Blind channel identification based on cyclic statistics

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Indexing terms: Linear prediction algorithms, Subspace fitting

Abstract: Use of cyclic statistics in fractionally sampled channels in subspace fitting and linear prediction for channel identification is proposed, possibly for multiuser and multiple antennas. Identification schemes are based on cyclic statistics using the stationary multivariate representation, leading to the use of all cyclic statistics. Compared with classical approaches, the methods proposed have an equivalent performance for subspace fitting, and an enhanced performance for linear prediction.

1 Introduction

Major impairments of most wireless communication channels, especially in mobile environments, are intersymbol interference (ISI), cochannel interference (CCI) and adjacent channel interference (ACI). In wireless networks the latter is solved by source separation techniques and ISI by equalisation techniques. In the past three decades, so-called blind channel identification and equalisation techniques flourished, where blind really means based on the outputs of the channel only, and some assumptions on the nature of the input and/or channel. Among these techniques those based on second-order statistics only are very attractive because they need few samples to allow channel identification compared with other methods (implicitly or explicitly based on higher order statistics).

Recognising that communication (continuous time) signals are cyclostationary shows the cyclostationarity of the oversampled discrete time signals (w.r.t. data rate) and under mild conditions leads to the identifiability of the channel. The optimal method is the covariance matching [1]. The two other families of methods are subspace fitting and linear prediction introduced with noncyclic statistics [2], which are suboptimal but do not need complex numerical searches as the covariance matching method.

In this paper we introduce a new multichannel model derived from the stationary multivariate representation introduced in [3]. This representation allows the derivation of subspace fitting and linear prediction methods using cyclic statistics. Algebraic considerations show that the cyclic subspace fitting has in theory the same performance as the noncyclic subspace fitting, although the cyclic approach is characterised by fewer parameters for the channel, leading to some enhancement w.r.t. the noncyclic method. For linear prediction, basing the prediction on more samples leads to better performance.

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2 Data model

We consider a spatial division multiple access (SDMA) communication system with p emitters and a receiver constituted of an array of M antennas (Fig. 1). The signals received are oversampled by a factor m w.r.t. the symbol rate. The channel is FIR of duration NT/m where T is the symbol duration.

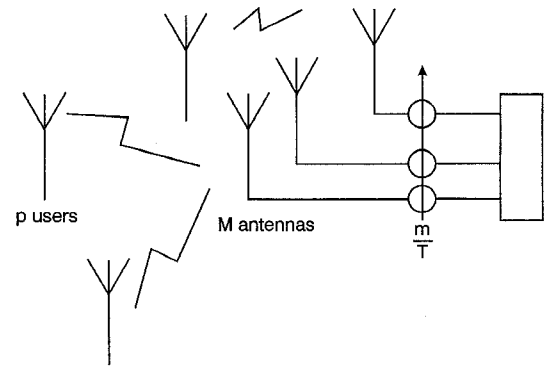

Fig. 1 *Schematic SDMA situation*

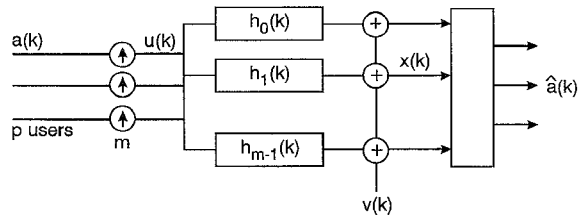

Fig. 2 *Channel model*

The received signal can be written as

$$\mathbf{x}(n) = \sum_{k=0}^{N-1} \mathbf{h}(k)\mathbf{u}(n-k) + \mathbf{v}(n) = \sum_{k=\lceil \frac{n}{m} \rceil}^{\lfloor \frac{n+N-1}{m} \rfloor} \mathbf{h}(n-km)\mathbf{a}_k + \mathbf{v}(n) \quad (1)$$

where $\mathbf{u}(n) = \Sigma_{k=-\infty}^{\infty} \mathbf{a}_k \delta(n-km)$ (Fig. 2). The received signal $\mathbf{x}(n)$ and noise $\mathbf{v}(n)$ are $M \times 1$ vectors. $\mathbf{x}(n)$ is cyclostationary with period m whereas $\mathbf{v}(n)$ is assumed not to be cyclostationary with period m. $\mathbf{h}(k) = [\mathbf{h}_1(k)^T \ldots \mathbf{h}_M(k)^T]^T$ has dimension $M \times p$, $\mathbf{a}(k)$ and $\mathbf{u}(n)$ have dimensions $p \times 1$.

