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[1] Time compression approximation (TCA) is a practical and often quite accurate tool to predict postponding infiltration for field applications. A modified approximation (MTCA) can be used just as easily and, in general, will reduce the error by about 50%. This is based on two results: (1) After ponding, TCA and MTCA predict very close infiltration rates; and (2) MTCA, but not TCA, uses the actual cumulative infiltration up to the ponding time. Thus, TCA has an additional error in its prediction of postponding infiltration. Previously, those results, including the 50% reduction in error, were observed numerically for linear and Burger's soils. They are illustrated here numerically with an actual soil (a Grenoble sand). More importantly, we developed a general analytical approximation for this problem and showed that it can provide a very convenient predictive tool which can then be used for arbitrary soil properties.

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1. Introduction

[2] Time compression (sometimes "condensation") approximation (sometimes "analysis") or TCA postulates that infiltration after ponding depends only on the total cumulative infiltration at ponding not on the details of the rainfall rate [*Brutsaert*, 2005]. Thus, when TCA applies, one can replace the true rainfall rate before ponding by its average value. It follows that if one knows the cumulative infiltration I as a function of flux q for saturated surface water content, then for the average rainfall rate q_p this relation will give the cumulative infiltration at ponding and thus provide an estimate of ponding time t_p. After this ponding time estimate, the saturated solution is continued.

[3] However, if the average value of rainfall rate is known until ponding, then the ponding time must be known fairly accurately as well as the cumulative rainfall amount, which is also the cumulative infiltration, at ponding time. Thus, MTCA assumes knowledge of ponding time t_p and cumulative infiltration at that time I_p and does not assume that the average flux before ponding is the flux at ponding.

[4] To further extend our present understanding of TCA and MTCA see *Liu et al.* [1998], *Parlange et al.* [2000], *Basha* [2002], *Brutsaert* [2005], and *Barry et al.* [2007]. We will analyze numerically and analytically infiltration for constant flux and for constant surface water content for nonlinear soils, revisiting earlier papers [*Parlange et al.*, 1985; *Hogarth et al.*, 1991; *Parlange et al.*, 1997; *Parlange et al.*, 1999] which compared numerical results with analytical results. The analytical approach was refined by *Barry et al.* [2007] and is used here to reanalyze the numerical results of *Parlange et al.* [1985] and *Hogarth et al.* [1991] obtained for a Grenoble sand. The sand's hydraulic properties are fully reported in those two papers [*Parlange et al.* 1985; *Hogarth et al.*, 1991] and will be used here to illustrate our results. The earlier numerical solutions have been reproduced using COMSOL numerical software. The converged COMSOL finite element solutions agreed with the original solutions presented by *Hogarth et al.* [1991].

2. Analysis

[5] The method is based on a double integration of Richards' equation [*Parlange and Haverkamp*, 1989], yielding

$$z(\theta, t) = \int_{\theta}^{\theta_s} \frac{D(\bar{\theta})d\bar{\theta}}{\partial \int_0^{\bar{\theta}} z d\tilde{\theta}/\partial t - k(\bar{\theta})}. \tag{1}$$

In equation (1) D and k are the soil water diffusivity and hydraulic conductivity, respectively, and z is the distance from the surface (positive downward), t is the time, θ is the water content at z, and θ_s is θ for $z = 0$ (the surface). The expression $\partial \int_0^{\bar{\theta}} z d\tilde{\theta}/\partial t$ is the flux, which does not vary

