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[1] We show that the two-point cross-correlation of self-potential field recordings is equal to the electric resistivity between the two points. This holds under the condition that spatially and temporally uncorrelated noise sources exist throughout the volume. These sources should have a known amplitude spectrum and their correlated strengths should be proportional to the dissipative medium property function. Natural fluctuations, such as thermal noise, may occur that satisfy the necessary conditions. When these fluctuations are random deviations from a state of thermal equilibrium, the fluctuation-dissipation theorem can be used to describe these sources. Other types of sources may exist, such as the ones creating the self-potential field through coupling with fluctuations in pressure, temperature and chemical potential gradients. **Citation:** Slob, E., R. Snieder, and A. Revil (2010), Retrieving electric resistivity data from self-potential measurements by cross-correlation, *Geophys. Res. Lett.*, *37*, L04308, doi:10.1029/2009GL042247.

1. Introduction

[2] Many of the underlying theories for Green's function extraction, which in the seismological community is referred to as seismic interferometry, have in common that the medium is assumed to be lossless to exploit time-reversal invariance [*Scherbaum*, 1987; *Schuster*, 2001; *Campillo and Paul*, 2003; *Wapenaar*, 2004; *Shapiro et al.*, 2004; *Snieder*, 2004]. It has been shown [*Snieder*, 2006, 2007] that a volume distribution of uncorrelated noise sources, with source strengths proportional to the dissipation parameters of the medium, precisely compensates for the energy losses. This allowed for extracting Green's functions of diffusive and wave fields in dissipative media. *Wapenaar et al.* [2006], *Snieder et al.* [2007], and *Weaver* [2008] showed that Green's function extraction by cross-correlation, including its extensions for wave fields and diffusive fields in dissipative media, can be represented in a unified form. Thermal noise in dissipative fluids gives rise to the required volume distribution of sources to extract the corresponding acoustic Green's function [*Godin*, 2007]. Recently, this has been extended to elastic waves in dissipative media [*Godin*, 2009]. The macroscopic electrodynamic theory was developed by *Rytov* [1956] from the fluctuation-dissipation theorem [*Callen and Welton*, 1951]. Rytov found that under thermal equilibrium conditions the cross-correlation product of the electric field, generated by thermal noise and measured at two locations, is proportional to the real part of the electric field Green's function in the frequency domain between these two locations. The question of whether potential fields can also be retrieved by cross-correlation of noise measurements is investigated here. Both low-frequency induced polarization methods and Self-Potential methods use quasi-static electric fields [*Revil et al.*, 2003]. In practice, potential field values are obtained by integrating measured time varying field values. The resulting average value is then stored as the value corresponding to the potential field. The noise average is zero, but the cross-correlations of the noise is not. We show that these cross-correlations are equal to the noise power spectrum times the electric potential Green's function.

[3] Here we derive identities for the sum and difference of the Green's function and its time-reversed version between two points, from cross-correlations of electric potential field fluctuations generated by sources throughout the volume and measured at the same two points. The expression for the sum is based on a fundamentally different formulation of Green's function extraction by cross-correlations (R. Snieder et al., Lagrangian Green's function extraction, with applications to potential fields, diffusion, and acoustic waves, submitted to *New Journal of Physics*, 2010) than the ones used in seismic interferometry. The identity is valid in an arbitrary heterogeneous anisotropic medium and is derived for point sources. Then we show that the results obtained by *Rytov* [1956] for thermal noise sources can be used in our result to determine the properties of such sources. We point out that thermal noise sources may be of interest for laboratory scale experiments, but for field experiments other types of noise sources may dominate. Other source mechanisms can be random variations in subsurface fluid pressure, temperature and chemical potential gradients in porous soils and rocks. These other source mechanisms need to couple into electric fields and we argue that by virtue of the fluctuation-dissipation theorem the strength of the corresponding electric sources is expected to have the desired cross-correlation functions.