3 Cyclic statistics

Following the foregoing assumptions the correlations

$$\mathbf{R}_{xx}(n,\tau) = \mathrm{E}\{\mathbf{x}(n)\mathbf{x}^H(n-\tau)\} \tag{2}$$

are cyclic in n with period m (^{H} denotes complex conjugate transpose and E{ } denotes the expectation operator) [4]. One can easily express them as

$$\mathbf{R}_{xx}(n,\tau) = \sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty} \mathbf{h}(n-\alpha m)\mathbf{R}_{aa}(\beta) \times \mathbf{h}^H(n-\alpha m+\beta m-\tau) + \mathbf{R}_{vv}(\tau) \tag{3}$$

We then express the kth cyclic correlation as

$$\mathbf{R}_{xx}^{\{k\}}(\tau) \equiv \frac{1}{m}\sum_{l=0}^{m-1}\mathbf{R}_{xx}(l,\tau)e^{-j\frac{2\pi lk}{m}} \equiv \mathrm{E}^{\{k\}}\{\mathbf{x}(l)\mathbf{x}^H(l-\tau)\} \tag{4}$$

whose value is

$$\mathbf{R}_{xx}^{\{k\}}(\tau) = \frac{1}{m}\sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty} \mathbf{h}(\alpha)\mathbf{R}_{aa}(\beta) \times \mathbf{h}^H(\alpha+\beta m-\tau)e^{-j\frac{2\pi\alpha k}{m}} + \mathbf{R}_{vv}(\tau)\delta(k) \tag{5}$$

with k integer and $\mathbf{R}_{xx}^{\{k\}}(\tau) = \mathbf{R}_{xx}^{\{k+m\}}(\tau)$ (usually one uses $k \in [-\lfloor m/2\rfloor, \ldots, \lfloor m/2\rfloor]$). Let us denote $T_K(\mathbf{H}_N)$ as the convolution matrix of $\mathbf{H}_N = [\mathbf{h}(0)^T\mathbf{h}(1)^T\ldots\ \mathbf{h}(N-1)^T]^T$ and

$$\mathbf{D}_{DFT}^{\{k,p\}} = \mathrm{blockdiag}\left[\mathbf{I}_p \left| e^{-j\frac{2\pi k}{m}}\mathbf{I}_p \right| \ldots \left| e^{-j\frac{2\pi(N-1)k}{m}}\mathbf{I}_p\right.\right] \tag{6}$$

where blockdiag[A ... Z] denotes a matrix with its block diagonal elements as A ... Z and zero elsewhere.

4 Gladyshev's theorem and Miamee process

Gladyshev's theorem [3] states that

Theorem: Function $\mathbf{R}_{xx}(n, \tau)$ is the correlation function of some periodically correlated sequence (PCS) if and only if the matrix-valued function

$$\underline{\mathbf{R}}(\tau) = [\mathbf{R}_{xx}^{\{kk'\}}(\tau)]_{k,k'=0}^{m-1} \tag{7}$$

$$\mathbf{R}_{xx}^{\{kk'\}}(\tau) = \mathbf{R}_{xx}^{\{k-k'\}}(\tau)e^{2\pi jk\tau/m} \tag{8}$$

is the matrix correlation function of some m-variate stationary sequence.

Remembering that $\mathbf{R}_{xx}^{\{k\}}(\tau) = \mathbf{R}_{xx}^{\{k-m\}H}(-\tau)$,the matrix

$$\underline{\mathbf{R}} \equiv \begin{bmatrix} \underline{\mathbf{R}}(0) & \underline{\mathbf{R}}(1) & \cdots & \underline{\mathbf{R}}(K-1) \\ \underline{\mathbf{R}}(-1) & \underline{\mathbf{R}}(0) & \cdots & \underline{\mathbf{R}}(K-2) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\mathbf{R}}(1-K) & \underline{\mathbf{R}}(2-K) & \cdots & \underline{\mathbf{R}}(0) \end{bmatrix} \tag{9}$$