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much, unlike k. *Parlange* [1972] suggested that a first approximation to solving equation (1) for z is to replace the flux term by $q\theta/\theta_s$, where q is the surface flux. That substitution has the desirable property that it gives the exact result, usually called the traveling wave solution, when q/θ_s is constant [*Fleming et al.*, 1984]. In the long time limit equation (1) reproduces the so-called "profile at infinity" for θ_s constant [*Philip*, 1969]. A straightforward iterative scheme replacing the resulting value of z from equation (1) in the integrand has not proved convenient. Another approach is to generalize the method of *Heaslet and Alksne* [1969] and to expand instead the first approximation in terms of z or [*Parlange et al.*, 1997; *Barry et al.*, 2007]

$$\int_{\theta}^{\theta_s} \frac{D d\bar{\theta}}{q\bar{\theta}/\theta_s - k(\bar{\theta})} = z + Mz^2 + \cdots. \quad (2)$$

In practice excellent accuracy is obtained keeping only the first two terms on the right side of equation (2). $M(t)$ satisfies [*Barry et al.*, 2007]

$$2M = \frac{q}{\theta_s D_s} - \frac{1}{q - k_s}\frac{d\theta_s}{dt}, \quad (3)$$

where D_s and k_s are, respectively, the values of D and k at θ_s. Note that near saturation, D_s is basically undefined and, from the short time limit, could be estimated by [*Parlange et al.*, 1999]

$$\frac{1}{\theta_s D_s} = \frac{\int_0^{\theta_s}(\theta_s - \theta)D d\theta}{\int_0^{\theta_s} D d\theta \int_0^{\theta_s} \theta D d\theta}. \quad (4)$$

If the relationship between θ_s and q is known, then equation (3) yields M. Integrating equation (2) provides the additional equation

$$\int_0^{\theta_s} \frac{D\theta d\theta}{q\frac{\theta}{\theta_s} - k(\theta)} = I + M\int_0^{\theta_s} z^2 d\theta, \quad (5)$$

where $I(t)$ is the cumulative infiltration

$$I = \int_0^{\theta_s} z d\theta. \quad (6)$$

As it is only a small correction, the last term in equation (5), $\int z^2 d\theta$, can be evaluated roughly, assuming a Green and Ampt-type flow, or

$$\int_0^{\theta_s} z^2 d\theta \approx I^2/\theta_s, \quad (7)$$

in which case equation (5) becomes

$$\int_0^{\theta_s} \frac{D\theta d\theta}{q\frac{\theta}{\theta_s} - k} = I + M\, I^2/\theta_s. \quad (8)$$

Up to now the analysis applies whether q or θ_s is imposed. However, equation (3) leads to very different results depending on whether q or θ_s is constant.

2.1. Constant Flux Analysis

[6] Differentiation of equation (8) yields

$$q + \frac{dMI^2/\theta_s}{dt} = \frac{\theta_s D_s}{q - k_s}\frac{d\theta_s}{dt} - \int_0^{\theta_s} \frac{D\theta^2 d\theta}{\left(q\frac{\theta}{\theta_s} - k\right)^2} q\frac{d1/\theta_s}{dt}, \quad (9)$$

and, combining with equation (3),

$$2M\theta_s D_s + \frac{dMI^2/\theta_s}{dt} = -\int_0^{\theta_s} \frac{D\theta^2 d\theta}{\left(q\frac{\theta}{\theta_s} - k\right)^2} q\frac{d1/\theta_s}{dt}, \quad (10)$$

we can estimate the order of magnitude of the second term as

$$\frac{MI^2 d1/\theta_s}{dt} = O\left[M\int_0^{\theta_s} \frac{D\theta^2 d\theta}{\left(q\frac{\theta}{\theta_s} - k\right)^2}\int_0^{\theta_s} D d\theta\frac{d1/\theta_s}{dt}\right]. \quad (11)$$

Thus, if this second term were of the order of the third term in equation (10), we would have

$$M\int_0^{\theta_s} D d\theta = O[q]. \quad (12)$$

However, in that case, the first term in equation (10), $2M\theta_s D_s$, would be an order of magnitude greater than all the other terms in that equation and it could not be balanced by any other term. Hence, the second term in equation (10) can be neglected giving

$$2M\theta_s D_s = q\int_0^{\theta_s} \frac{D(\theta/\theta_s)^2 d\theta}{\left(q\frac{\theta}{\theta_s} - k\right)^2}\frac{d\theta_s}{dt}. \quad (13)$$

