

Optimizing microalgal production in raceway systems

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Abstract

The industrial exploitation of microalgae is characterized by the production of high value compounds. Optimization of the performance of microalgae culture systems is essential to render the process economically viable. For raceway systems, the task of optimization is rather challenging since the process is by essence periodically forced and, as a consequence, optimization must be carried out in a periodic framework. In this paper, we propose a simple operational criterion for raceway systems that when integrated in a strategy of closed-loop control allows to attain biomass productivities very near to the maximal productivities. The strategy developed was tested numerically by using a mathematical model of microalgae growth in raceways. The model takes into account the dynamics of environmental variables temperature and light intensity and their influence on microalgae growth.

Keywords: control, microalgae, modeling, optimization, photobioreactors, raceway

1. Introduction

Microalgae assimilate inorganic carbon through photosynthesis, a process that takes place in two phases, namely light-dependent stage (light reactions) and light-independent stage (dark reactions). The second phase comprises a series of reactions catalyzed by the enzyme ribulose biphosphate carboxylase oxygenase (RuBisCO). In this phase, CO₂ is incorporated into organic material, leading to the formation of the carbon building blocks that are further synthesized into carbohydrates, proteins, lipids and nucleic acids [38].

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8 Due to its biochemical properties, microalgae have been raised as promising feed-
9 stocks for the production of high value compounds. The commercial use of microalgae
10 includes applications in food industry and cosmetics [35]. Moreover, microalgae have been
11 identified as a renewable source for biodiesel production [5, 18]. However, despite these
12 favorable characteristics, microalgae production in a sustained and large scale basis is
13 probably carried out far from an optimal working mode. Here, we mainly refer to the
14 technology of raceways (high rate open ponds), which are nowadays the systems generally
15 used for large microalgae production.

16 The difficulty of achieving optimal productivities of microalgae in outdoor systems
17 results from the high interaction of phenomena that take place during growth and the low
18 level of control that we have on them. This factor makes the whole process inefficient under
19 an environment that is fluctuating by nature. The challenge of optimizing microalgal
20 culture systems is a broad endeavour that includes reactor design [24, 32] and strain
21 selection [33]. Furthermore, once the reactor configuration and the microalgal strain have
22 been chosen, optimal performances can be achieved by acting on operational variables,
23 such as temperature, pH and nutrient feeding rate. In this aspect, mathematical models
24 offer a powerful tool to be exploited. Indeed, optimization can be model driven. Since
25 microalgal metabolism is mainly influenced by nutrient availability, light intensity and
26 temperature, several models have been developed to account for these factors [8, 14, 15,
27 29, 26, 3]. A work of synthesis has been performed to provide a mathematical model that
28 incorporates the main factors that influences microalgal dynamics by keeping a relative
29 simple structure [2] that might be suitable for control processes.

30 The task of bringing a process close to optimality by acting on the inputs of the system
31 is the realm of optimal control. The optimal control problem that we are considering
32 consists in finding the time evolution of the manipulated variables maximizing a given
33 criterion on a finite time horizon. This problem can be solved by indirect methods such
34 as the Pontryagin's maximum principle or by direct methods (numerical optimization).
35 The advantage of using Pontryagin's maximum principle is that of providing an analytic
36 solution. In this respect, a theoretical study on a simplified model of microalgae growth
37 provided guidelines on the form of the controller to reach an optimal productivity [16].

38 This study was the first dedicated to optimal control of microalgae in a periodically
39 varying environment. A series of simplifications were needed to provide a very simple
40 model suitable for applying the Pontryagin’s maximum principle. The distance between
41 the proposed optimal strategy and the optimal strategy for a more realistic model is
42 therefore difficult to assess.

43 When a mathematical model is complex, the application of the Pontryagin’s maxi-
44 mum principle is not straightforward. Hence, methods based on numerical optimization
45 are, in practice, the most used. In the standard form, the numerical approach takes
46 place in open loop fashion, without taking into account the real state of the system. For
47 a real implementation, however, available measurements (either online or offline) must
48 be used to compensate for disturbances and to correct model mismatches (see, *e.g.*, [4]
49 for a survey of methods). An example of this type of strategy is the predictive con-
50 trol approach, in which the optimal control problem is solved online. This strategy was
51 developed for the optimization of biomass and oil productivities for the heterotrophic
52 microalgae *Auxenochlorella protothecoides* [11]. This approach, however, is computa-
53 tionally expensive and requires sophisticated algorithms of adaptation and proof of process
54 stability is lacking.

55 A practical alternative for optimizing system performance is to translate the optimiza-
56 tion problem into a regulatory (tracking) problem. The objective then consists in finding
57 a variable (or a combination of variables) that when regulated maintains the system close
58 to optimality [34]. In the case of photobioreactors, the phenomenon of light transfer
59 to the culture governs the performance of the system [28]. Based on this principle, it
60 has been proposed that under constant light, maximal productivities can be attained by
61 maintaining the light intensity at the bottom of the reactor at a constant value. This
62 value corresponds to the compensation irradiance (G_c), defined as the minimum value
63 of light intensity required to guarantee a positive net growth rate (strict compensation
64 condition) [7, 6, 37]. By defining the working illuminated fraction χ as the fraction of
65 the reactor volume with light intensities higher than the compensation irradiance, it has
66 been demonstrated mathematically that an optimal biomass productivity requires the
67 condition $\chi = 1$.