is an hermitian $K \times K$ block Toeplitz matrix of $Mm \times Mm$ blocks, then Miamee [5] gives the explicit expression of the multivariate associated stationary process

$$\underline{\mathbf{Z}} = [\mathbf{Z}_n^k]_{k=0}^{m-1} \text{ where } \mathbf{Z}_n^k = \bigoplus_{j=0}^{m-1}\mathbf{x}(n+j)e^{2\pi jk(n+j)/m} \tag{10}$$

where $\oplus$ is the direct sum, i.e. noting $w = e^{2\pi j/m}$.

$$\mathbf{Z}_n^k = w^{kn}[\mathbf{x}(n), \mathbf{x}(n+1)w^k, \ldots; \mathbf{x}(n+m-1)w^{k(m-1)}] \tag{11}$$

is defined in an Hilbert space, where the correlation is the Euclidean product

$$\langle \mathbf{Z}_n^k, \mathbf{Z}_{n+l}^{k'}\rangle = \sum_{j=0}^{m-1}\mathrm{E}\{\mathbf{Z}_n^k(j)\mathbf{Z}_{n+l}^{k'\ H}(j)\} \tag{12}$$

and $\underline{\mathbf{Z}}_n = [\mathbf{Z}_n^{0T}\ \mathbf{Z}_n^{1T}\ \ldots\ \mathbf{Z}_n^{m-1T}]^T$, with the classical correlation for multivariate stationary processes. On the other hand, Miamee gives the link between the linear prediction $\underline{\mathbf{Z}}_n$ and the cyclic AR model of $\mathbf{x}(n)$, which we use to derive an efficient way of computing the linear predictor.

5 Expression of $\underline{\mathbf{Z}}_n$ w.r.t. $\mathbf{u}(n)$ and $\mathbf{h}(n)$

From $\mathbf{Z}_n^k = \oplus_{j=0}^{m-1}\mathbf{x}(n+j)\,e^{2\pi jk(n+j)/m}$, where

$$\mathbf{x}(n+j) = \sum_{k=0}^{L-1}\mathbf{h}(k)\mathbf{u}(n+j-k) + \mathbf{v}(n+j) = \mathbf{H}_N\begin{bmatrix}\mathbf{u}(n+j)\\ \mathbf{u}(n+j+1)\\ \vdots \\ \mathbf{u}(n+j-N+1)\end{bmatrix} + \mathbf{v}(n+j) \tag{13}$$

and defining $\mathbf{U}_{n+j} = [\mathbf{u}(n+j)^T\ \ldots\ \mathbf{u}(n+j-N+1)^T]^T$ and $\mathbf{H}_N^{\{k\}} = [w^{-kj}\mathbf{h}(j)]_{j=0}^{N-1}$ we express the Miamee process as

$$\mathbf{Z}_n^k = \bigoplus_{j=0}^{m-1}\left(\mathbf{H}_N^{\{-k\}}w^{kn}\mathbf{U}_{n+j} + \mathbf{v}(n+j)e^{2\pi jk\frac{n+j}{m}}\right) = \mathbf{H}_N^{\{-k\}}w^{kn}[\mathbf{U}_n\quad \mathbf{U}_{n+1}\quad \cdots\quad \mathbf{U}_{n+m-1}] + \bigoplus_{j=0}^{m-1}\mathbf{v}(n+j)e^{2\pi jk\frac{n+j}{m}} \tag{14}$$

$$\Rightarrow \underline{\mathbf{Z}}_n = \mathbf{H}_{tot}\hat{\mathbf{U}}(n) + \hat{\mathbf{V}}(n) \tag{15}$$

where $\mathbf{H}_{tot} = [\mathbf{H}_N^{\{0\}T}\ \mathbf{H}_N^{\{1\}T}\ \ldots\ \mathbf{H}_N^{\{1-m\}T}]$,

$$\hat{\mathbf{U}} = \mathbf{D}_{DFT}^{\{n,pN\}}[\mathbf{U}_n\quad \mathbf{U}_{n+1}\quad \cdots\quad \mathbf{U}_{n+m-1}] \tag{16}$$

