, as suggested by equation (9).

4.3.3. Variable-Slope Topography

The criterion of validity of the Dupuit solution is not expected to be a function of the land surface topography. Examples of concave and convex topographies (cases C and D, respectively) confirm that the developed criterion applies well (Figures 8c and 8d). These examples moreover demonstrate that in the range of validity, the Dupuit solution also performs well for variable-slope topographies.

In order to generalize the results of the relatively "deep" systems to cases with variable-slope topography, the concept of topographic slope has to be generalized. Since in relatively "deep" systems the flow field is fundamentally two-dimensional (horizontal-vertical), it is expected that the topography all along the seepage area influences the result. For this reason, generalization of the parameter s in equation (9) by the average slope along the seepage area is suggested. The generalized form of equation (9) is then

$$\frac{L_S^\infty}{L} = \frac{R}{\left\langle \frac{\partial z_T}{\partial x} \right\rangle_{L_S^\infty} K} \tag{13}$$

In cases C and D, this expression gives a very good estimation of L_S^∞/L (Figures 8c and 8d).

5. Conclusion

In this study, a Dupuit solution for seepage area prediction in hillslope cross sections was tested against a full-depth solution of saturated groundwater flow. Compared to previous works that tackled a similar problem, the originality of the study primarily lies in the use of a no-flow downhill boundary, which represents the groundwater divide below the stream. In homogeneous hillslopes with horizontal impervious base and constant-slope topography, the comparison reveals that the validity of the Dupuit solution depends not only on the ratio of depth to hillslope length d/L, but also on other parameters of the system such as the ratio of hydraulic conductivity to recharge K/R and the topographic slope s. Although $sKd/(RL) \ll 1$ gives a first-order criterion for the validity of the Dupuit solution, the relative difference between the full-depth solution and the Dupuit solution is not uniquely determined by the value of $sKd/(RL)$. Instead, the relative difference is shown to be a unique function of another ratio, the ratio of depth to seepage length d/L_S (except beyond a cutoff value of 0.2, in which case K/R and s also become important). A ratio of $d/L_S < 0.2$ ensures a relative difference smaller than 14% for the wide range of parameter values tested, and is suggested as a criterion of validity for the Dupuit solution. For ratios of $d/L_S > 0.2$, the relative difference increases dramatically; the vertical flow component at $x = L_S$ cannot be neglected. In practice, since L_S is required to assess this criterion, a Dupuit solution would have to be tested a posteriori. Note that this is correct since the Dupuit solution is valid up to the cutoff value, and beyond it gives an overestimation of d/L_S.

When d/L_S increases beyond the cutoff value, the ratio of seepage length to hillslope length L_S/L given by the full-depth solution tends toward a nonzero asymptotic value. Remarkably, this asymptotic value is shown to be controlled by $R/(sK)$. Moreover, the equality $L_S^\infty/L = R/(sK)$ holds for $R/(sK) < 0.4$. In other words, the equality holds for cases where $L_S^\infty/L < 0.4$, which probably encompasses most natural situations.

The findings of this paper are expected to form a solid base for the analysis of more complex systems. Cases featuring heterogeneity, nonhorizontal impervious base, and variable-slope topography have been discussed above in some extent, but more examples should be examined. Future research should also explore the effects of hillslope convergence/divergence, catchment asymmetry, deep groundwater leakage, spatially distributed recharge, and transient behaviors.

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