2. Quasi-Static Electric Field Equations

[4] When the time variations in the magnetic field are negligible, the electric field is curl-free, and hence the electric field can be written as the gradient of a space-time dependent scalar electric potential function; this is known as the quasi-static electric field. The macroscopic space-time quasi-static electric field is determined by the scalar electric

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field potential $V(\mathbf{r}, t)$, the electric conduction current density $\boldsymbol{J}(\mathbf{r}, t)$, the anisotropic electric-conductivity 3 × 3 tensor time-convolution operator, $\boldsymbol{\sigma}(\mathbf{r}, t)$. The time-dependence of the conductivity allows for dispersion effects, which corresponds in the frequency domain to a complex-valued frequency-dependent conductivity. The external electric-charge injection or extraction rate is denoted $\dot{q}^e(\mathbf{r}, t)$, and the external electric field strength, $\mathbf{E}^e(\mathbf{r}, t)$. We develop the theory in the time-domain and use the following notation for time-convolution $f(\mathbf{r}, t) \star g(\mathbf{r}, t) = \int_{\tau=-\infty}^{\infty} f(\mathbf{r}, \tau) g(\mathbf{r}, t-\tau)\, \mathrm{d}\tau$, and use the subscript notation for vectors and tensors, while the summation convention applies to repeated lower case Latin subscripts, ranging from one to three. The electric conductivity is given by $\sigma_{kr}(\mathbf{r}, t)$. The effects of all possible time-relaxation mechanisms are incorporated in the time-dependent conductivity tensor. The reciprocal of the conductivity tensor is the resistivity tensor $\rho_{rs}(\mathbf{r}, t)$, such that $\sigma_{kr} \star \rho_{rs} = \delta_{ks}\delta(t)$, where δ_{ks} denotes the 3 × 3 identity matrix. Note that both σ_{kr} and ρ_{rs} are symmetric tensors and contain causal tensor elements. Starting from Maxwell's equations the quasi-static electric field equations can be written as

$$\partial_k J_k = -\dot{q}^e, \tag{1}$$

$$\partial_k V + \rho_{kr} \star J_r = -\rho_{kr} \star J_k^e, \tag{2}$$

where the external electric current density, J_k^e is introduced for later convenience as the time-convolution of the medium electric conductivity and the applied external electric field strength

$$J_k^e(\mathbf{r}, t) = \sigma_{kr}(\mathbf{r}, t) \star E_r^e(\mathbf{r}, t). \tag{3}$$

These field equations can be combined into a single equation given by

$$\partial_k(\sigma_{kr} \star \partial_r V) = -\mathcal{J}, \tag{4}$$

where the source term, $\mathcal{J}$ is the effective volume density of electric current that is injected or extracted; it is given by

$$\mathrm{J}(\mathbf{r}, t) = \dot{q}^e(\mathbf{r}, t) + \partial_k J_k^e(\mathbf{r}, t). \tag{5}$$

Henceforth the dependency on time is omitted from function arguments where possible. For Green's functions we use point sources $\mathcal{J}(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}')\delta(t)$, and the electric potential becomes a Green's function, $V(\mathbf{r}) = G(\mathbf{r}, \mathbf{r}')$, which satisfies

$$\partial_k\left[\sigma_{kr}(\mathbf{r}) \star \partial_r G\left(\mathbf{r}, \mathbf{r}'\right)\right] = -\delta\left(\mathbf{r} - \mathbf{r}'\right)\delta(t). \tag{6}$$

Two source types are given in equation (5) and we can define the monopole response

$$G^{Vq}\left(\mathbf{r}, \mathbf{r}'\right) \stackrel{\text{def}}{=} G\left(\mathbf{r}, \mathbf{r}'\right), \tag{7}$$

where the left superscript denotes the field type, in this case the electric potential V, and the right superscript denotes the source type, in this case the charge injection or extraction rate. The dipole response $\boldsymbol{G}^{VJ}$ is then the electric potential field (first superscript) impulse response generated by an electric dipole current source (second superscript). It is a vector function whose components (subscript) can be written as

$$G_k^{VJ}\left(\mathbf{r}, \mathbf{r}'\right) = -\partial_k G\left(\mathbf{r}, \mathbf{r}'\right). \tag{8}$$

For a distributed external electric field occupying the domain $\mathbb{D}$, the electric potential is given by

$$V^J(\mathbf{r}) = -\int_{\mathbf{r}' \in \mathbb{D}} \partial_r G\left(\mathbf{r}, \mathbf{r}'\right) \star J_r^e\left(\mathbf{r}'\right) \mathrm{d}^3\mathbf{r}'. \tag{9}$$