$$\hat{\mathbf{V}}(n) = \begin{bmatrix}\mathbf{v}(n) & \cdots & \mathbf{v}(n+m-1)\\ \mathbf{v}(n)w^n & \cdots & \mathbf{v}(n+m-1)w^{n+m-1}\\ \vdots & \ddots & \vdots \\ \mathbf{v}(n)w^{n(m-1)} & \cdots & \mathbf{v}(n+m-1)w^{(n+m-1)(m-1)}\end{bmatrix} \tag{17}$$

$$\Rightarrow \hat{\mathbf{Z}} = T_{L+N-1}(\mathbf{H}_{tot})\hat{\mathbf{U}}_L + \hat{\mathbf{V}}_L \tag{18}$$

$_L = [\ (n)]_{n=L-1}^{0}$ is clearly a stationary process whose correlation matrix can easily be deduced from $\mathbf{R}_{aa}$. Refer to Figs. 3 and 4. Based on eqn. 18 we apply the classical subspace fitting and linear prediction channel identification schemes as follows.

5.1 Identifiability

Provided that the data collected are numerous enough the rank condition on $T_{L+N-1}(\mathbf{H}_{tot})$ leads to the usual identifiability conditions, i.e. $\mathbf{H}_{tot}(z)$ must be minimum phase, which is equivalent to the condition that $\mathbf{h}_i(z)$ may not have $2\pi/m$ equispaced zeros and that $\mathbf{h}_i(z)$ and $\mathbf{h}_j(z)$ may not have common zeros for all $i \neq j$.

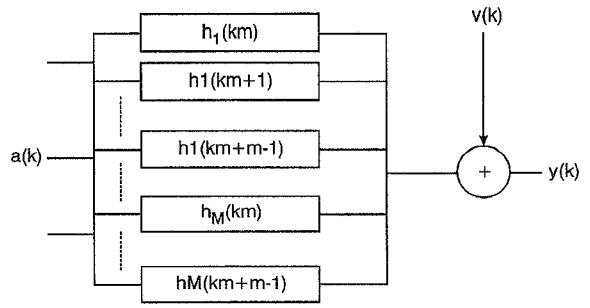

Fig. 3 *Classical time series channel model representation*

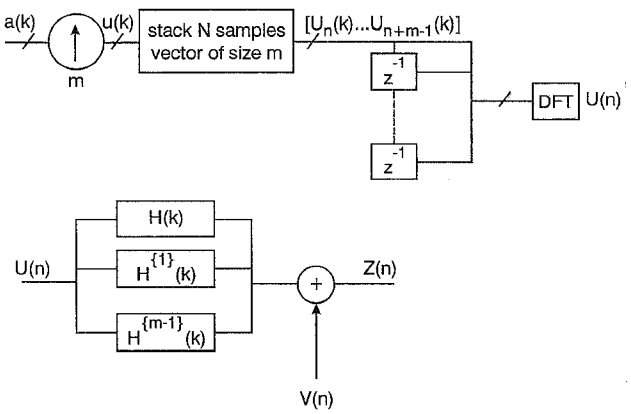

Fig. 4 *New channel model*

5.2 Signal subspace fitting

Recall the signal subspace fitting [6, 7] (noise subspace based) blind channel identification algorithm. One can write the SVD of the cyclocorrelation matrix $\underline{\mathbf{R}} = \mathbf{ABC}^H$ with the relations

$$\text{range}\{\mathbf{A}\} = \text{range}\{\mathbf{C}\} = \text{range}\{T_K(\mathbf{H}_{tot})\} \quad (19)$$

We then solve the classical subspace fitting problem

$$\min_{\mathbf{H}_{tot}} \|T_K(\mathbf{H}_{tot}) - \mathbf{A}T\|_F^2 \quad (20)$$