Given that M is an order of magnitude smaller than suggested by equation (12), then M can be obtained from equation (13), replacing $d\theta_s/dt$ by $q(q - k_s)/\theta_s D_s$ from equation (3), where M has been dropped, or

$$2M\theta_s^2 D_s^2 = (q - k_s)q^2\int_0^{\theta_s} \frac{D(\theta/\theta_s)^2 d\theta}{\left(q\frac{\theta}{\theta_s} - k\right)^2}. \quad (14)$$

As $q \to k_s$, the integral is singular since $q - k_s \to 0$. We remove the singularity by using a Gardner-type soil obeying [*Barry et al.*, 2007]

$$D \approx \theta_s\int_0^{\theta_s} D d\bar{\theta}\frac{dk/\theta}{d\theta}/k_s. \quad (15)$$

Although not exact, such a D introduces only a small error on the value of M giving

$$2M\theta_s D_s = q\left(\frac{\int_0^{\theta_s} Dd\theta}{\theta_s D_s}\right). \tag{16}$$

For a rapidly increasing D, the term in the parentheses is much less than unity, as can be estimated from equation (4). This also shows that in equation (3) the M term is much smaller than the other two terms, which basically balance each other. According to equation (16), M approaches a constant when $t \to \infty$. This, of course, means that the MI^2/θ_s correction in equation (8) becomes increasingly large if $t \to \infty$. For q sufficiently larger than k_s, ponding will occur for short times and the correction remains small. However, for q less than or close to k_{sat}, the contribution of dI^2/dt in equation (10) has to be considered, so that $2M\theta_s D_s$ in equation (10) is replaced by $2M(\theta_s D_s + 2Iq/\theta_s)$ and equation (14) is replaced by the more accurate

$$2M\theta_s D_s(\theta_s D_s + Iq/\theta_s) = q\int_0^{\theta_s} Dd\theta. \tag{17}$$

In *Barry et al.* [2007] this additional term was not kept as only $q > k_{sat}$ was considered and ponding occurred, so in that case this term is normally negligible.

2.2. Infiltration Analysis With Surface Saturation

[7] This case is especially important for using the TCA technique as it serves as a reference. Of course, for $\theta_s = \theta_{sat}$ the $d\theta_s/dt$ term drops out of equation (3) and M is given by

$$2M = q/\theta_{sat}D_{sat}. \tag{18}$$

[8] Under constant flux the $d\theta_{sat}/dt$ term and $q/\theta_{sat}D_{sat}$ largely balanced each other giving $M << q/\theta_{sat}D_{sat}$. This cannot happen here for $\theta_s = \theta_{sat}$, so the M term introduces an order of magnitude larger correction. With such an M, equation (8) holds and relates I and q.

[9] As noted by *Sivapalan and Milly* [1985], TCA, to be exact, would require the same $I(q)$ relation for an arbitrary dependence of the flux q on time. Obviously this is impossible. For instance, we have shown that for constant q the M term has essentially no effect on ponding; here, on the other hand, the $I(q)$ relationship is affected as M is much larger.

[10] As noted earlier, $1/\theta_{sat}D_{sat}$ is not a very meaningful parameter, which means that our condition is unreliable but the estimate of equation (4) holds in the short-time limit. If we use that estimate for all times in equation (8), there is an obvious difficulty for the long-time case as the last term, no matter how small M is, will eventually dominate and cease to be a small correction. An alternative is to apply equation (8) in the short-time limit only so that equation (4) leads to

$$MI^2/\theta_{sat} = \int_0^{\theta_{sat}} (\theta_{sat} - \theta)\, Dd\theta/2q. \tag{19}$$