3. Global Field Interactions

[5] A state is a particular measurement situation in a spatial and temporal domain. It is determined by its source type and location, its receiver type and location, and by the medium parameters in the domain. We consider two different states, labeled A and B. The states can differ in their source mechanisms, their medium parameters, and their spatial and temporal locations. Here we take $\boldsymbol{\sigma}_A = \boldsymbol{\sigma}_B = \boldsymbol{\sigma}$, and assume that $\boldsymbol{\sigma}$ is symmetric. Equation (4) for state B can be time-convolved with the electric potential V_A, resulting in

$$V_A \star \partial_k[\sigma_{kr} \star \partial_r V_B] = -V_A \star \mathcal{J}_B, \tag{10}$$

and then be integrated over an arbitrary bounded spatial domain $\mathbb{D}$, with outer boundary $\partial\mathbb{D}$, which has a continuous outward pointing unit normal vector $\boldsymbol{n}^T = \{n_1, n_2, n_3\}$, to give

$$\int_{\mathbb{D}} V_A \star \mathcal{J}_B \mathrm{d}^3\mathbf{r} = \int_{\mathbb{D}} (\partial_k V_A) \star \sigma_{kr} \star (\partial_r V_B) \mathrm{d}^3\mathbf{r} - \oint_{\partial\mathbb{D}} n_m V_A \star \sigma_{mn} \star \partial_n V_B \mathrm{d}^2\mathbf{r}. \tag{11}$$

To arrive at equation (11), integration by parts and Gauss' divergence theorem has been used in the integral containing the divergence operator. The resulting boundary integral runs over the outer boundary, where continuity conditions or explicit conditions of the Dirichlet and/or Neumann types are assumed to apply. In the latter case the boundary integral vanishes. Similarly, equation (4) for state A can be time-convolved with V_B and integrated over the domain $\mathbb{D}$, resulting in

$$\int_{\mathbb{D}} V_B \star \mathcal{J}_A \mathrm{d}^3\mathbf{r} = \int_{\mathbb{D}} (\partial_k V_A) \star \sigma_{kr} \star (\partial_r V_B) \mathrm{d}^3\mathbf{r} - \oint_{\partial\mathbb{D}} n_m V_B \star \sigma_{mn} \star \partial_n V_A \mathrm{d}^2\mathbf{r}. \tag{12}$$

Notice that the volume integral in the right-hand side of equation (11) is equal to the volume integral in the right-hand side of equation (12).

[6] We now use delta functions for the sources, $\mathcal{J}_A = \delta(\mathbf{r} - \mathbf{r}_A)\delta(t)$, and a similar expression for $\mathcal{J}_B$, the potentials become Green's functions as defined by $V_A = G(\mathbf{r}, \mathbf{r}_A)$ and a similar expression for V_B. For these sources and fields equation (11) and (12) become

$$G(\mathbf{r}_B, \mathbf{r}_A) = \int_{\mathbb{D}} (\partial_k G(\mathbf{r}, \mathbf{r}_A)) \star \sigma_{kr} \star (\partial_r G(\mathbf{r}, \mathbf{r}_B)) \mathrm{d}^3\mathbf{r} - \oint_{\partial\mathbb{D}} n_m G(\mathbf{r}, \mathbf{r}_A) \star \sigma_{mn} \star \partial_n G(\mathbf{r}, \mathbf{r}_B) \mathrm{d}^2\mathbf{r}, \tag{13}$$

$$G(\mathbf{r}_A, \mathbf{r}_B) = \int_{\mathbb{D}} (\partial_k G(\mathbf{r}, \mathbf{r}_A)) \star \sigma_{kr} \star (\partial_r G(\mathbf{r}, \mathbf{r}_B)) \mathrm{d}^3\mathbf{r} - \oint_{\partial\mathbb{D}} n_m G(\mathbf{r}, \mathbf{r}_B) \star \sigma_{mn} \star \partial_n G(\mathbf{r}, \mathbf{r}_A) \mathrm{d}^2\mathbf{r}. \tag{14}$$

Here we have assumed that both points $\mathbf{r}_A$, $\mathbf{r}_B$ are inside $\mathbb{D}$. Subtracting equation (13) from (14) gives

$$G(\mathbf{r}_A,\mathbf{r}_B) - G(\mathbf{r}_B,\mathbf{r}_A) = \oint_{\partial\mathbb{D}} (n_m G(\mathbf{r},\mathbf{r}_A) \star \sigma_{mn} \star \partial_n G(\mathbf{r},\mathbf{r}_B) - n_k G(\mathbf{r},\mathbf{r}_B) \star \sigma_{kr} \star \partial_r G(\mathbf{r},\mathbf{r}_A)) \mathrm{d}^2\mathbf{r}. \quad (15)$$