Introducing $\mathbf{A}^\perp$ such that $\lfloor \mathbf{A}\mathbf{A}^\perp \rfloor$ is a unitary matrix leads to

$$\min_{\mathbf{H}_{tot}} \mathbf{H}_{tot}^t \left[\sum_{i=D^\perp}^{KMm} T_N(\mathbf{A}_i^{\perp Ht}) T_N^H(\mathbf{A}_i^{\perp Ht}) \right] \mathbf{H}_{tot}^{Ht} \quad (21)$$

where $\mathbf{A}_i^\perp$ is a $KMm^2 \times 1$ matrix and $D^\perp = N + K$ and superscript ^{t} denotes the transposition of the blocks of a block matrix. Under constraint $\|\mathbf{H}_{tot}\| = 1$, $\hat{\mathbf{H}}_{tot}^t$ is then the eigenvector corresponding to the minimum eigenvalue of the matrix between brackets. Similar work has been done by Schell [4] for the direction-of-arrival estimation problem. One can reduce the computational burden by using $D^\perp > N + K$ [7], which leads to loss of performance. A reduced-complexity signal subspace fitting without loss of performance is described in [8]. The case $p > 1$ can be (partially) solved as in [9, 10].

5.3 Linear prediction

Consider the denoised case. The correlation matrix is then computed as $\mathbf{R}_{xx,sb}^{\{0\}} = \mathbf{R}_{xx}^{\{0\}} - \mathbf{R}_{VV}(\tau)$, where

$$[\mathbf{R}_{vv}(\tau)]_{i,j} = \sum_{l=0}^{m-1} \mathrm{E}\{\mathbf{v}(n+l)\mathbf{v}^H(n+l+\tau)\} \times w^{i(n+l)} w^{-j(n+l+\tau)}$$

$$= \mathbf{R}_{vv}(\tau) w^{n(i-j)-j\tau} \sum_{l=0}^{m-1} w^{(i-j)l}$$

$$= \mathbf{R}_{vv}(\tau) w^{n(i-j)-j\tau} m\delta_{ij}$$

$$= m\delta_{ij}\mathbf{R}_{vv}(\tau) w^{-j\tau} \quad (22)$$

Hence $\mathbf{R}_{VV}(\tau) = \text{blockdiag}\lfloor \mathbf{R}_{VV}(\tau)\ w^\tau \mathbf{R}_{VV}(\tau)\ \ldots\ w^{(m-1)\tau}\mathbf{R}_{VV}(\tau) \rfloor$, which in $\mathbf{R}$ corresponds to the noise contribution of the zero cyclic frequency cyclic correlation. From eqn. 18 and noting $\hat{\mathbf{Z}}_K(n-1) = \lfloor \underline{\mathbf{Z}}_j \rfloor_{j=n-1}^{j=n-K}$, the predicted quantities are

$$\hat{\underline{\mathbf{Z}}}(n)|_{\hat{\mathbf{Z}}_K(n-1)} = \mathbf{p}_1 \underline{\mathbf{Z}}_{n-1} + \cdots + \mathbf{p}_K \underline{\mathbf{Z}}_{n-K} \quad (23)$$

$$\tilde{\underline{\mathbf{Z}}}(n) = \underline{\mathbf{Z}}(n) - \hat{\underline{\mathbf{Z}}}(n)|_{\hat{\mathbf{Z}}_K(n-1)} \quad (24)$$

Following [11], we rewrite the correlation matrix as

$$\underline{\mathbf{R}} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{r}_K \\ \mathbf{r}_K^H & \mathbf{R}_{K-1} \end{bmatrix} \quad (25)$$

this yields the prediction filter

$$\mathbf{P}_K \equiv [\mathbf{p}_1 \cdots \mathbf{p}_K] = -\mathbf{r}_K \mathbf{R}_{K-1}^{-1} \quad (26)$$

and the prediction error variance

$$\sigma_{\tilde{\underline{\mathbf{Z}}},K} = \mathbf{R}_0 - \mathbf{P}_K \mathbf{r}_K^H = \mathbf{H}_{tot}(0)\sigma_{\underline{U},K+N-1}^2 \mathbf{H}_{tot}^H(0) \quad (27)$$

where the inverse might be replaced by the Moore–Penrose pseudoinverse and still yield a consistent channel estimate. Another way of being robust to order overestimation would be to use the Levinson–Wiggins-Robinson (LWR) algorithm to find the prediction quantities and estimate the order with this algorithm.