68 For outdoor raceways, the definition of a strategy to guarantee maximal productivities
69 is not trivial due to the diurnal light cycle. It has been conjectured that the compensation
70 point should also be reached for natural light in order to maximize productivity [25].
71 Inspired on this premise, an experimental study with *Chlorella sorokiniana* was carried
72 out on an artificial lightened photobioreactor with planar geometry mimicking the daily
73 cycle of light [9]. The experimental set-up was conceived to maintain the photon flux
74 density leaving the reactor at a constant value. The luminostat operation did not exhibit
75 significant improvements of productivity compared to an operation at constant dilution
76 rate. From these results it can be drawn that keeping a constant light intensity at the rear
77 of the reactor might not be an optimal strategy for varying light conditions. Accordingly,
78 an imminent question is thus how to attain maximal productivities when light is varying.
79 In the present work, we proposed a simple operational criterion which when regulated
80 to an adequate set point maintains the system near to optimal operation. The proposed
81 strategy has the advantage to be straightforward to implement in a classical closed loop
82 control.

83 As a basis, we use the model proposed by [2] for a planar culturing device in combi-
84 nation to a model describing lipid production under nitrogen limitation [19, 20]. These
85 models have shown to reproduce experimental data of lab scale systems. Here, we extend
86 such models to account for characteristics of raceway systems. Our *in silico* case study
87 takes the configuration of a pilot-scale open raceway (Algotron) located at INRA-LBE,
88 France.

89 **2. Modeling**

90 Under the assumption that nitrogen and light are the limiting factors for the growth
91 of microalgae, we combined the biomass model from [2] to the lipid production model
92 proposed in [19, 20]. It results in the following mass balance equations for a completely
93 mixed reactor at constant volume V

$$\dot{s} = f_i s_{in}/V - f_o s/V - \rho x, \quad (1)$$

$$\dot{q}_n = \rho - (\mu - R)q_n, \quad (2)$$

$$\dot{x} = (\mu - f_o/V - R)x, \quad (3)$$

$$\dot{x}_l = \beta q_n \mu x - \gamma \rho x - r_0 \phi_T x_l - f_o x_l/V, \quad (4)$$

$$\dot{x}_f = (\alpha + \gamma) \rho x - r_0 \phi_T x_f - f_o x_f/V, \quad (5)$$

94 where s is the extracellular nitrogen concentration and q_n is the internal nitrogen quota.
 95 The concentration of the total carbon biomass x is the sum of three carbon pools, namely
 96 storage lipids (x_l), carbohydrates (x_g) and a functional pool (x_f), mainly formed by
 97 proteins and phospholipids. Note that the dynamics of x_g can be easily deduced since
 98 $x_g = x - (x_l + x_f)$. The influent nitrogen concentration is s_{in} .

99 The temperature exerts a strong influence on the behaviour of microalgae systems, in
 100 particular in outdoor raceways [27]. This effect is included in the model in two manners.
 101 Firstly, it is assumed, in line with [14], that temperature has an homogeneous effect on
 102 uptake, growth and respiration rates. Secondly, following the work of [13], the Chl a:N
 103 ratio was set to be dependent on the temperature and light. The equations are detailed
 104 later on.

105 To model the growth rate, the following is assumed:

106 (i) Microalgal growth is uncoupled dynamically to nutrient uptake. Growth kinetics
 107 follows the cell quota model of Droop [12].

108 (ii) Light intensity is distributed spatially in the raceway. The absorption of light in the
 109 raceway follows the Lambert-Beer law. Thus, for a given depth z , the corresponding
 110 light intensity I_z satisfies

$$I_z = I_0 \exp(-\xi z), \quad (6)$$

111 where I_0 is the incident light and ξ is the light attenuation factor, expressed as

$$\xi = a \text{Chl} + b. \quad (7)$$

112 At the bottom of the reactor $z = L$. The term ξL is known as optical depth (λ).

113 It should be noted that I_0 varies in time in an oscillatory fashion. Its amplitude
 114 depends on the season and the geographical location. For a given day, I_0 follows an
 115 increasing behaviour until noon, then decreases until midnight.

116 (iii) Light intensity affects the growth rate. This effect is described by a Monod type
 117 kinetics. For a given depth z ($0 \leq z \leq L$) with intensity I_z , the growth rate at
 118 hypothetical infinite nitrogen quota is

$$\mu_z = \tilde{\mu} \frac{I_z}{I_z + K_{sI}}. \quad (8)$$

119 Finally, the growth rate is represented by an average growth rate obtained by in-
 120 tegration of (8) along the raceway depth. The resulting equation for the growth rate
 121 reads

$$\mu = \bar{\mu} \phi_T \left(1 - \frac{Q_0}{q_n} \right), \quad (9)$$

with

$$\bar{\mu} = \frac{\tilde{\mu}}{\xi L} \ln \frac{I_0 + K_{sI}}{I_0 e^{-\xi L} + K_{sI}},$$

$$\phi_T = \frac{(T - T_{\max})(T - T_{\min})^2}{(T_{\text{opt}} - T_{\min}) [(T_{\text{opt}} - T_{\min})(T - T_{\text{opt}}) - (T_{\text{opt}} - T_{\max})(T_{\text{opt}} + T_{\min} - 2T)]}.$$

122

123 The term ϕ_T represents the temperature effect. It is described by the model developed
 124 for bacteria by [30] and validated for microalgae by [3].