For two locations, $\mathbf{r}_A$ and $\mathbf{r}_B$, the left-hand side of equation (15) is independent of the choice of $\mathbb{D}$, therefore the right-hand side is also independent of the choice of $\mathbb{D}$. The right-hand side of equation (15) is therefore independent of the size and shape of $\mathbb{D}$ as long as the points $\mathbf{r}_{A,B}$ are both inside the volume. When the volume is taken as infinite space the surface integral goes to zero. This is because the integrand goes to zero proportionally to inverse distance cubed and the surface area of a sphere is proportional to distance squared. This establishes the well-known source-receiver reciprocity relation $G(\mathbf{r}_A, \mathbf{r}_B, t) = G(\mathbf{r}_B, \mathbf{r}_A, t)$.

4. Green's Function Retrieval

[7] For state B we now take the time-reversed causal state. Using this in equation (11) and (12) leads to expressions involving cross-correlations of quantities in the time-domain. Cross-correlations of two functions $f(\mathbf{r}, t)$ and $g(\mathbf{r}, t)$ are denoted as $f(\mathbf{r}, \pm t) \star g(\mathbf{r}, \mp t)$. Using source-receiver reciprocity, we directly obtain the global interactions by taking $-t$ in the conductivity tensor and the Green's function for $\mathbf{r}_B$ in equations (13) and in the Green's function for $\mathbf{r}_B$ in (14), to obtain

$$G(\mathbf{r}_B,\mathbf{r}_A,t) = \int_{\mathbb{D}} (\partial_k G(\mathbf{r},\mathbf{r}_A,t)) \star \sigma_{kr}(\mathbf{r},-t) \star (\partial_r G(\mathbf{r},\mathbf{r}_B,-t)) \mathrm{d}^3\mathbf{r} - \oint_{\partial\mathbb{D}} n_m G(\mathbf{r},\mathbf{r}_A,t) \star \sigma_{mn}(\mathbf{r},-t) \star \partial_n G(\mathbf{r},\mathbf{r}_B,-t) \mathrm{d}^2\mathbf{r}, \quad (16)$$

$$G(\mathbf{r}_B,\mathbf{r}_A,-t) = \int_{\mathbb{D}} (\partial_k G(\mathbf{r},\mathbf{r}_A,t)) \star \sigma_{kr}(\mathbf{r},t) \star (\partial_r G(\mathbf{r},\mathbf{r}_B,-t)) \mathrm{d}^3\mathbf{r} - \oint_{\partial\mathbb{D}} n_m G(\mathbf{r},\mathbf{r}_B,-t) \star \sigma_{mn}(\mathbf{r},t) \star \partial_n G(\mathbf{r},\mathbf{r}_A,t) \mathrm{d}^2\mathbf{r}. \quad (17)$$

We extend the domain $\mathbb{D}$ to three-dimensional infinite space $\mathbb{R}^3$ and use the fact that in that case each surface integral goes to zero. We then obtain two integral relations for the Green's function as

$$G(\mathbf{r}_B,\mathbf{r}_A,\pm t) = \int_{\mathbb{R}^3} (\partial_k G(\mathbf{r}_A,\mathbf{r},t)) \star \sigma_{kr}(\mathbf{r},\mp t) \star (\partial_r G(\mathbf{r}_B,\mathbf{r},-t)) \mathrm{d}^3\mathbf{r}. \quad (18)$$

Interestingly, the only difference in the right-hand side of equation (18) is in the time-dependence of the conductivity. By adding and subtracting the result of equation (18) for both signs of the time-dependence, we obtain

$$G(\mathbf{r}_B,\mathbf{r}_A,t) \pm G(\mathbf{r}_B,\mathbf{r}_A,-t) = \pm \int_{\mathbb{R}^3} (\partial_k G(\mathbf{r}_A,\mathbf{r},t)) \star [\sigma_{kr}(\mathbf{r},t) \pm \sigma_{kr}(\mathbf{r},-t)] \star (\partial_r G(\mathbf{r}_B,\mathbf{r},-t)) \mathrm{d}^3\mathbf{r}. \quad (19)$$