There are many ways to proceed from the prediction quantities to the channel estimate [12, 13]. We use the optimal solution here. For $K = \underline{K} = \lceil N - 1/Mm - 1 \rceil$, eqn. 27 allows one to find $\mathbf{H}_{tot}(0)$ up to a scalar multiple. Let $\mathbf{H}_{tot}^\perp$ be $M.m \times (M.m - 1)$ of rank $M.m - 1$ such that $\mathbf{H}_{tot}^\perp \mathbf{H}_{tot}(0) = 0$, then

$$\mathbf{F}_{\underline{K}+1}^b = \mathbf{H}_{tot}^{\perp H} \mathbf{P}_{\underline{K}} \quad (28)$$

is a set of $M.m - 1$ blocking equalisers, since $\mathbf{F}^b \underline{\mathbf{Z}} = 0$. Due to the commutativity of convolution,

$$\mathbf{F}_K^b T_K(\mathbf{H}_{tot}) = 0 \Leftrightarrow \mathbf{H}_{tot}^t T_N(\mathbf{F}_L^{bt}) \quad (29)$$

Now

$$\dim(\text{Range}^\perp\{T_N(\mathbf{F}_{\underline{K}+1}^{bt})\}) = 1 \quad (30)$$

so that we identify the channel $\mathbf{H}_{tot}^{tH}$ as the last right singular vector of $T_N(\mathbf{F}_{\underline{K}+1}^{bt})$.

6 Computational aspects

It is obvious that the correlation matrix $\underline{\mathbf{R}}$ built from the cyclic correlations is bigger (in fact each scalar in $\mathbf{R}$ is replaced by a $m \times m$ block in $\underline{\mathbf{R}}$) than the corresponding matrix built from the classical time series representation of oversampled stationary signals. This fact must be balanced with the stronger structure that is cast in our correlation matrix. In fact, one can show that the estimates $\hat{\mathbf{H}}_N^{\{-k\}}$ are strictly related (i.e. $\hat{\mathbf{H}}_N^{\{-k\}} = [w^{-kj}\hat{\mathbf{h}}(j)]_{j=0}^{N-1}$ for all k), which indicates that this structure should lead to reduced complexity algorithms w.r.t. the original ones. When developing the expressions in detail this is particularly obvious in linear prediction, where the prediction filter has some strong structure, which is also visible in [14]. Moreover, as noted in [13], the multichannel linear prediction prob-

lems correspond to a block triangular factorisation and to an orthogonalisation of the block components of the vector $\underline{\mathbf{Z}}$.

Returning to the original channel model, we alternatively introduce sequential processing and orthogonalise the elements of the vector $\mathbf{X} = [\mathbf{x}(n) \ldots \mathbf{x}(n+K)]$ scalar component by scalar component. This leads to the cyclic prediction filters whose explicit relations to the multivariate predictions filters are known, and results from a true (nonblock) triangular factorisation.

7 Simulations

In our simulations we restrict ourselves to the $p = 1$ case, using a randomiy generated real channel of length $6T$, an oversampling factor of $m = 3$ and $M = 3$ antennas. We draw the NRMSE of the channel, defined as

$$MRMSE = \sqrt{\frac{1}{100}\sum_{t=1}^{100} \|\hat{\mathbf{h}}^{(l)} - \mathbf{h}\|_F^2 / \|\mathbf{h}\|_F^2} \quad (31)$$

where $\hat{\mathbf{h}}^{(l)}$ is the estimated channel in the lth trial. The correlation matrix is calculated from a burst of 100 QAM-4 symbols (if we used real sources we would have used the conjugate cyclocorrelation, which is another means of getting rid of the noise, provided it is circular). For these simulations we used 100 Monte-Carlo runs.

7.1 Subspace fitting

The estimations of 25 realisations for an SNR of 20dB are reproduced in Fig. 5.