Writing the correction in this form has the great advantage that if we apply it for long times (even though it was derived for short times), it remains finite in the long times when $q \to k_s$, and as a result, is negligible in that limit, when $I \to \infty$ in equation (8). Equation (8) then becomes

$$\int_0^{\theta_{sat}} \frac{D\theta d\theta}{\left(q\dfrac{\theta}{\theta_{sat}} - k\right)} = I + \int_0^{\theta_{sat}} (\theta_{sat} - \theta)\, Dd\theta/2q, \tag{20}$$

which, for a given q, gives I quite easily. Note that time not appear in equation (20), and I is only a function of q for given soil properties. Figure 1 gives various $I(q)$ for the Grenoble sand. First, for q constant, I corresponds to its value at ponding obtained numerically and from equation (2) dropping the M term altogether, the agreement is obviously excellent. The figure also gives $I(q)$ when θ at the surface is saturated for all times and from equation (20). Again, the agreement is quite good, up to higher order terms neglected in equation (20).

[11] In the figure the numerical results for the case q = 50 cm h^{-1} until ponding, followed by θ_{sat} at the surface is also given. Of course, as q decreases, with increasing time this $I(q)$ approaches the results when θ_{sat} at the surface holds for all times. The figure also indicates the relationships assumed by TCA, (BACF) and MTCA (BCF) (see also sketches of Figure 2). In that sketch, point F represents the long time limit when all the $q(t)$ merge as $q \to k_{sat}$. The

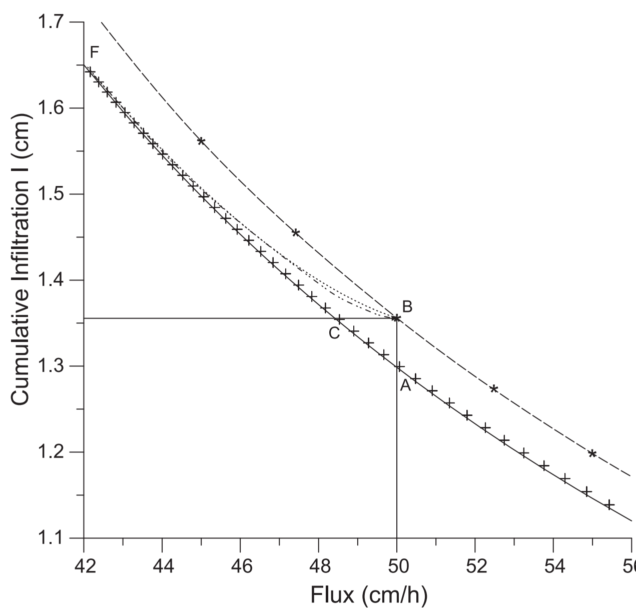

Figure 1. Cumulative infiltration is given as a function of flux, showing the relationship between the cases of q constant and $\theta = \theta_{sat}$. The dashed line is equation (2) with M neglected, the asterisks give the numerical values, the solid line is equation (20), the crosses give the numerical values, the dashed-dotted line is equation (21) with $\alpha = 18.105$, and the dotted line gives the numerical values for the transition from q constant to $\theta = \theta_{sat}$. BACF is the TCA and BCF is the MTCA.