Equation (19) shows that either the sum or difference of the Green's function between $\mathbf{r}_A$ and $\mathbf{r}_B$ and its time-reversed version is obtained from cross-correlation products of the Green's function measured at $\mathbf{r}_A$ and Green's function measured at $\mathbf{r}_B$. Both are related to external electric-current point sources scaled with the medium conductivity tensor at $\mathbf{r}$. Integration over all sources in $\mathbb{R}^3$ finally gives the Green's function. The sum of the conductivity tensors corresponds in the frequency domain to twice the real part of the complex conductivity tensor, whereas the difference would correspond to 2i times the imaginary part. We substitute equation (8) in equation (19) to obtain the right-hand side as contributions from dipole responses

$$G(\mathbf{r}_B,\mathbf{r}_A,t) \pm G(\mathbf{r}_B,\mathbf{r}_A,-t) = \pm \int_{\mathbb{R}^3} G_k^{VJ}(\mathbf{r}_A,\mathbf{r},t) \star [\sigma_{kr}(\mathbf{r},t) \pm \sigma_{kr}(\mathbf{r},-t)] \star G_r^{VJ}(\mathbf{r}_B,\mathbf{r},-t) \mathrm{d}^3\mathbf{r}. \quad (20)$$

For this equation we can find suitable source distributions that allow for practical Green's function extraction, which is discussed in the next section.

5. Field Fluctuations

[8] In the time-domain the Green's matrix is causal, hence $G(\mathbf{r}_B, \mathbf{r}_A, t) = 0$ for $t < 0$, and the time-reversed Green's function is time-reversed causal, hence $G(\mathbf{r}_A, \mathbf{r}_B, -t) = 0$ for $t > 0$. Because of this difference in the time dependence, both $G(\mathbf{r}_B, \mathbf{r}_A, t)$ and $G(\mathbf{r}_A, \mathbf{r}_B, -t)$ can be retrieved from the left-hand side of equation (20). Apart from the usefulness of equation (20) for modeling and inversion [*Wapenaar*, 2007], and for validation of numerical codes, here the primary interest is in possible applications of remote sensing without active sources.

5.1. Fluctuations About a Thermodynamic Equilibrium State

[9] *Rytov et al.* [1989] worked out the full electrodynamic theory for field fluctuations in piecewise continuous macroscopic systems in thermal equilibrium. We write their results here for the quasi-static electric potential Green's function, which is in our notation given by

$$\langle V(\mathbf{r}_A,t) \star V(\mathbf{r}_B,-t) \rangle = 2k_B T [G(\mathbf{r}_A,\mathbf{r}_B,t) + G(\mathbf{r}_A,\mathbf{r}_B,-t)], \quad (21)$$

where k_B is Boltzmann's constant and T is absolute temperature. The condition for the validity of this equation is that the medium is unbounded, or that homogeneous boundary conditions occur at the boundary of the domain. Equation (21) states that the cross-correlation of the electric potentials at the two locations $\mathbf{r}_A$ and $\mathbf{r}_B$ is equal to the thermal energy, $k_B T$, times the sum of the Green's function and its time-reversed version between $\mathbf{r}_A$ and $\mathbf{r}_B$. We observe that the right-hand side of equation (21) is, apart from twice the thermal energy, the same as the left-hand side of equation (20) with a plus-sign. We can now use this fact to establish the cross-correlation of the thermal noise sources. We first write the observation of the electric-potential at $\mathbf{r}_A$ or $\mathbf{r}_B$ as a response to thermal noise electric dipole sources distributed throughout the volume, in accordance with equation (5), as

$$V(\mathbf{r}_A) = \int_{\mathbf{r}\in\mathbb{R}^3} G_r^{VJ}(\mathbf{r}_A,\mathbf{r},t) \star J_r^e(\mathbf{r},t) \mathrm{d}^3\mathbf{r}, \quad (22)$$

$$V(\mathbf{r}_B) = \int_{\mathbf{r}'\in\mathbb{R}^3} G_k^{VJ}\left(\mathbf{r}_B,\mathbf{r}',t\right) \star J_k^e\left(\mathbf{r}',t\right) \mathrm{d}^3\mathbf{r}'. \quad (23)$$