125 Nitrogen uptake rate (ρ) is modeled by a modified Michaelis-Menten kinetics [19].

$$\rho = \bar{\rho} \phi_T \frac{s}{s + K_s} \left(\nu + (1 - \nu) \frac{\bar{I}^m}{\bar{I}^m + \epsilon_I^m} \right) \left(1 - \frac{q_n}{Q_l} \right). \quad (10)$$

126 The nitrogen uptake rate is expressed as a function of the average irradiance in the raceway
 127 \bar{I} . Here, it is considered that nutrient uptake is regulated by the internal nitrogen quota,
 128 *i.e.*, when the cells are nutrient saturated, uptake rate stops. Additionally, the equation
 129 includes a light regulating factor (in the form of a Hill-type function). Therefore, when
 130 the cells enter to the dark period, the nutrient uptake rate exhibits a slowdown.

131 The model includes an overall respiration rate R , that gathers maintenance respiration
 132 and biosynthesis cost (assumed to be proportional to nitrogen uptake rate):

$$R = r_0\phi_T + \varphi\rho, \quad (11)$$

133 where r_0 is the maintenance respiration and φ is a biosynthesis cost coefficient. Model
 134 notation is given in Table 1.

135 Additionally, it is assumed that chlorophyll concentration (Chl) is correlated to par-
 136 ticulate nitrogen (xq_n) [2]. The Chl:N ratio (θ_N) is influenced by light and temperature
 137 following [13]

$$\theta_N^{-1} = (g_1 - g_2T) + g_3\bar{I}\exp(-g_4T). \quad (12)$$

138 In this equation, it is implicitly assumed that the cells are photoacclimated at the average
 139 light intensity \bar{I} .

140 Environmental variables, notably light intensity (solar irradiance) and temperature
 141 govern reactor performance. These two variables incorporated in the kinetics of growth
 142 and nitrogen uptake can be accessible from online sensors or meteorological stations.
 143 In addition, mathematical models have been developed to predict light intensity [23] and
 144 raceway temperature [1] for a given location. In the present study, mathematical modeling
 145 supported by meteorological data was used for the location of Narbonne, France (see
 146 Fig. 1).

147 Model parameters were taken from studies on the microalgae *Isochrysis* aff. *galbana*,
 148 when available. The parameters describing ϕ_T are those obtained for *Nannochloropsis*
 149 *oceanica* [3].

150 3. Driving raceway operation to optimal performance

151 3.1. Optimal problem statement

152 In this study, we are interested in designing a control law on the input flow rate (f_i)
 153 that allows to bring either the biomass productivity (P_x) or the lipid productivity (P_l)
 154 very close to the maximal productivities that can be attained in the raceway. For a given
 155 time horizon t_f , the maximal productivities can be obtained by solving an optimal control
 156 problem that can be formulated as follows

$$\begin{aligned}
& \max_{f_i(t)} \int_{t_0}^{t_f} \psi(t, \mathbf{x}(t), f_i(t)) dt. \\
& \text{s.t.} \\
& 0 \leq f_i(t) \leq f_{\max} \\
& \dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, f_i, t), \quad \mathbf{x}(0) = \mathbf{x}_0.
\end{aligned} \tag{13}$$

157 With \mathbf{x} the state vector and f_{\max} the upper bound of the input flow rate. If the
158 purpose of the controller is to optimize biomass productivity P_x , then

$$\psi(t, \mathbf{x}(t), f_i(t)) = f_o(t) x(t). \tag{14}$$

159 If the objective is to optimize lipid productivity P_l , then

$$\psi(t, \mathbf{x}(t), f_i(t)) = f_o(t) x_l(t). \tag{15}$$

160 For the sake of clarity, we will call CP_x the optimal controller that maximizes biomass
161 productivity and CP_l the optimal controller that maximizes lipid productivity.

162 The model equations (1)-(5) were used for the optimization study. The influent nitro-
163 gen concentration s_{in} was set to 50 g N m⁻³. This value is an operational concentration
164 used in Algotron. The volume was assumed to be constant, so $f_o = f_i$.

165

166 The Matlab toolbox DOTcvsSB [17] was used for solving the optimal control prob-
167 lems numerically. DOTcvsSB uses the approach of sequential discretization (control
168 vector parametrization) to solve the non-linear programming (NLP) problem. In the
169 optimization stage, the stochastic algorithms developed by [31] and [36] were used.

170 3.2. Quasi optimal closed loop control

171 As it was mentioned in the Introduction section, solving the optimal control problem
172 (13) (with the functional objective defined by (14) or (15)) might be computationally
173 expensive and difficult to implement in practice. For a real implementation, it will be
174 desirable to identify a controlled variable that when regulated towards a set point will
175 ensure that the system operates close to optimality. In this respect and since light transfer

176 is a crucial phenomenon of the process of microalgal growth, we propose the efficiency of
177 light absorption (η_L) to be such a controlled variable.

$$\eta_L = \frac{I_0 - I_L}{I_0} = 1 - \exp(-\xi L). \quad (16)$$

178 As it will be shown hereafter, this simple controller has a very good ability to maintain
179 the system close to the optimal solution. In fact, several strategies were tested based on
180 preliminary studies, and η_L turned out to present the best trade-off between simplicity
181 and efficiency.