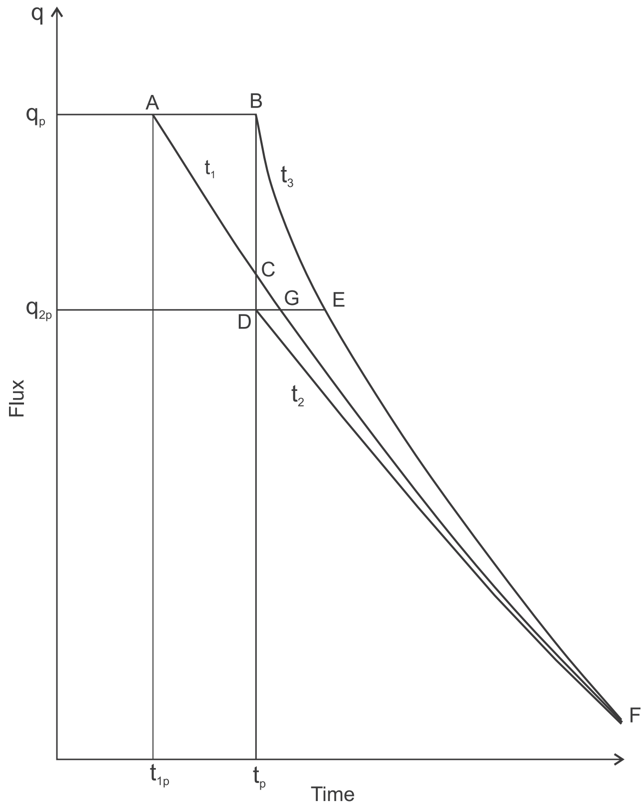

Figure 2. Sketch of fluxes versus time illustrating the relationship between TCA and MTCA.

other points (ABCDEG) are close together, as $q_{2p} - q_p$ must be small for TCA, and MTCA, to apply, and points (DCG) are even closer to each other as discussed below. TCA assumes that, at ponding, point A in Figures 1 and 2, q is continuous so that $I = I_{1p}$ given by equation (20) for $q = q_p$. Hence $I_{1p} = q_p t_{1p}$ is less than $I_p = q_p t_p$ with, from equation (20),

$$q_p(t_p - t_{1p}) = \int_0^{\theta_{sat}} (\theta_{sat} - \theta)\, D d\theta / 2q. \qquad (21)$$

MTCA rather assumes that $q = q_p$ until ponding time, point B, then q drops discontinuously to q_{2p}, to point C in Figure 1 and point D in Figure 2. Equation (20) yields $q = q_{2p}$ taking $I = I_p = q_p t_p$.

[12] The $I(q)$ curve when $q = 50$ cm h^{-1} at the surface until ponding followed by infiltration with the surface saturated obviously shows on the figure as an interpolation between the two cases of q constant and $\theta_s = \theta_{sat}$. An analytical interpolation is now guessed. The expression

$$\int_0^{\theta_{sat}} \frac{D\theta d\theta}{\left(q\frac{\theta}{\theta_{sat}} - k\right)} = I + \frac{1}{2q}\int_0^{\theta_{sat}} (\theta_{sat} - \theta) \cdot D d\theta \left[\left(\frac{q}{q_p}\right)^{\alpha} - 1\right] / \left[\left(\frac{k_{sat}}{q_p}\right)^{\alpha} - 1\right] \qquad (22)$$

is chosen because it goes to the right limits, i.e., $M = 0$ at $q = q_p$ and $M = \int_0^{\theta_{sat}} (\theta_{sat} - \theta)\, D d\theta / 2q$ when $q = k_{sat}$. In addition, we introduce a parameter α in equation (22) which allows us to satisfy another condition which is available in the transition. As $q_p \to \infty$ the transition is instantaneous so we impose the condition $dI/dq = 0$ in that limit, giving

$$\alpha \approx \int_0^{\theta_{sat}} D d\theta / \int_0^{\theta_{sat}} D(\theta_{sat} - \theta) D d\theta, \qquad (23)$$

where we neglected $(k_{sat}/q_p)^{\alpha}$ compared to 1, since we assume that q_p is not too close to k_s and equation (23) shows that $\alpha >> 1$.

[13] In our illustration, $\alpha \approx 18$. Figure 1 also gives the transition curve based on equations (22) and (23)—the agreement is obviously quite good.