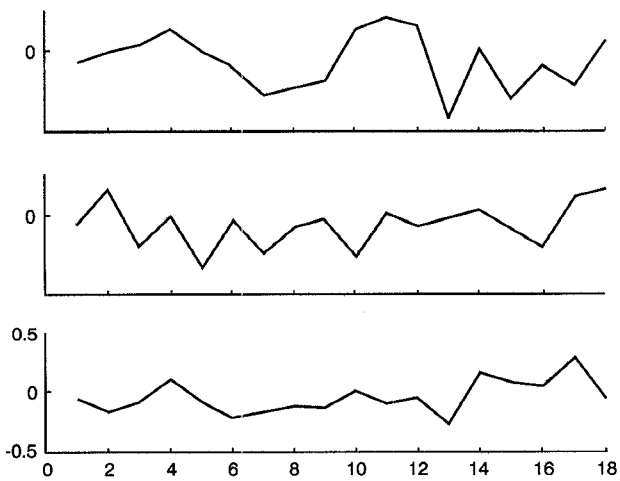

Fig. 5 *Simple channel estimate*

For comparison we used the same algorithm for the classical time series representation of the oversampled signal. The results, Fig. 6, show a better performance for the classic approach due to the fact that we used the same complexity for both algorithms (same matrix size), which results in a lower noise subspace size for the cyclic approach. In theory, when one uses the same subspace size, as there is a one-to-one correspondence between the elements of the classic correlation matrix and the elements of the cyclic correlation matrix, the performances should be equal. The third curve illustrates this.

In more realistic cases, when one uses a channel model where the transmitter and receiver filters are incorporated, the cyclic approach gives better results. This is mostly due to the fact that one can better refine

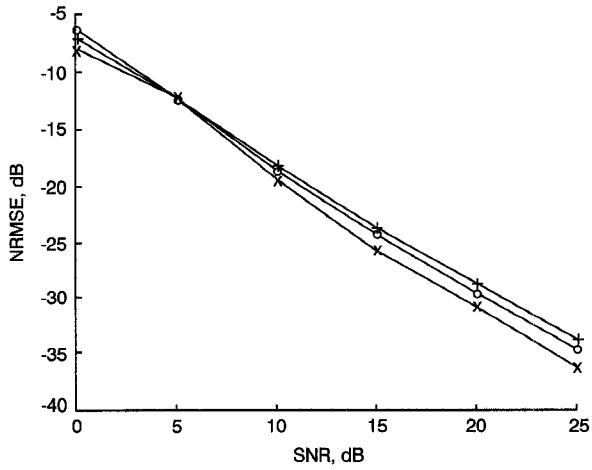

Fig. 6 *Subspace fitting estimation error*
+ cyclic correlation
O classic correlation
× cyclic correlation, increased complexity

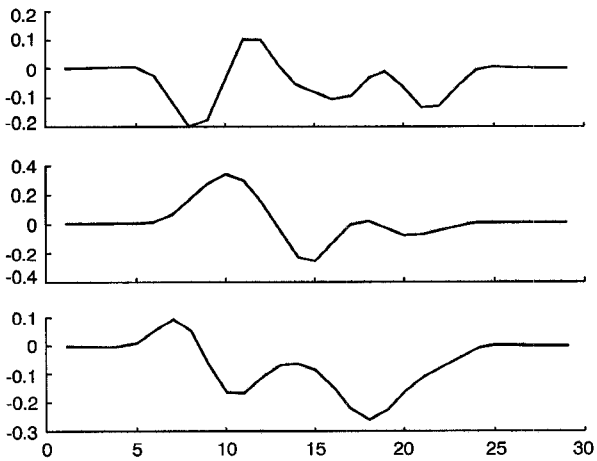

Fig. 7 *Combined channel and transmission/reception filter*

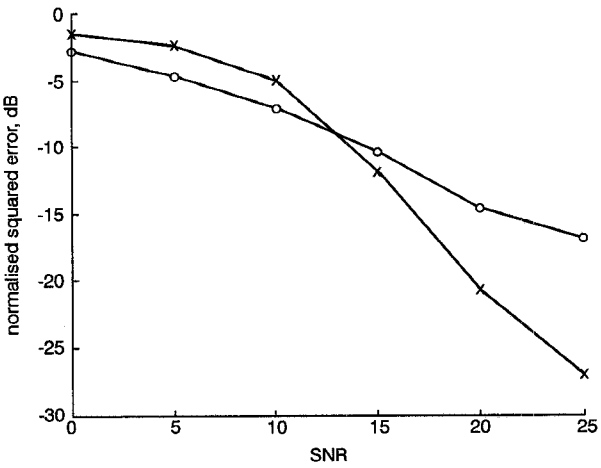

Fig. 8 *Subspace fitting estimation error*
O stationary statistics
× cyclic statistics

