Generation of mono-site Rainfall time series from micro to large scale

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Context

- Rainfall is a very complex naturally occurring phenomenon.
- It is highly variable in time and space especially for high resolutions.
- Complexity of the modelling of rainfall is related on some characteristics namely intermittency, rain extremes, high rain rate variability and multiple scaling regimes.
- Modeling rainfall at high resolution influence number of areas (hydrology, meteorology, ...)
- It is important to have high resolution data.
Two approaches are used to model precipitations:

**Top-down:**
- availability of data at coarse resolution
- Simulate data at coarse resolution
- Downscaling

**Bottom-up:**
- Lack of data
- Generate data at high resolution (at least 5 minutes in existing models)
- Aggregation

Our work:
- Generate intra rain events (parameters observed at high scale)
- Generate rain support and restore its characteristic at high resolution

Simulate high resolution long time-series of rainfall
Objective 1/2 : The basic idea

• generating rainfall time-series with a very high temporal resolution:
  – mono-site (Palaiseau France)
  – Spatial resolution (sensor capture : 100cm²).
  – temporal resolution (15 seconds)

• Rain properties are checked from small to large scales (for several weeks / months).
Objective 2/2: more specifically

- Realistic statistical properties of rainfall and support:
  - Occurrence frequency rain/no rain durations (intermittency)
  - Fractal dimension of rain support
  - Power spectrum,
  - (Rain rate) scale invariance properties
  - (Extreme rain) Non-Gaussian tail distribution

- Realistic CDF of rain rate and rain durations at different resolution (15 sec, 5 min, 1 hour)
Plan

• **Data set**
  • Data set properties

• **Modeling (generator)**
  • Step 1: Generate rain support
  • Step 2: Generate rain rate (using multifractal model)
  • Step 3: Calibrate/forcing rain rate

• **Validation**

• **Conclusion and perspectives**
Data set (What are we trying to model?)

- Dual-Beam Spectropluviometer (DBS)
  - Time series resolution is 15s
  - The existing literature does not provide a model for this order of scale
- Rain rate measurements in Palaiseau (France) from 01 July 2008 to 31 December 2009.
Data set properties: Power spectrum (scale invariance properties)
Data set properties: Observed rain support (precipitation occurrence)

• Intermittence rain/no rain (rain support) at resolution $\lambda$

$$I_\lambda(x) = \begin{cases} 0 & \text{si} \quad R_\lambda(x) = 0 \\ 1 & \text{si} \quad R_\lambda(x) > 0 \end{cases}$$

• Support properties: Support is fractal if:

$$\Pr(I(x) = 1) \propto \lambda^{-C_f}$$

is satisfied.

![Graphs showing rain durations and no rain durations with thresholds for $d_r < 5\text{min}$, $d_r > 5\text{min}$, $d_{nr} < 5\text{min}$, and $d_{nr} > 5\text{min}$]
Rainfall generator

**Step 1**
Generate rain support

**Step 2**
Generate rain events using FIF model

**Step 3**
Calibrate mean rain rate per event
Step 1: generate rain support

• Alternating rain ($d_r$)/no rain ($d_{nr}$) duration

- Short: $d_r < 5\text{ min}$
- Long: $d_r > 5\text{ min}$

- Generalized pareto distribution is used to model duration short/long rain/no rain durations:

$$f(d/k, \sigma, \theta) = \left(\frac{1}{\sigma}\right) \left(1 + k \frac{(d - \theta)}{\sigma}\right)^{-1 - \frac{1}{k}}$$

• Three parameters are estimated on each short/long rain/no rain durations $\Rightarrow 12$ parameters
Step1: generate rain support

- Percentage of rain in observed time series: 4.46%
- Mean percentage of rain in 100 simulated series: 4.11%
Rainfall generator

Step 1: Generate rain support

Step 2: Generate rain events using FIF model

Step 3: Calibrate mean rain rate per event
Step 2: Generate rain events

• Models and techniques for simulation
  • Poisson processes...
  • Geostatistique techniques...
  • Multifractal techniques ...