The correlation product of $V(\mathbf{r}_A)$ and $V(\mathbf{r}_B)$, can then be written as

$$\langle V(\mathbf{r}_A,t) \star V(\mathbf{r}_B,-t)\rangle = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \langle G_r^{VJ}(\mathbf{r}_A,\mathbf{r},t) \star J_r^e(\mathbf{r},t) \star J_k^e(\mathbf{r}',-t) \star G_k^{VJ}(\mathbf{r}_B,\mathbf{r}',-t)\rangle \mathrm{d}^3\mathbf{r}'\,\mathrm{d}^3\mathbf{r}. \tag{24}$$

In the right-hand side of equation (21) we substitute equation (20) to obtain

$$\langle V(\mathbf{r}_A,t) \star V(\mathbf{r}_B,-t)\rangle = 2k_BT \int_{\mathbb{R}^3} \langle G_r^{VJ}(\mathbf{r}_A,\mathbf{r},t) \star [\sigma_{kr}(\mathbf{r},t) + \sigma_{rk}(\mathbf{r},-t)] \star G_k^{VJ}(\mathbf{r}_B,\mathbf{r},-t)\rangle \mathrm{d}^3\mathbf{r}. \tag{25}$$

Comparing the right-hand side of equation (24) with the right-hand side of equation (25) we find that cross-correlation of the thermal noise electric dipole sources are given by

$$\int_{\mathbb{R}^3} \langle J_k^e(\mathbf{r},t) \star J_r^e\left(\mathbf{r}',-t\right)\rangle \mathrm{d}^3\mathbf{r}' = 2k_BT[\sigma_{kr}(\mathbf{r},t) + \sigma_{rk}(\mathbf{r},-t)]. \tag{26}$$

Equation (26) states that the cross-correlation product of thermal noise electric dipole sources is equal to the sum of the electric conductivity and its time-reversed version, times the thermal energy in the system. This result is also given by *Landau and Lifshitz* [1960], where it is derived from the fluctuation-dissipation theorem.

[10] When the medium is not dissipative everywhere in space, $\sigma(\mathbf{r}, t) = 0$ in some part of $\mathbb{D}$, the non-dissipative part of the medium can be excluded from the volume integration to allow the boundary to run at the intersection of the dissipative and non-dissipative domain. Under quasi-static electric field conditions, the earth surface is such an interface, and Dirichlet conditions apply to the normal component of the electric current, so that the boundary integrals present in equations (16) and (17) are still zero and equation (19), with a plus-sign, and equation (21) remain valid in the reduced volume where the medium is dissipative. This implies that the electric response of the earth as a heterogeneous half space, or sphere, can be determined by cross-correlations of thermal fluctuational electric noise recordings.

5.2. Strengths of Electric Thermal Noise Signals

[11] The expected strength of the thermal noise fields depends on the thermal energy. Boltzmann's constant is $k_B \approx 1.4 \times 10^{-23}$ J/K, hence for a temperature of $T \approx 300$ K, $k_BT \approx 4.2 \times 10^{-21}$ J. The total energy delivered between two electrodes is equal to the energy dissipated in the medium by the resistance of the medium, which is related to the location of the two electrodes. This results in $\langle V\rangle^2/R = 4k_BT\Delta f$, where R is the apparent resistance of the medium and Δf is the frequency bandwidth of the measurement. The energy depends on the bandwidth of the instrument used in the experiment. For a bandwidth of $\Delta f = 10$ *kHz*, the power is $4k_BT\Delta f \approx 1.7 \times 10^{-16}$ Watt. If the apparent resistance of the measurement is 0.6 Ω, the mean fluctuation of the squared electric potential difference between these two points is $\langle V\rangle^2 \approx 10^{-16}$ which corresponds to measuring a voltage difference between the two electrodes of $V \approx 10$ *n*V. The bandwidth of the measurement can be increased by increasing the receiver bandwidth, because the mean energy of the signal is proportional to the bandwidth. Finally, the apparent resistance of a measurement is given by $V/I = R_{\mathrm{app}} = \rho_{\mathrm{app}}/(4\pi|\mathbf{r} - \mathbf{r}'|)$, where ρ_{app} is the apparent resistivity of an equivalent homogeneous infinite medium, $\mathbf{r}$ and $\mathbf{r}'$ are the positions between which the potential difference is measured. For the assumed 0.6 Ω apparent resistance in our example, the apparent resistivity is $\rho_{\mathrm{app}} = 7|\mathbf{r} - \mathbf{r}'|\Omega$m. For a distance between the two electrodes of $|\mathbf{r} - \mathbf{r}'| = 10$ m, the apparent resistivity is $\rho_{\mathrm{app}} = 70\Omega$m. For a certain distance between the electrodes the potential fluctuations increase if the conductivity decreases. We can always find electrode pairs that will lead to a high quality virtual measurement from cross-correlations of thermal noise potential difference recordings. While this may be sufficient in the laboratory as demonstrated by *Johnson* [1928], in the field there will be many other sources of electric noise.