[14] We are now going to give analytical expressions to estimate $q(t)$. Differentiation of equation (20) gives

$$dt = -\frac{dq}{q}\left[\int_0^{\theta_{sat}} \frac{D\theta^2 d\theta}{\theta_{sat}(q\theta/\theta_{sat} - k)^2} - \frac{1}{2q^2}\int_0^{\theta_{sat}} (\theta_{sat} - \theta)\, D d\theta\right] \qquad (24)$$

and by integration imposing the condition that $q \to \infty$ as $t \to 0$:

$$t = \int_0^{\theta_{sat}} \frac{D\theta^2}{k^2\theta_{sat}} \ln\left(\frac{q\theta/\theta_{sat} - k}{q\theta/\theta_{sat}}\right) d\theta + \int_0^{\theta_{sat}} \frac{D\theta^2 d\theta}{k\theta_{sat}(q\theta/\theta_{sat} - k)} - \frac{1}{4q^2}\int_0^{\theta_{sat}} (\theta_{sat} - \theta) D d\theta. \qquad (25)$$

In addition to giving a sketch of the fluxes as a function of time for TCA (curve ACF) and MTCA (curve DF), Figure 2 also shows the interpolation case (curve BEF). The curves $t_1(q)$ for TCA and $t_2(q)$ for MTCA are based on a translation of $t(q)$ in equation (25) or

$$t_1 - t_{1p} = t(q) - t(q_p) \qquad (26)$$

and

$$t_2 - t_p = t(q) - t(q_{2p}). \qquad (27)$$

Repeating the same procedure with equation (22) as with equation (20), differentiation and integration gives $t_3(q)$ for the interpolation curve (BEF) in Figure 2 or

$$t_3 = t_1 + \frac{1}{2}\frac{(q/q_p)^{\alpha}}{q^2}\int_0^{\theta_{sat}} D(\theta_{sat} - \theta) D d\theta \qquad (28)$$

for α large.

[15] We are now going to prove two general results observed previously for linear soils and for Burgers' soils. First, we are going to show that points C, G, and D are practically the same (as observed by *Basha* [2002, Figure 4] for a Burgers' soil) so that in the $q(t)$ plane the MTCA curve and the TCA curve are effectively the same for $t > t_p$.

[16] Second, we are going to show that the area ABC and the area BDF are of the same order of magnitude, which means that the error of TCA in predicting the cumulative infiltration is about twice the error of MTCA.

[17] First we want to show that $t_1 - t_2$ for $q = q_{2p}$ is much smaller than $t_{1p} - t_p$ so that DG << AB in Figure 2 and the three points CDG are essentially indistinguishable. Equations (26) and (27) give

$$t_1 - t_2 = t_{1p} - t_p + t(q_{2p}) - t(q_p), \tag{29}$$

but $q_p(t_{1p} - t_p) = I_{1p} - I_p$ and $I_{1p} - I_p = I(q_p) - I(q_{2p})$, with I given by equation (20). Since $dI = qdt$, thus $(I_{1p} - I_p)/q_p \approx [t(q_p) - t(q_{2p})]\bar{q}/q_p$ where $q_{2p} < \bar{q} < q_p$. Finally

$$t_1 - t_2 = [t(q_{2p}) - t(q_p)](1 - \bar{q}/q_p), \tag{30}$$

which is small compared to $t_{1p} - t_p$ since $(1 - \bar{q}/q_p)$ is small for TCA, and MTCA, to be applicable.

[18] Second, the area BAF is obtained as

$$\int_{k_{sat}}^{q_p} (t_3 - t_1)dq \approx q_p^3(t_p - t_{1p})^2 / \left(\theta_{sat} \int_0^{\theta_{sat}} D d\theta\right) \tag{31}$$

from equation (28), and the ABC area is given by $\frac{1}{2}(t_p - t_{1p})(q_p - q_{2p})$ or

$$\text{ABC area} \approx \frac{1}{2} q_p^3(t_p - t_{1p})^2 / \left(\theta_{sat} \int_0^{\theta_{sat}} D d\theta\right) \tag{32}$$