➤ Multifractal
  ✓ Multiplicative cascades
  ✓ Scale invariance properties.
  ✓ Non-conservativity
Theory: Multifractal field: properties 1/2

- Multifractal field $\phi$ is characterized by a fractal codimension function $c(\gamma)$ which extends the usual notion of topological dimension for non-integer values.

$$\text{Pr}(\phi_\lambda > \lambda^\gamma) \propto \lambda^{-c(\gamma)}$$

- Moment scaling function characterizes the statistical properties at different resolutions.

$$E(\phi_\lambda^q) \propto \lambda^{K(q)}$$
Theory: Multifractal field 2/2: Multiplicative cascades

- Construction of multifractal fields by a self-similar, stochastic iterative procedure
  \[ \Phi_{\lambda_n} = \Phi_{\lambda_{n_0}} \cdot \prod_{i=1}^{n} \mu \Phi \]
  → Convergence to multifractal fields
  → Scale invariance properties

- Universal multifractal (UM) parameters:
  - \( \alpha \) (multifractality, < 2)
  - \( C_1 \) (homogeneity)

- **FIF (integrated UM model) parameters**:
  - \( \alpha \) & \( C_1 \)
  - \( H \) (no conservative field)

Graph from Schertzer & Lovejoy (2002)
Step 2: Multifractal analysis of observed data

Estimation of parameters 1/2:

- moment scaling function: \( K(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q) \)

For each event:

Histogram of 1200 events extracted from observed data

\( \Rightarrow \) Retained FIF parameters: \( \alpha = 1.6 \quad C_1 = 0.10 \)
Step 2: Multifractal analysis of observed data

Estimation of parameters 2/2:

- **Structure function**

  \[ \langle |R_\lambda(\Delta t)| \rangle \sim \lambda^H \]

- **H parameter** is estimated on all observed events

  ➔ Retained FIF parameter \( H = 0.42 \)
Rainfall generator

Step 1: Generate rain support

Step 2: Generate rain events using FIF model

Step 3: Calibrate mean rain rate per event
Two classes are considered (less or more than 5 minutes)
• $\alpha$-stable distributions are used to model mean rain rates corresponding to both short and long durations.

$\Rightarrow$ 4 parameters.
Step3: calibration

Events rain rate/duration relationship 2/2

- $\alpha$-stable distribution parameters
  - Location parameter (fixed) $\delta = 0$
  - Asymmetry Parameter (fixed) $\beta = 1$
  - Stability and scale parameters $\alpha_r = 0.78$, $\gamma = 0.17$ long events
  - and $\alpha_r = 0.91$, $\gamma = 0.01$ are estimated from short events
Step 3: calibration

Multifractal analysis (example of 2 years simulated series)
Rainfall generator

1. Generate rain support using Generalized Pareto distribution
   - Intermitency
   - Rain/no rain durations probability distribution
   - Support co-dimension

2. Generate rain events using FIF model
   - Scale invariance properties

3. Calibrate mean rain rate per event using $\alpha$-stable distribution
   - Mean rain rate distribution

Simulate 100 two-year time series at 15s resolution
validated properties

It remains to verify the global properties of the complete series:

- Power spectrum
- Rain rate distribution checked from small to large scales.
Rain rate power spectrum (100 simulated series)
Survival function (100 simulated series)
Survival function (100 simulated series)
Conclusion:

Our model simulates yearly time series with a 15s resolution in agreement with the observed data, and it is able to restore various aspect (intensity, support, variability ...):

Support properties

Events

Scale invariance properties
+ mean rain rate

Global properties

✓ Simulated & observed power spectrum show the same regime of scaling.
✓ From small to large scale:
   ✓ Simulated & observed rain rate distribution are coherent
   ✓ Simulated & observed rain durations distribution are coherent
Perspectives

- Generalization to other climatic area
  - Study the variability of mean rain rate / duration relationship parameters on other data sets.
  - 10 parameters are needed to adapt the simulator to a particular climatic area.
  - Improve the modeling (parametric method)

- Include the concept of seasonality in the simulation process
  - Require longer observed time series.

- Extension to 2D rain maps (work in progress)
Thank you for your attention
Extension to 2D: work in progress

- The used dataset are collected for 6 years with a 5 minutes temporal resolution and 1km×1km spatial resolution.
- Rain support is simulated using a Sequential Indicator Simulation (SIS) algorithm.
- Calibration of rain rate events is based on the relationship between the mean rain rate of events and theirs surfaces.