5.3. Other Types of Random Electrical Noise Sources

[12] Randomly generated electric fluctuations at the microscopic level are caused by random motion of charge carriers, such as electrons, ions, and small colloidal particles. At the macroscopic level these correspond to fluctuations in the electric field, which then act as the source. According to the fluctuation-dissipation theorem, energy that is dissipated is also radiated to maintain thermal equilibrium. In a linear model for natural sources, obtained from assuming linearity in the vicinity of thermodynamic equilibrium, the source electric current density is given as the sum of coupling coefficients multiplied by gradients in fluid pressure, temperature, and chemical potential of the pore water [*Revil and Linde*, 2006]. Fundamentally, random fluctuations in gradients of temperature, fluid pressure, and chemical potential of the pore water generate fluctuations in the source current density that create small radiating dipoles. These random fluctuations are through the coupling coefficients transformed into electrical currents. The coupling coefficients also contribute to the strengths of these sources. These coupling coefficients can be explicitly determined for a porous material by upscaling the corresponding Nernst-Planck equation [*Revil*, 2007]. Other possible sources of noise are related to drainage processes. When the pore water is replaced by a non-wetting fluid, electric bursts are generated that are associated with Haines jumps. Haines jumps are sudden jumps in the position of the meniscus during drainage and imbibitions. During drainage, the associated electrical potential fluctuations can be higher than 10 mV [*Haas and Revil*, 2009]. These are available subsurface sources and their effects are measured in self-potential methods for subsurface characterization and monitoring applications. Here we propose that these self-potential measurements be cross-correlated to retrieve electric potential data as if active resistivity surveys were carried out, because equation (20) remains valid in absence of thermal noise and can be used when the other noise sources are available.

[13] For thermal noise sources the strength of these sources can be determined from the fluctuation-dissipation theorem as formulated in equation (21) in combination with the reciprocity relation of the correlation type with a plus-sign as expressed in equation (20). This means that the thermal noise cross-correlations are determined by the basic equations. This is not necessarily true for the other source

mechanisms. They can be either thermal non-equilibrium or non-thermal in nature. Then we will still have

$$\langle V(\mathbf{r}_A,t) \star V(\mathbf{r}_B,-t)\rangle \propto [G(\mathbf{r}_A,\mathbf{r}_B,t) + G(\mathbf{r}_A,\mathbf{r}_B,-t)], \quad (27)$$

An important aspect for equation (27) to hold is that these sources are uncorrelated. When they have a structural component, non-zero spatial correlation lengths may result. Even if the random deviations do not depend on this structural component there may be some correlation length present in these sources. When the correlation lengths are small compared to the measurement scale, they are not important, but when they are of the same length scale or larger than the measurement scale they cannot be neglected. In that case the statistics of the sources need to be known or determined by independent means. This can be achieved by measuring simultaneously the noise in the electric potential, fluid pressure and temperature fields, and analyzing their noise characteristics.

6. Conclusions

[14] We have derived an identity that relates cross-correlations of fluctuations in the self-potential, recorded at two locations, to the electric resistivity between these two points. We have used the fluctuation-dissipation theorem to arbitrary linear dissipative systems, which can be used for Green's function extraction. This leads to the identity for the cross-correlation of thermal noise measurements at two locations being proportional to the Green's function between those two locations, with thermal energy as the proportionality constant. This is possibly of interest in laboratory investigations.

[15] For possible field applications, thermal noise may not be the major source of fluctuations. For other uncorrelated sources of noise the mechanisms that couple into the electric fields have been identified as possible noise sources and the additional requirement is that their coupling coefficients are known or can be determined by independent means. This may lead to field procedures that allow for obtaining electric resistivity data from cross-correlations of self-potential noise recordings.

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