using equation (24), as long as k_s is not close to q. Thus the area of BAF is roughly twice the area of ABC, or BCF has about half the area of BAF. Since those areas correspond to the errors in cumulative infiltration of MTCA and TCA, the latter has roughly twice the error of MTCA as already observed for linear and Burgers' soils. The same improvement of 50% was also observed by *Parlange et al.* [2000] for a power law diffusivity in the absence of gravity to allow analytical treatment with the tools available at that time. *Parlange et al.* [2000] obtained some analytical results with gravity; however, it was not possible to extend them to predict infiltration after ponding. Here we predict analytically that the reduction of the error in the cumulative infiltration with MTCA should apply to any soil. Figure 3 gives the various $q(t)$ obtained numerically for our example and analytically from equations (26)–(28). Not surprisingly, the agreement is quite good, and we cannot distinguish points D, C, and G on Figure 3 as expected.

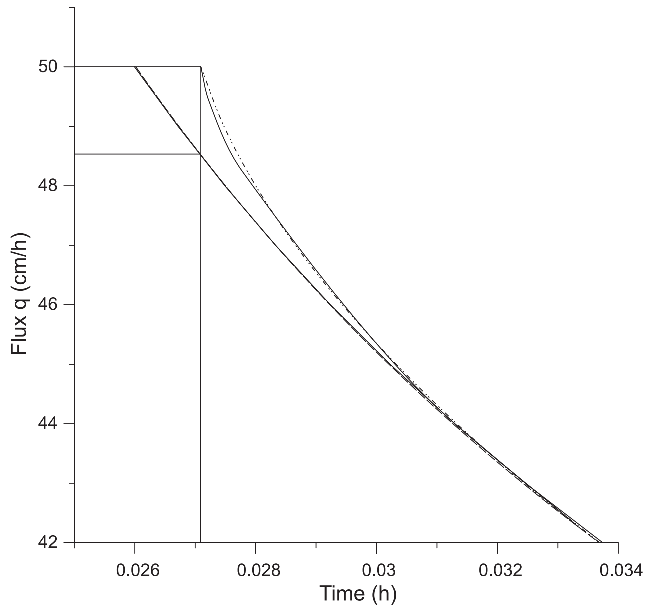

Figure 3. Flux versus time showing the numerical results (solid line) and the analytical results of equations (26) and (27) (dashed line) and equation (28) (dashed-dotted line).

3. Conclusion

[19] One practical advantage of MTCA over TCA is that its application requires a knowledge of ponding time rather than rainfall rates. However, when TCA and MTCA are accurate tools, they both assume that using average rainfall rates, rather than the actual values, does not lead to large errors in predicting postponding infiltration. We assumed that this is the case here, i.e., we did not discuss those situations when the use of an average flux leads to large predictive errors. Rather, we showed that when TCA is a good predictor of postponding infiltration then, MTCA, which is as easy to apply, reduces the error of cumulative infiltration by about 50%.

[20] We derived analytically two results valid for any soil property. First, infiltration rates after ponding are the same for TCA and MTCA. Second, the error of the predicted cumulative infiltration for MTCA is about half of what it is for TCA. Both results were obtained previously for linear and Burger's soils and are checked here for a Grenoble sand. More importantly, we predict that they should hold for any soil.

[21] We are able to model TCA, MTCA analytically and the transition from constant flux to constant surface water content for arbitrary soil properties. Small corrections to the cumulative infiltration in equation (5) had to be estimated. Being small, we could use rough, i.e., Green and Ampt or Gardner, approximations that affect the small corrections to a higher order which are negligible. We illustrated the accuracy of the analytical model by comparison with the numerical results for the Grenoble sand.

[22] The analytical results presented here apply potentially to any soil, which is more general than previous analytical results that use specific forms of the soil water properties. As a consequence, those results could be used as a predictive tool under field conditions, when the soil properties are known but do not follow specific forms.

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