182 For a given microalgae, there exists a set point η_L^* that maintains the system near to
183 optimal productivities. The value η_L^* depends on the characteristics of the microalgae,
184 namely optical properties and light affinity. In this study a set point $\eta_L^* = 0.95$ was
185 selected. Note that regulating η_L implies the regulation of the optical depth λ . Given the
186 form of the attenuation factor (7), regulating the optical depth is equivalent to regulating
187 the Chlorophyll concentration. For $\eta_L^* = 0.95$, the set point for Chlorophyll concentration
188 is $\text{Chl}^* = 4.95 \text{ g Chl m}^{-3}$. This result is very convenient because during darkness the
189 efficiency of light absorption can not be defined but we can still regulate the Chlorophyll
190 concentration to Chl^* in such a way that when $I_0 > 0$ the efficiency of light absorption
191 will be close to η_L^* .

192 In the following, we show by means of numerical simulation, the performance of the
193 raceway by regulating η_L to the set point η_L^* . This regulation can be achieved by any
194 adequate feedback controller. In this work, we use a standard PI controller. Since our
195 premise is that this controller brings the system to work almost optimally, we call it a
196 quasi optimal (QO) controller.

197 4. Results

198 4.1. Comparison of the QO controller to optimal strategy

199 Figure 2 displays the responses of the state variables and the lipid and biomass pro-
200 ductivities (P_l , P_x) when applying the QO controller and the optimal CP_x controller for
201 a time period of 30 days. To calculate the productivities, it was assumed that carbon
202 contributes to the 56% of ash-free dry weight [38]. The productivities are divided by the

203 surface of the raceway and the time. The QO control controller brings the efficiency of
204 light absorption very close to the defined set point. At $t = 6.5$ d, η_L is 95% of η_L^* . The
205 maximal biomass productivity obtained with the optimal controller CP_x is 168 tons dry
206 weight $\text{ha}^{-1} \text{a}^{-1}$. This value is consistent with productivities reported in the literature
207 [5, 38]. Importantly, the biomass and lipid productivity provided by the QO controller
208 were both 98% of those given by the controller CP_x (Table 2). After 25 d, the final lipid
209 quota ($q_l = x_l/x$) oscillates with a maximal value of 16%. This relative low level of lipids
210 is due to the fact that many of the parameters used in the model were taken from studies
211 with *I. galbana*, which is known to have a low lipid content.

212 To have an assessment of the maximal lipid productivity that can be attained, the
213 optimal controller CP_l was calculated. For the model parameters used in our case study,
214 it resulted that optimizing lipid productivity was equivalent to optimizing biomass pro-
215 ductivity. Thus, the response of the system behaviour when applying the CP_l controller
216 was very similar to the response obtained when applying the CP_x controller. This result
217 is interesting because it is often claimed the conflict between optimizing lipid productivity
218 and optimizing biomass productivity. Indeed, such a conflict occurs when light is con-
219 stant. Figure 3 shows the system response for the optimization of the two performance
220 indexes (P_x, P_l) when the system was set to operate at constant temperature (21°C) and
221 constant incident light ($250 \mu\text{mol photons m}^{-2}\text{s}^{-1}$). It is observed that while the optimal
222 strategy for biomass productivity provides a higher biomass concentration, the optimal
223 strategy for lipid productivity drives the microalgae to increase its lipid content, which
224 in turn is detrimental for attaining high concentration of biomass. While the difference
225 between the biomass productivities obtained by the two controllers is very small, the CP_l
226 controller provides a lipid productivity that is 10% higher than that obtained with the
227 CP_x controller. These differences may become more important for microalgae with high
228 potential of lipid accumulation.

229 For a diurnal light cycle, however, our results suggest that there is not discrepancy
230 between optimizing lipid productivity and optimizing biomass productivity. For both
231 performance indexes, the cells should growth as much as they can in the light period
232 to accumulate enough carbon. The higher the biomass the concentration, the higher

233 the available carbon source that can be potentially directed to the lipid pool. To as-
234 sess whether our indication was independent on the properties of lipid accumulation of
235 the microalgae, the coefficients of fatty acid synthesis (β) and fatty acid mobilization
236 (γ) were modified to represent a microalgae with high lipid level ($\approx 40\%$ of the total
237 carbon). The optimal controllers CP_x and CP_l were further calculated with the new pa-
238 rameters. The system dynamics were specific to each applied controller CP_x and CP_l , *e.g.*,
239 the biomass concentration for the CP_x exhibited a different dynamics that the biomass
240 concentration provided by the CP_l . However, such differences were not significant and
241 both controllers provided similar biomass and lipid productivities. Hence, we confirm the
242 previous suggestion that optimizing lipid productivity is almost equivalent to optimizing
243 biomass productivity for a photobioreactor with diurnal light cycle.

244 The results presented here are very promising. We show that the QO controller per-
245 forms as well as the optimal controllers, confirming our hypothesis that controlling the
246 efficiency of light absorption (η_L) makes it possible to attain high productivities both in
247 lipid and biomass. The response of the QO controller suggests that an optimal strategy
248 consists in driving the biomass concentration to a certain value and to allow it oscillate
249 around this point. This result is consistent with the work developed by [6] and the the-
250 oretical results presented by [16], where an optimal controller was developed by forcing
251 the biomass concentration to fulfill a periodicity condition.