the channel-length estimate (and should be preceded by a good channel-length estimation algorithm). Indeed, if we use a channel of the form ($M = 2$ and $m = 2$)

$$\begin{pmatrix} \varepsilon & * & * & * & * & * & \varepsilon \\ \varepsilon & * & * & * & * & * & \varepsilon \end{pmatrix} \quad (32)$$

where ε is a near zero value, the cyclocorrelation approach can afford to restrict to the central part of the channel, but the classical approach will try to find the $M.m$ multichannel

$$\begin{pmatrix} \varepsilon & * & * \\ * & * & * \\ \varepsilon & * & * \\ * & * & * \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} * & * & * \\ * & * & \varepsilon \\ * & * & * \\ * & * & \varepsilon \end{pmatrix} \tag{33}$$

with two more (near zero) parameters to estimate, which will globally give a worse estimation. Fig. 8 illustrates, for moderate SNR, the performance enhancement for a $5T$ channel combined to a 90% excess bandwidth raised-cosine filter (we continue to use $M = 3$ and $m = 3$).

7.2 Linear prediction

For the linear prediction we expect to have a slightly better performance in the cyclic approach than in the classic approach. Indeed, in the classic approach, if we use for example an $M = 1$ antenna and an oversampling factor of $m = 3$, we predict $[x(n)\ x(n-1)\ x(n-2)]^T$ based on $[x(n-3)\ x(n-4)\ \ldots\ ]^T$, whereas in the cyclic approach we predict the scalar $x(n)$ based on $[x(n-1)\ x(n-2)\ x(n-3)\ \ldots\ ]^T$. The corresponding prediction filter thus captures little more prediction features in the cyclic case.

On the other hand, the noise contribution being only present in the zero cyclic frequency cyclic correlation (see eqn. 5), we expect a better behaviour of the method if we do not take noise into account in the correlation matrix (i.e. do not estimate the noise variance before doing the linear prediction). Those expectations are confirmed by the simulations shown in Fig. 9 (The label 'LP on cyclic statistics' refers to the use of $\underline{\mathbf{R}}$ where the noise contribution has been removed, whereas 'LP on cyclic statistics, no denoising' refers to the use of the plain correlation matrix).

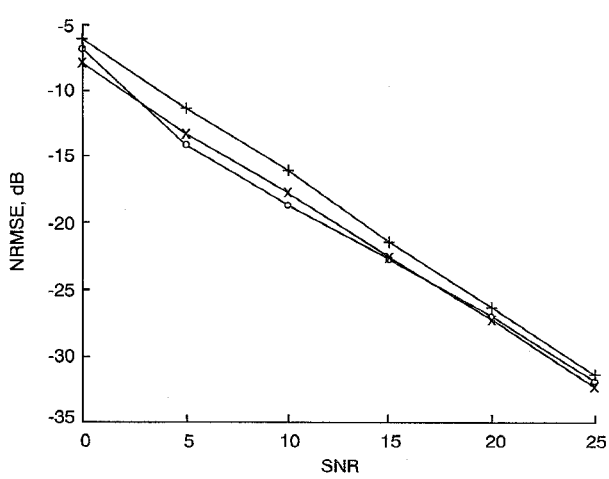

Fig. 9 *Linear prediction estimation error*
+ LP on stationary statistics
× LP on cyclic statistics
○ LP on cyclic statistics, no denoising

8 Conclusions

Using the stationary multivariate representation introduced in [3, 5, 14], we have explicitly expressed this process. It can be seen as the output of a system with transfer channel $\mathbf{H}_{tot} = [\mathbf{H}_N^{\{0\}T}\ \mathbf{H}_N^{\{1\}T}\ \ldots\ \mathbf{H}_N^{\{1-m\}T}]$, and input easily related to the actual system input. Once these quantities are expressed, application of the classical subspace fitting and linear prediction algorithms is straightforward.

For the subspace fitting, one has essentially the same performance as in the time series representation [11]. The only advantage one could expect is some refinement in the channel order estimation prior to the subspace fitting. The main drawback is the increase of the computational burden.

For the linear prediction, there is a better performance owing to the fact that we take the very near past into account. What is more, use of modular multichannel linear prediction algorithms such as those described in [15] provide fast algorithms and adaptive implementations.

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