Reply to comment by A. Fiori et al. On "Asymptotic dispersion in 2D heterogeneous porous media determined by parallel numerical simulations"

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[1] The comment by A. Fiori, G. Dagan, and I. Jankoviæ [*Fiori et al.*, 2008] compares numerical results [*de Dreuzy et al.*, 2007] on the longitudinal asymptotic dispersion coefficient to a self-consistent solution [*Fiori et al.*, 2003]. The comparison given by the Figure 1 of the comment displays a good agreement for $\sigma^2 \leq 6.25$ but the self-consistent approach overestimates the numerical result by a factor of 2 for $\sigma^2 = 9$. For $\sigma^2 = 9$, the self-consistent approach shows a critical influence of the very low velocity zones on dispersivity. The low-velocity zones induce large residence and delay times and thus increase dispersivity. Undersampling them leads to a much lower dispersivity. In this reply, we first analyze the sampling of the low velocity zones and secondly test its effect on dispersion for the lognormally correlated fields of *de Dreuzy et al.* [2007]. We use for these tests one of the realizations used for determining the asymptotic dispersion coefficient for $\sigma^2 = 9$. The domain is a rectangle of longitudinal length $L_x = 1638{,}4.\lambda$ and transversal length $L_x = 819{,}2.\lambda$ where the correlation length λ is equal to 10 grid cells. The domain contains thus around $N_c = 135 \times 10^6$ cells. The width of the injection window is fixed to 655 λ. The lognormal permeability mean $\mu = <\ln K>$ is set at 0. More details on parameters and simulation procedure can be found in the work by *de Dreuzy et al.* [2007].

[2] Undersampling can come first from an absence of very low velocity zones either because of an absence of very low permeability zones or because of their removal by the flow and velocity computations. The permeability distribution obtained numerically extends to $Y_{min} = \ln(K_{min}) = -15$ (Figure 1). This is consistent with the theoretical expectation according to which the order of the minimal permeability of a field containing N_c cells can be obtained by $F(Y_{min}) = 1/Nc$ where F is the cumulative Gaussian distribution function. For $N_c = 135 \times 10^6$, $Y_{min} = -15{,}24$. The discretization of the flow equation has been performed according to a finite volume scheme with a harmonic mean for the interblock permeability. The harmonic mean keeps the small permeability values and at the maximum increases the lowest log permeability Y by a factor of $\ln 2 \sim 0.69$. Once the head computed, the velocity distribution consistently extends to $\ln v_{min} = -12$ (Figure 1, dash-dotted line).

[3] Undersampling can come secondly from an insufficient number of particles traduced by the absence of particles going into the smallest velocity zones. To check this, we have computed the velocity distribution sampled by the particles at a given time $t = 1000$ at which all particles are still within the domain for number of particles n_p increasing from 10^3 to 5×10^5 (Figure 1). As expected the sampled velocity distributions follow the Eulerian velocity distribution (dash-dotted line) and the sampling of the lowest velocities increases with more particles.

[4] So far, we have checked first that the velocity distribution extends to values as low as $\ln v_{min} = -12$ and secondly that increasing the number of particles leads to a better sampling of the very low velocity zones. We finally look at the evolution of the longitudinal dispersion $D_L(t)$ according to the number of particles (Figure 2). The number of particles does not change fundamentally the behavior of $D_L(t)$. We do not observe any marked tendency with the

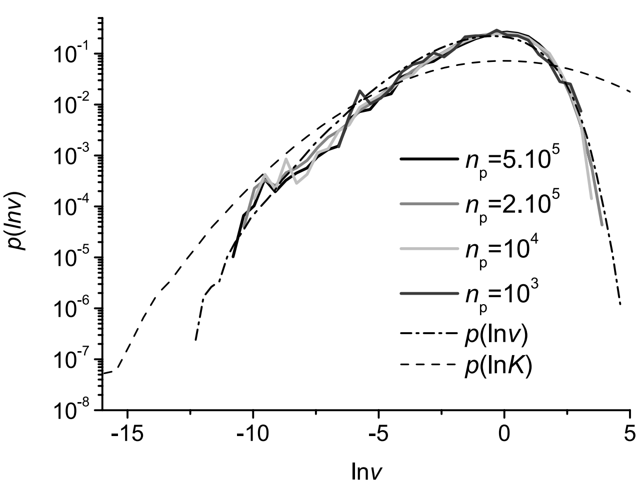

Figure 1. Distributions of permeability (dashed line), Eulerian velocity (dash-dotted line), and sampled velocity at time $t = 1000$ for different particle number n_p (solid lines) for one of the realizations used by *de Dreuzy et al.* [2007] for determining the asymptotic dispersion coefficient for $\sigma^2 = 9$.

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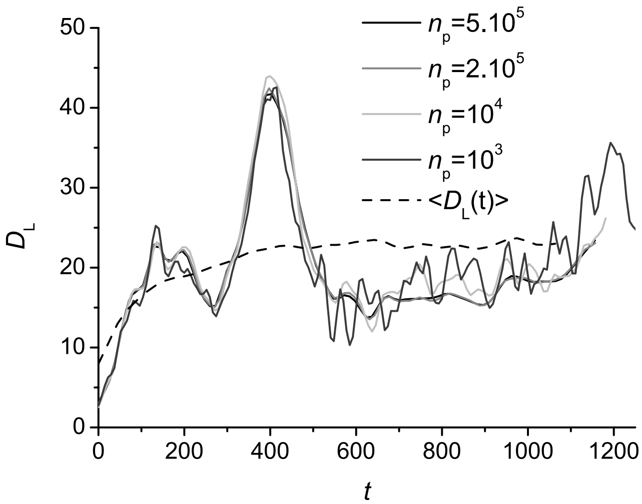

Figure 2. Normalized longitudinal dispersion coefficient for single realizations D_L(t) for different particle numbers n_p (solid lines) and their average over 100 realizations (dashed line).

number of particles. We thus conclude that the observed better sampling of the lower-velocity zones obtained by the increase of the particle number does not lead to an increase of the dispersion coefficient. In the permeability field structure studied in *de Dreuzy et al.* [2007], the smallest velocity zones for $\sigma^2 = 9$ do not lead to a dramatic increase of dispersion, as opposed to within the self-consistent approach of *Fiori et al.* [2003]. As mentioned in the comment, this may be due to the differences in the conductivity structures at high order of σ^2 or by the lesser relevance of the effective medium approximation in 2D than in 3D.

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