252 4.2. Comparison of open loop configurations to optimal strategy

253 We were interested to assess the performance of the raceway in open loop (OL) config-
254 uration. To this end, the model was simulated initially with an input flow rate $f_i = 5.13$
255 $\text{m}^3 \text{d}^{-1}$ (dilution rate $D = 0.30 \text{d}^{-1}$), which is a typical value [5, 22]. The lipid and biomass
256 productivities were, respectively, 54% and 59% of those obtained with the optimal con-
257 trollers CP_x and CP_l . This means that for an open loop configuration, a wrong choice
258 of the input flow rate will imply an unsatisfactory performance. To perform a fair com-
259 parison, in addition to the optimal controllers previously calculated, the optimal control
260 problem (14) was solved by setting a constant flow rate. The optimal flow rate was found
261 to be $3.22 \text{m}^3 \text{d}^{-1}$ ($D = 0.19 \text{d}^{-1}$) and the resulting biomass productivity was 93% of the

262 one obtained with the optimal controller CP_x . Table 2 summarizes the comparison of the
263 performances of different controllers and configurations evaluated (including the QO con-
264 troller) relative to the optimal productivities. Additionally, Fig. 4 shows the dependency
265 of the biomass and lipid productivities on the dilution rate. The curves are normalized
266 to illustrate that the both productivities reach their maximal value at the same dilution
267 rate.

268 The relative high productivity obtained with the optimal constant flow rate is not
269 surprising, since experimental studies on artificial photobioreactors [9, 10] have shown
270 that with an adequate constant flow rate it is possible to attain high productivities. This
271 result may suggest that, when the microalgae are not nutrient limited, the environmen-
272 tal conditions, namely light intensity and temperature exert such a strong influence on
273 the system behaviour that the improvement of the performance that can be reached by
274 manipulating the dilution rate is only marginal. This finding might, at first sight, dis-
275 courages the endeavour of developing any control strategy for raceways systems, since it
276 appears that even with a constant flow rate, a satisfactory performance can be attained.
277 This result, however, must be taken with caution. Indeed, we argue in favor of the QO
278 controller over the other control strategies and, of course, over open loop operation. The
279 arguments supporting our decision are developed in the following.

280 *4.3. Robustness of optimal strategies and QO controller*

281 The previous results that indicate that an adequate constant flow rate leads to near
282 optimal operation are derived from very well controlled systems (closed photobioreac-
283 tors) or from mathematical models that do not take into account model uncertainty and
284 potential disturbances. Since the QO controller operates in closed loop fashion, it has
285 many practical advantages. For instance, it can be easily tuned for a real scenario that is
286 subjected to disturbances and technical failures. The optimal controllers can also be in
287 closed loop fashion. However, its implementation is more demanding than that of the QO
288 controller. If the optimal controllers are used in open loop, the system can be directed to
289 suboptimal operation. Figure 6 displays the productivities given by the QO controller and
290 the optimal controller CP_x considering uncertainty in the model parameters. The value

291 of $\tilde{\mu}$ was decreased 30% of the value used originally to calculate the optimal controller.
292 It is observed that the QO controller provided a biomass productivity that is 17% higher
293 than that provided by the controller CP_x . It should be noted that this result was achieved
294 with a simple PI controller. We expect that by using a nonlinear controller based on the
295 light efficiency, the productivity might be even better. The design of such a nonlinear
296 controller is one of the perspectives of this work.

297 *4.4. Is the strict compensation condition relevant for diurnal light cycles?*

298 The strict compensation condition (implying that the working illuminated fraction
299 $\chi = 1$) has been identified as a necessary condition to attain maximal productivities in
300 closed photobioreactors under light constant regime [37]. The light intensity at which the
301 compensation occurs is often expressed as a constant parameter. Indeed, if the incident
302 light intensity is constant, there exists one value of light intensity at the bottom of the
303 reactor at which the productivity is maximal [21]. However, we might notice that for a
304 varying light system, the light of compensation depends on the actual state of the system
305 and thus there is not a fixed value that will bring the system to operate under the strict
306 compensation condition.

307 For outdoor raceways, where microalgae are exposed to long periods of darkness,
308 respiration affects negatively growth. It is clear that in the dark period, the compensation
309 condition do not play any role on the reactor performance. When the incident light is
310 higher than zero, the strict compensation condition is such that $\mu_L = R$. The light of
311 compensation is thus a dynamic operational variable that depends of the state of the
312 system. The optical depth of the reactor must then be adjusted accordingly to reach
313 the light of compensation at the rear of the reactor. Note that if the reactor volume is
314 constant, the regulation acts on the attenuation factor ξ . This strategy, however, may
315 suffer of reachability problems, as experienced in the study of [9], where the light at the
316 bottom of the photobioreactor could not be maintained at the defined set point due to the
317 dynamic boundary imposed by the growth rate. To enlarge the discussion in this point,
318 we assess by means of simulations if the strict compensation condition could be fulfilled
319 in a diurnal light cycle and if it is relevant to attain such a condition to achieve maximal

320 productivities. The following optimal control problem was defined

$$\min_{f_i(t)} \int_{t_0}^{t_f} (\mu_L - R)^2 dt. \quad (17)$$

321 The controller optimizing (17) is called CP_c .

322 Figure 5 shows the ratio between the growth rate at the rear of the raceway (μ_L)
323 and the respiration rate (R). The results are given for the optimal controller CP_c . It is
324 observed that, for the light period, the growth rate at the rear of the raceway is higher
325 than the respiration rate and that the compensation condition is not strictly fulfilled all
326 the time $\chi > 1$. The results indicate that attaining the strict compensation throughout
327 the day might be no physically possible due to the dynamic bound imposed by the growth
328 rate.

329 The optimal controller CP_c resulted in biomass and lipid productivities that were,
330 100% of those obtained with the optimal controllers CP_x and CP_l . Our results suggest that
331 the closest the system is to the compensation condition the closest the system operates
332 optimally. However, the results also indicate that for a photobioreactor subject to the
333 diurnal light cycle, the strict compensation condition is not a necessary condition to be
334 fulfilled for achieving maximal productivities. We have also verified that trying to impose
335 a compensation condition valid around the midday light peak could be inefficient resulting
336 in low productivities.

337 We must note that when the respiration rate is very low, the strict compensation
338 condition became $\mu_L \approx 0$ implying that $I_L \approx 0$. Here, the compensation condition im-
339 plies almost full absorption of light which is rather difficult to maintain throughout the
340 day. Due to the limitation of reachability of the strict compensation condition and the
341 difficulty associated to the online determination of μ_L and R, we suggest that the strict
342 compensation condition is not a practical criterion for the design of control strategies. By
343 contrary, the strategy that we proposed of controlling the efficiency of light absorption
344 η_L is technically feasible for real implementation and provides almost optimal productiv-
345 ities. In the near future, an optimal framework of harvesting strategies will be proposed
346 complementary to the QO control.

347 **5. Conclusion**

348 To summarize, controlling the efficiency of light absorption makes it possible to attain
349 maximal productivities. The overall performance of the QO controller developed here and
350 its practical advantages for real implementation makes it a suitable control strategy for
351 optimizing microalgae production in raceways.

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357 **References**

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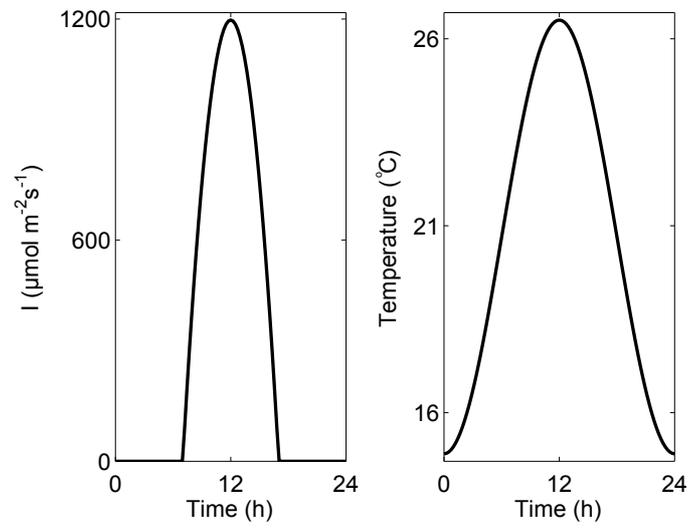


Figure 1: Typical diel variation of light intensity and temperature for the location of Narbonne in June.

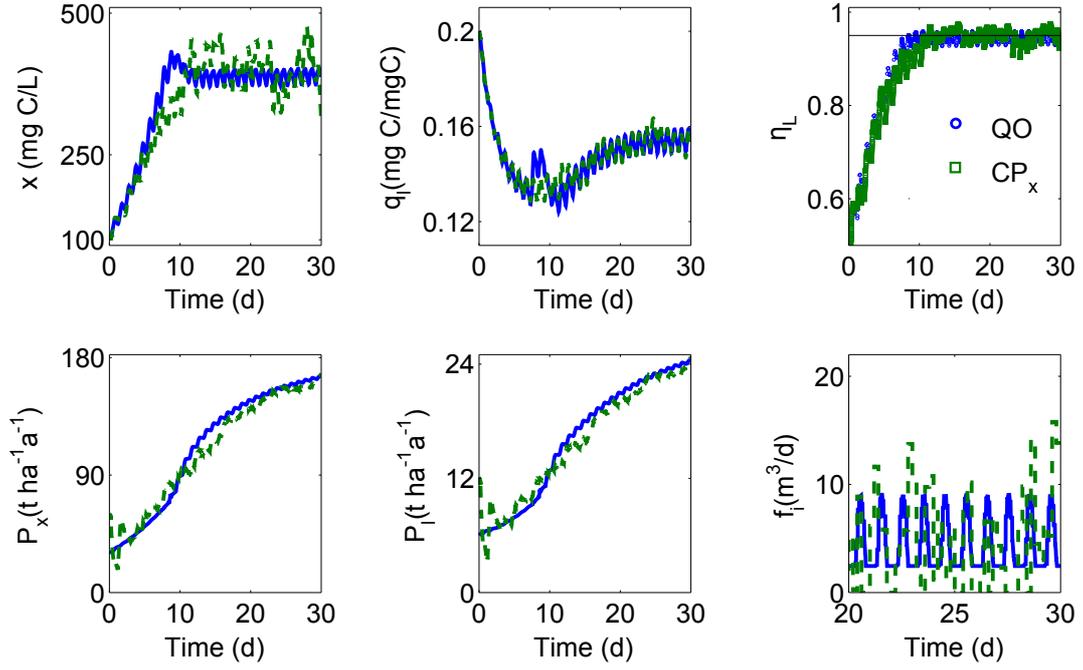


Figure 2: Trajectories of state variables and productivities given by the QO controller (solid blue line) and by the optimal controller CP_x (dashed green line). The productivities are given in dry weight basis. The biomass and lipid productivities given by the QO controller are both 98% of those provided by CP_x . The top right plot shows the evolution η_L during the light period. The horizontal continuous line corresponds to the set point $\eta_L^* = 0.95$.

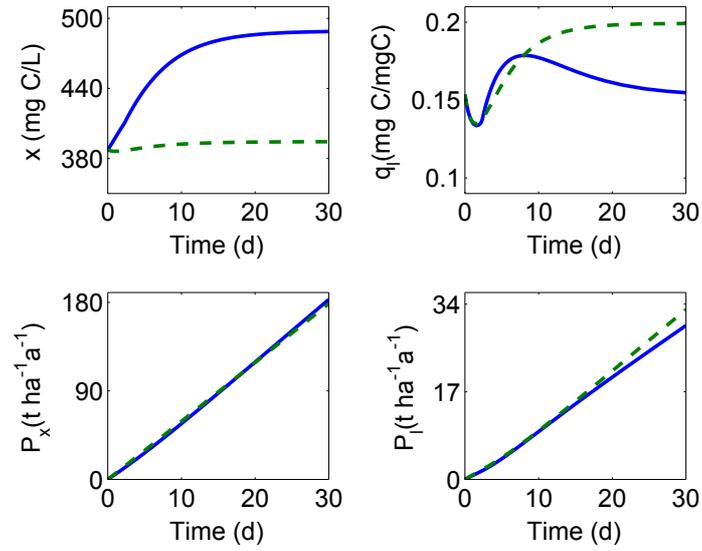


Figure 3: In a photobioreactor with constant incident light, the response of the system when applying a controller to optimize lipid productivity (dashed green line) differs of the response provided by a controller optimizing biomass productivity (solid blue line). Maximizing biomass productivity favors high biomass concentration, while maximizing lipid productivity favors high lipid quota.

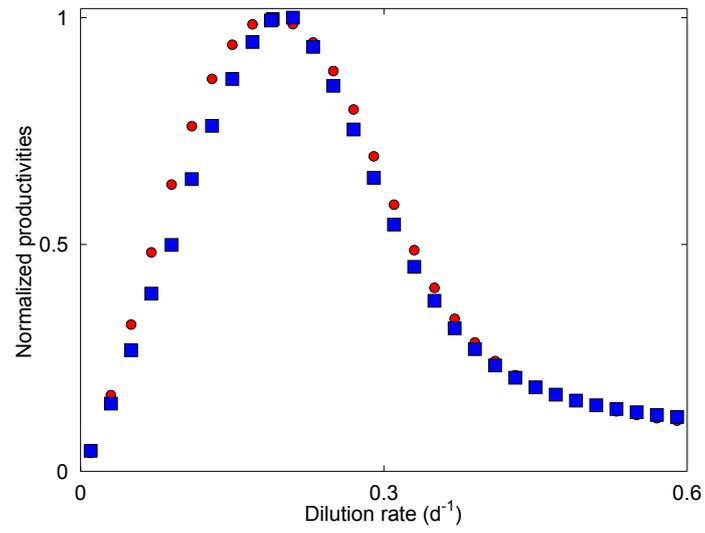


Figure 4: Normalized productivities at different dilution rates. In a raceway reactor, the biomass productivity (circles) and the lipid productivity (squares) reach their maximal value at almost the same dilution rate.

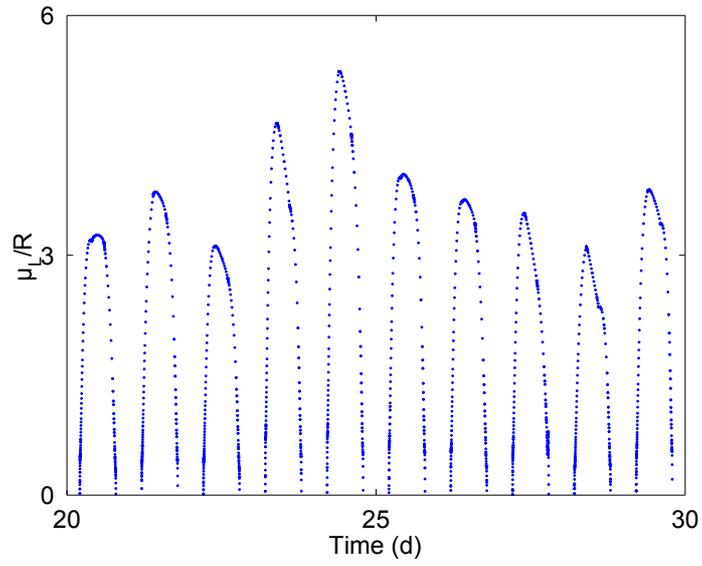


Figure 5: The strict compensation condition ($\mu_L/R=1$) is not attained along the day. Response obtained for the optimal controller CP_c .

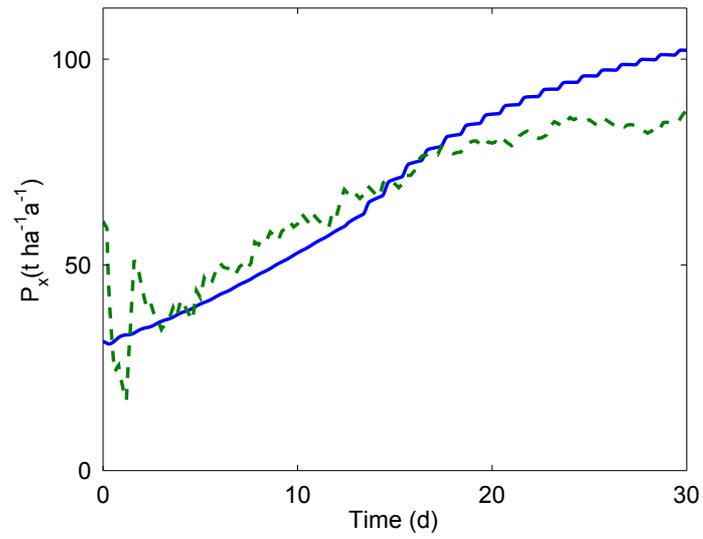


Figure 6: Productivities provided by the QO controller (solid blue line) and the optimal controller CP_x (dashed green line) under parameter uncertainty. The value of $\tilde{\mu}$ was decreased 30%.

Table 1: Model notation and parameter values.

	Definition	Units	Value
Variables			
s	Nitrogen concentration	g N m^{-3}	
q_l	Neutral lipid quota	g C (g C)^{-1}	
q_n	Nitrogen quota	g N (g C)^{-1}	
x	Carbon biomass concentration	g C m^{-3}	
x_f	Functional carbon concentration	g C m^{-3}	
x_g	Carbohydrates carbon concentration	g C m^{-3}	
x_l	Lipid carbon concentration	g C m^{-3}	
Chl	Chlorophyll concentration	g Chl m^{-3}	
G_c	Compensation light intensity	$\mu\text{mol photons m}^{-2}\text{s}^{-1}$	
I_0	Incident light intensity	$\mu\text{mol photons m}^{-2}\text{s}^{-1}$	
\bar{I}	Average light intensity	$\mu\text{mol photons m}^{-2}\text{s}^{-1}$	
I_L	Light intensity at the bottom of the raceway	$\mu\text{mol photons m}^{-2}\text{s}^{-1}$	
I_z	Light intensity at depth z	$\mu\text{mol photons m}^{-2}\text{s}^{-1}$	
T	Raceway temperature	$^{\circ}\text{C}$	
Functions and parameters			
χ	Working illuminated fraction		
η_L	Efficiency of light absorption		
ϕ_T	Temperature factor affecting growth kinetics		
λ	Optical depth		
μ	Growth rate	d^{-1}	
$\bar{\mu}$	Average growth rate	d^{-1}	
ρ	Nitrogen uptake rate	g N (g C d)^{-1}	
θ_N	Chl:N ratio	g Chl (g N)^{-1}	
ξ	Attenuation factor	m^{-1}	
D	Dilution rate	d^{-1}	
f_i	Feeding flow rate	$\text{m}^3 \text{d}^{-1}$	
f_o	Effluent flow rate	$\text{m}^3 \text{d}^{-1}$	
f_r	Rain flow rate	$\text{m}^3 \text{d}^{-1}$	
f_v	Evaporation flow rate	$\text{m}^3 \text{d}^{-1}$	
R	Overall respiration rate	d^{-1}	

α	Protein synthesis coefficient	g C (g N)^{-1}	3.0
β	Fatty acid synthesis coefficient	g C (g N)^{-1}	3.80
ϵ_I	Dissociation light constant.	$\mu\text{mol photons m}^{-2}\text{s}^{-1}$	50
φ	Biosynthesis cost coefficient	g C (g N)^{-1}	1.30
γ	Fatty acid mobilization coefficient	g C (g N)^{-1}	2.90
ν	Reduction factor of nitrogen uptake during night		0.19
$\tilde{\mu}$	Theoretical maximum specific growth rate	d^{-1}	2.11
$\bar{\rho}$	Maximum uptake rate	g N (g C d)^{-1}	0.10
a	Light attenuation due to chlorophyll	$\text{m}^2(\text{g Chl})^{-1}$	2.0
b	Light attenuation due to background turbidity	m^{-1}	0.087
g_1	Coefficient Eq. (12)	g N (g Chl)^{-1}	16.74
g_2	Coefficient Eq. (12)	$\text{g N (g Chl } ^\circ\text{C)}^{-1}$	0.39
g_3	Coefficient Eq. (12)	$\text{g N (g Chl } \mu\text{mol photons m}^{-2}\text{s}^{-1})^{-1}$	0.0014
460 g_4	Coefficient Eq. (12)	$(^\circ\text{C})^{-1}$	0.0015
K_s	Nitrogen saturation constant	g N m^{-3}	0.018
K_{sI}	Light saturation constant	$\mu\text{mol photons m}^{-2}\text{s}^{-1}$	150
L	Pond depth	m	0.30
m	Hill coefficient		3.0
Q_l	Saturation cell quota	g N (g C)^{-1}	0.20
Q_0	Minimal nitrogen cell quota	g N (g C)^{-1}	0.05
r_0	Maintenance respiration rate	d^{-1}	0.01
s_{in}	Influent nitrogen concentration	g N m^{-3}	50
S	Pond surface	m^2	57
T_{\min}	Lower temperature for microalgae growth	$^\circ\text{C}$	-0.20
T_{\max}	Upper temperature for microalgae growth	$^\circ\text{C}$	33.30
T_{opt}	Temperature at which growth rate is maximal	$^\circ\text{C}$	26.70
V	Raceway volume	m^3	17.10

Table 2: Productivity performance of open loop (OL) configuration and closed loop controller. The resulting performance is presented relative to the optimal strategy.

	$100P_x/P_x^*$	$100P_l/P_l^*$
QO	98%	98%
CP _c	100%	100%
OL: $f_i^* = 3.22 \text{ m}^3 \text{ d}^{-1}$	93%	91%
OL: $f_i = 5.13 \text{ m}^3 \text{ d}^{-1}$	54%	59%

* stands for the productivities obtained by the optimal controllers CP_x and CP_l. ★ optimal constant flow rate.